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Candidate instructions

ECON4510 - Finance Theory

This is some important information about the written exam in ECON4510. Please read this carefully before you start answering the exam.

Date of exam: Monday, May 13, 2019
Time for exam: 09.00 - 12.00 (3 hours)
The problem set: The problem set consists of 3 problems. They will be given weight as indicated.

Some advice: Start by reading through the whole exam, and make sure that you allocate time to answering problemss you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve. It is better to try to do something on each problem than to get bogged down with one problem. If you find you are spending too much time on one problem, stop working on it and plan to get back to it if you have time at the end. Make sure you state any assumptions you make.

Sketches: You may use sketches on all questions. You are to use the sketching sheets handed to you. You can use more than one sketching sheet per question. See instructions for filling out sketching sheets at the bottom. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each problem under the problem text. You will NOT be given extra time to fill out the "general information" on the sketching.

Access: You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

Resources allowed: No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).

Grading: The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Grades are given: Monday, June 3, 2019

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Problem 1 - 35%

Assume that all investors have linear-quadratic preferences and suppose they can invest their wealth in a set of risky assets. Moreover, the returns on the assets are imperfectly correlated and the expected return and variance differs across assets.

(A) [5%] Suppose there is no risk free asset. Illustrate, using a diagram, the set of portfolios that investors will choose in equilibrium.

(B) [5%] Explain how the optimal portfolios change when a risk free asset is introduced

(C) [10%] Suppose a risky asset i has an expected return equal to the risk free rate $(E(r_i) = r_f)$ and the same standard deviation as the return on the market portfolio.

i. Explain why all individuals want to hold this asset in equilibrium even though it offers a low equilibrium return.

ii. What must the covariance be between the rate of return on the asset, r_i, and the return on the market portfolio in equilibrium?

(D) [15%] Empirical testing of CAPM:

i. Explain how CAPM can be tested empirically.

ii. Review some empirical evidence that CAPM should be rejected.

iii. How could a die-hard CAPM believer hold on to CAPM despite the evidence above?

Fill in your answer here and/or on sketching paper

(C) i. All of the individuals will want to hold a portfolio made out of only 2 assets, the risk free asset and the market portfolio. Since the market portfolio is comprised of all assets in the market (as long as the expected return of the asset is not smaller than the expected return of the risk free asset) all individuals will hold the risky asset i through the market portfolio.

ii. We use the CAPM equation : $E\left(ri\right)-rf=\beta\left(E\left(rm\right)-rf\right)$
Since the risky asset has the same return as the risk free asset the left hand side of the equation equals 0.
We then have: $\beta\left(E\left(rm\right)-rf\right)=0$
Since in equilibrium the return of the market portfolio cannot be the same as the return of the risk free asset the only way that the equation holds is if $\beta=0$.
We find that in equilibrium $\beta=0$ and since $\beta\ =\frac{Cov(ri,rm)}{\sigma_{rm}}$ the covariance between the risky asset and the market portfolio is equal to 0.

(D) i.The CAPM can be tested using the ß of all the portfolios. The CAPM model verifies the equation $E\left(ri\right)=rf+$ ß $\left(E\left(rm\right)-rf\right)$ where ri is the return of portfolio i and rm the return of the market portfolio. Using empirical data we can evaluate the portfolios, if some of them do not verify the equation there could be an arbitrage opportunity.

ii. CAPM should be rejected considering that it is very hard to find the true market portfolio. To replace/reject the model we could use the factor model where we can create 0 investment portfolios using price/book ratio, small/big companies or low/high beta. For example we would buy stocks with low price/book ratio and short stocks with high price/book ratio.Same for the other variables, we can create another 0 investment where we go long on small cap stocks and short on big cap. These 0 investment portfolios have empirically an excess return so they are a source of arbitrage and a good argument to reject the CAPM.

iii. He could argue that theoretically the CAMP holds but empirically it is very hard to find the true market portfolio and thus it is still very useful in the theoretical case but not empirically.

Besvart.

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

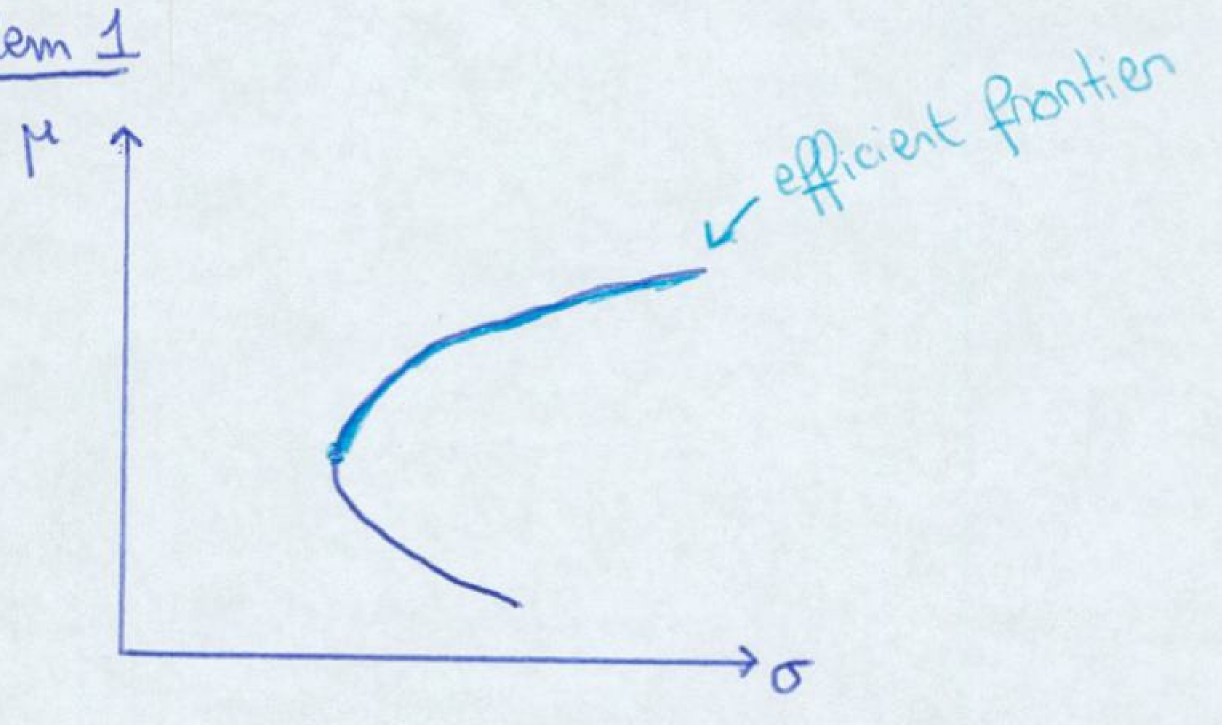
2 4 3 2 1 6 4

Oppgavekode / Question code	Dato / Date	Emnekode / Subject code	Kandidatnummer / Candidate number	Oppgavenummer / Question number	Sidetall / Page number
2432164	13/05/19	ECON4510	17773	~~[illegible]~~ 1	1

Tegneområde Drawing area

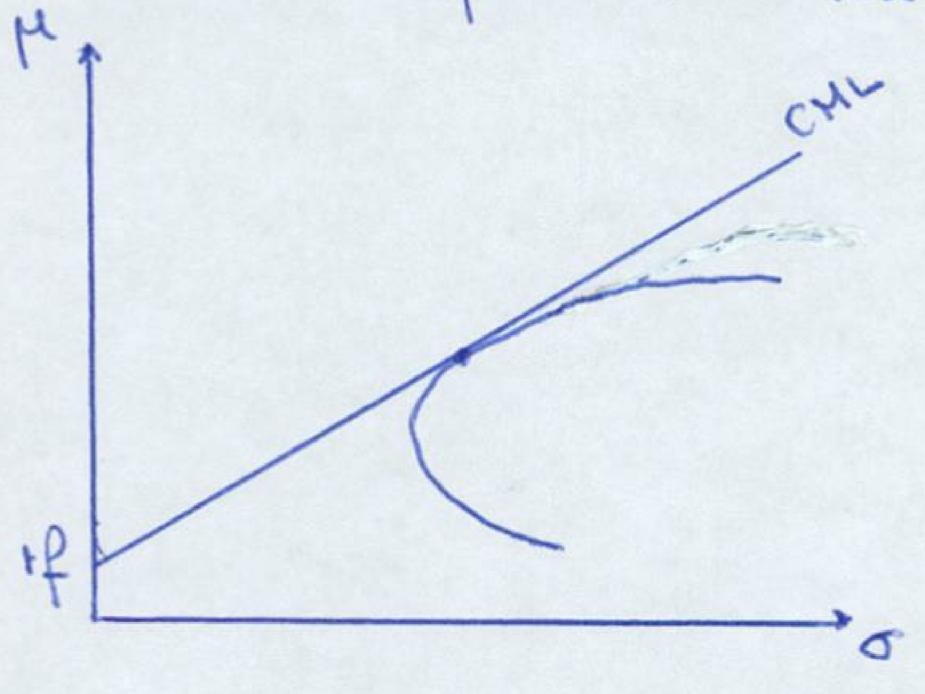
Problem 1

(A)



(We assume possibility of short selling)

(B) When a risk free asset is introduced individuals will hold a portfolio containing only 2 assets: the risk free asset and the market portfolio (tangency ~~portf~~ point between the capital market line and the efficient frontier. The portfolios choosen by the investors will be situated on the CML.



2 Problem 2 - 45%

Consider an individual with preferences $U(c_1, c_2) = u(c_1) + \beta u(c_2)$, where the utility function $u(c)$ is constant relative risk aversion (CRRA);

$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$

where $\gamma \geq 0$. The individual has an initial wealth W and has no other income.

(A) [5%] Show that the parameter γ is the relative risk aversion for this individual.

(B) [10%] Assume that in period 1 the individual can invest in stocks and bonds, where stocks pay a risky return r_s in period 2 and bonds have a safe return r_f in period 2.

i. Write down the optimization problem for the individual.

ii. Show that the first-order conditions for this individual can be expressed as

$$1 = E\left\{\frac{\beta u'(c_2)}{u'(c_1)} \cdot (1 + r_f)\right\}$$

$$1 = E\left\{\frac{\beta u'(c_2)}{u'(c_1)} \cdot (1 + r_s)\right\}.$$

(C) [15%] Assume that c_1 and c_2 are aggregate consumption and that all households have the same CRRA utility function U above. Moreover, assume that the aggregate consumption growth, c_{t+1}/c_t, and the risky return $1 + r_s$ are jointly log-normal. Show that the equity premium can be expressed as

$E(r_s) - r_f \approx \gamma \cdot cov\left(\ln\left(\frac{c_{t+1}}{c_t}\right), r_s\right)$

(D) [7%] Give an economic interpretation of the formula for the equity premium implied by the model, i.e., the equation in (2.C) above.

(E) [5%] Mehra and Prescott (1985) documented the following annual statistics for the 1889-1980 period for USA: $E(r_s) = 7\%$, $E(r_f) = 1\%$, and the following covariance matrix:

Variance-covariance matrix. USA 1889-1978

	$1 + r_{s,t+1}$	c_{t+1}/c_t	
$1 + r_{s,t+1}$	0.0274	0.0022	
c_{t+1}/c_t		0.0013	

Explain why these empirical observations are puzzling in light of the equity premium implied by the model, i.e., the equation in (2.C) above.

(F) [3%] Assume that a new asset i is introduced and that the return on this asset is negatively correlated with consumption growth, i.e., $corr(\frac{c_{t+1}}{c_t}, r_i) < 0$. Will the expected return on this asset be lower or higher than the safe interest rate r_f? Give thorough intuition for your argument.

Fill in your answer here and/or on sketching paper

(on paper)

Besvart.

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

2851942

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2851942	13/05/19	ECON 4510	17773	2	2

Tegneområde Drawing area

Problem 2:

(A) Relative risk aversion: CRRA: $-\frac{u''(c)c}{u'(c)}$

$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ $\quad u'(c) = \frac{1-\gamma}{1-\gamma} c^{-\gamma} = c^{-\gamma}$ $\quad u''(c) = -\gamma c^{-\gamma-1}$

CRRA: $-\frac{-\gamma c^{-\gamma-1} \cdot c}{c^{-\gamma}} = -\frac{-\gamma c^{-\gamma}}{c^{-\gamma}} = -(-\gamma) = \gamma$

we have that the relative risk aversion for the individual equals the parameter γ.

(B) (i) We assume the individual invests all of the wealth that he did not consume in period 1, to consume in period 2. He invests a quantity a_s in the risky asset and a_f in the risk free asset. He maximizes his utility:

$\text{Max } E(u(c_1, c_2))$ s.t. $c_1 = w - a_s - a_f$
$c_2 = a_s(1+r_s) + a_f(1+r_f)$

$\Leftrightarrow \text{Max } E(u(c_1) + \beta(c_2))$ st $c_1 = w - a_s - a_f$
$c_2 = a_s(1+r_s) + a_f(1+r_f)$

We plug in the constraints to have a simple max problem instead of a lagrang

$\Leftrightarrow \underset{\text{wrt } a_s, a_f}{\text{Max}}\ E\left(u(w - a_s - a_f) + \beta\left(u(a_s(1+r_s) + a_f(1+r_f))\right)\right)$

(ii) FOC: $\frac{\partial E(u)}{\partial a_s} = 0 \Leftrightarrow E\left(-u'(\overbrace{w - a_s - a_f}^{c_1}) + \beta(1+r_s)u'(\overbrace{a_s(1+r_s) + a_f(1+r_f)}^{c_2})\right) = 0$

$\Leftrightarrow E(-u'(c_1)) + E(\beta(1+r_s)u'(c_2)) = 0$

$\Leftrightarrow E(\beta(1+r_s)u'(c_2)) = E(u'(c_1))$

$\Leftrightarrow \frac{E(\beta(1+r_s)u'(c_2))}{E(u'(c_1))} = 1$

$\Leftrightarrow E\left(\beta(1+r_s) \cdot \frac{u'(c_2)}{u'(c_1)}\right) = 1$

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Tegneområde Drawing area

(ii) Same for af:

$$\frac{\partial E(u)}{\partial af} = 0 \iff E\left(-u'(c_1) + \beta(1+rf)u'(c_2)\right) = 0$$
$$\iff E\left(\beta(1+rf)u'(c_2)\right) = E\left(u'(c_1)\right)$$
$$\iff E\left(\beta(1+rf)\frac{u'(c_2)}{u'(c_1)}\right) = 1$$

We find the same FOC as the ones in the question.

(b) The formula shows that the risk payoff $(E(r_s) - rf)$ is equal to the relative risk aversion of the individual times the correlation between the growth of his consumption and the rate of return of the risky asset

(c) we equalize the FOC:

$$E\left(\beta\frac{u'(c_2)}{u'(c_1)}(1+rf)\right) = E\left(\beta\frac{u'(c_2)}{u(c_1)}.(1+r_s)\right)$$

$$\iff \beta\left(E\left(\frac{u'(c_2)}{u'(c_1)}.(1+rf)\right)\right) = \beta E\left(\frac{u'(c_2)}{u'(c_1)}.(1+r_s)\right) \qquad \Rightarrow \frac{u'(c_2)}{u'(c_1)} = \left(\frac{c_2}{c_1}\right)^{-\gamma}$$

$$\iff E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}.(1+rf)\right) = E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}.(1+r_s)\right) \qquad \Rightarrow E(XY) = E(X)E(Y) + Cov(XY)$$

$$\iff E\left[\left(\frac{c_2}{c_1}\right)^{-\gamma}\right]E(1+rf) + \underbrace{Cov\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}, 1+rf\right)}_{=0} = E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}\right)E(1+r_s) + Cov\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}, 1+r_s\right)$$

$E(1+rf) = 1 + E(rf)$

$=0$ bc consumption growth is completely uncorrelated to the wealth growth through risk free asset

$$\iff E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}\right)\left[E(rf) + 1\right] = E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}\right)\left(E(r_s) + 1\right) + Cov\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}, r_s\right)$$

$$\iff E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}\right)\left[E(rf) - E(r_s)\right] = Cov\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}, r_s\right)$$

We have $E(rf) = rf$ and $\left(\frac{c_2}{c_1}\right)^{-\gamma} \approx -\gamma \ln\left(\frac{c_2}{c_1}\right)$ thus:

$$\iff E\left(\left(\frac{c_2}{c_1}\right)^{-\gamma}\right)\left(rf - E(r_s)\right) \approx Cov\left(-\gamma \ln\left(\frac{c_2}{c_1}\right), r_s\right) \qquad \text{we assume } E\left(\frac{c_2}{c_1}\right)^{-\gamma} = 1$$

$$\iff rf - E(r_s) \approx -\gamma\, Cov\left(\ln\left(\frac{c_2}{c_1}\right), r_s\right)$$

Generalising

$$\iff E(r_s) - rf \approx +\gamma . Cov\left(\ln\left(\frac{c_{t+1}}{c_t}\right), r_s\right)$$

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Tegneområde Drawing area

(E) we have $E(r_S) - r_f = 6\%$

and $\gamma \cdot Cov(\ln \frac{C_{t+1}}{C_t}, 1+r_{S,t+1}) = \gamma . 0.0022$

so $0.06 \approx \gamma . 0.0022$

$\gamma \approx \frac{6}{0.22} \approx 33 >> 1$ we have a relative risk aversion that is way higher than 1. It is puzzling because with our model and assumptions the parameter γ should have a value higher than 0 but lower than or equal 1.

(F) Since $\gamma \geq 0$ and $Corr(\frac{C_{t+1}}{C_t}, r_i) < 0$ the left hand side of our (C) equation should be lower or equal to zero:

$E(r_i) - r_f \leq 0 \iff r_f \geq E(r_i)$

The expected return on this asset will be lower than the safe interest rate r_f.

3 Problem 3 - 20%

Consider a two-period binominal model for an asset. The price of the asset is $S = 1$ in the first period.

Assume that the interest rate is zero ($r = 0$). Moreover, between period 1 and 2 the share price of the asset will either increase with 10% (which happens with probability 0.6) or fall with 10% (which happens with probability 0.4). The asset does not pay any dividends. Consider a call option with a strike price $K = 1$ in the second period.

(A) [1%] When (i.e., in which states) should the option be exercised?

(B) [9%] Calculate the replicating portfolio. Explain why the replicating portfolio (for a call option) in the binominal model will always involve leveraged positions, i.e., debt ($B \leq 0$).

(C) [5%] Calculate the value of the call option in the first period.

(D) [5%] Why is the probability of a price increase irrelevant for the price of the option?

Fill in your answer here and/or on sketching paper

(on paper)

Besvart.

Knytte håndtegninger til denne oppgaven?

Bruk følgende kode:

4 9 9 5 8 5 9

Oppgavekode / Question code	Dato / Date	Emnekode / Subject code	Kandidatnummer / Candidate number	Oppgavenummer / Question number	Sidetall / Page number
4995859	13/05/19	ECON4510	17773	3	5

Tegneområde Drawing area

Problem 3:

(A) Options should be exercised at the strike date because they are worth more alive than dead (assuming no dividend payoff)

(B) We ~~can~~ replicate the option call using a portfolio including the underlying asset in quantity Δ and ~~the~~ a bond B.

We have:

$t=1$ $\quad$ $t=2$

$S_1 = 1$ → $uS = 1.1$; $dS = 0.9$

C → $uC = 1.1 - 1 = 0.1$; $dC = 0$

replicating portfolio in $t=2$:

$$\begin{cases} uS\Delta + e^r B = uC \\ d\Delta S + e^r B = dC \end{cases}$$

Calculating the replicating portfolio:

$$\begin{cases} uS\Delta + e^r B = uC \\ d\Delta S + e^r B = dC \end{cases} \iff \begin{cases} 1.1\Delta + 1.B = 0.1 \\ 0.9\Delta + B = 0 \end{cases}$$

$L1 - L2 \Rightarrow 0.2\Delta = 0.1$

$\Delta = 1/2 = 0.5$

$0.9 \times 0.5 + B = 0 \iff B = -0.45$

We have a replicating portfolio that holds 0.5 units of the underlying stock and -0.45 units of the bond.

Replicating a call option will always involve leverage because an option is much more risky than its underlying asset so to construct the same position with two less risky assets we need to short sell on the less risky (bonds) ~~and~~ to buy more of the other one (underlying stock) and create more risk.

(C) Call price in $t=1$:

$C_1 = \Delta . S + B \iff C_1 = 0.5 . 1 - 0.45 = 0.05$

The call option is worth 0.05 in period 1.

(D) The probability of a price increase is inrelevant because as shown in class it can be expressed using the other variables in the model so we never use it to calculate it the price of the option (it is already "priced in")