

# Comparing the welfare of growing economies

Geir B. Asheim\*

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## Abstract

Economies that currently have the same productive capacity may implement different growth rates. This entails that it is insufficient to base international comparisons of welfare solely on current well-being, or introducing the potential for future growth in an arbitrary manner. NNP-based measures trade off current well-being and the potential for future growth in a consistent manner. This paper shows that it matters for NNP-based measures whether different growth rates in different economies are due to different technological opportunities or different social preferences for development.

**Keywords and Phrases:** National accounting, Growth, Dynamic welfare.

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*Address:* Department of Economics, University of Oslo, P.O. Box 1095 Blindern, NO-0317 Oslo, Norway. *Tel:* +47 22855498 *Fax:* +47 22855035 *Email:* g.b.asheim@econ.uio.no

# 1 Introduction

International comparisons of welfare are needed for laying a normative foundations for transfers between different economies. Such comparisons may also be useful for international negotiations on trade, debt relief and climate control, since agreements on such issues result in implicit transfers.

In order to perform international comparisons of welfare, one needs information on some notion of “per capita welfare” in economies that differ in many respects, including having different growth rates. This paper discusses how to use NNP-based measures for such a purpose.

Economies that currently have the same productive capacity may implement different growth rates. This entails that it is insufficient to base international comparisons of welfare solely on current well-being, or introducing the potential for future growth in an arbitrary manner. An example in point is UNDP’s Human Development Index (HDI). This index mixes measures of current well-being (consumption, health, education) with the potential for future development and growth (cf. Dasgupta, 2001, C1–C2). In particular, the only forward-looking component of the HDI is the gross investment part of GDP, which makes no allowance for capital depreciation and resource depletion.

Fleurbaey and Gaulier (2009) present a careful welfare-economic analysis of how do correct per capita GDP for labor, risk of unemployment, health, household demography, inequalities and sustainability. However, they limit their analysis to the hypothetical case of a population permanently exposed to the current conditions, thereby abstracting from the problem being addressed here.

The present paper contributes to the literature on welfare comparisons based on national accounting aggregates, in the tradition of Weitzman (1976). This literature has primarily been concerned with developing and applying the theory of national accounting to the question of making over-time welfare comparison *within* economies (see, e.g., Aronsson and Löfgren, 1993; Arrow et al., 2003a,b; Asheim, 2004; Asheim and Weitzman, 2001; Dasgupta and Mäler, 2000; Kemp and Long, 1982; Pezzey, 2004; Sefton and Weale, 2006). The problems addressed include how to make accounting comprehensive by allowing for environmental degradation and natural resource depletion as well as technological progress and population growth. Measures include growth in comprehensive NNP and a positive *genuine savings indicator* (a term coined by Hamilton, 1994, p. 166) measuring the value of changes in capital stocks. By calculating the genuine savings indicator for different countries, one can compare to what extent they take

care of their own descendants. However, this is not welfare comparisons between different countries, an issue that has been scarcely treated in this line of literature (with Weitzman, 2001, and Asheim, 2009, being two exceptions).

The fact that the literature on national accounting aggregates has been concerned with over-time comparisons within economies, has naturally lead to a focus on the effects of changing productive capacity — as a result of investments, depletion and technological progress — while social preferences are assumed to remain unchanged through time. Usually, social preferences are taken to be of a discounted utilitarian type, where the economy maximizes the sum of discounted utilities, with both the discount rate and the instantaneous utility function being time-invariant.

On this background it is not surprising that — when the theory developed in this line of literature is applied to international comparison — it allows for different productive capacity and potential for growth, but assumes that different economies have identical social preferences. Indeed, this is what both Weitzman (2001) and Asheim (2009) do (see also Dasgupta and Mäler, 2000, Proposition 6).

In the present paper I allow for the possibility that different economies have different social preferences. I show that it matters for NNP-based measures whether different growth rates in different economies are due to different technological opportunities or different social preferences for development.

I make my analysis as simple as possible, using a one-sector Ramsey-type growth model (to be presented in Section 2) as the vehicle for my arguments. In Section 3 I reproduce results showing that a corrected measure of NNP can be used for international comparisons in the case where different growth rates are due to different technological constraints. In Section 4 I show that this result is no longer applicable in the case where different growth rates are caused by different social preferences for development. In Section 5 I offer some comments on the significance of these observations.

## 2 Model

Consider a world with different economies, indexed  $i = a, b$ , etc., each with population equal to 1. The analysis can easily be generalized to the case where different economies have populations of different size. Below I will comment on additional assumptions needed to interpret the variables and results on a per capita basis.

Denote by  $c_i(t)$  the non-negative *consumption* in economy  $i$  at time  $t$ . ‘Consumption’ should be interpreted as a scalar that indicates instantaneous well-being. No specific view on what constitutes instantaneous well-being is therefore implied by the subsequent analysis. Multiple consumption good models would allow for variation in preferences over consumption bundles, including differences in labor/leisure trade-offs. The simple one consumption good framework chosen here abstracts from such variation in preferences between economies. It also means that international trade is not an essential part of the model, and that the interesting problem of making international comparisons of instantaneous well-being (e.g., by means of consumer price indices) is not addressed.

At each time  $t$ , net national product in terms of consumption (*consumption-NNP*)  $y_i(t)$  in economy  $i$  depends on the stock of *capital*  $k_i(t)$ :

$$y_i(t) = f_i(k_i(t)).$$

The *production function*  $f_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is twice continuously differentiable, with  $f_i(0) = 0$ ,  $f'_i(k) > 0$  and  $f''_i(k) < 0$  for all  $k > 0$ , and  $\lim_{k \rightarrow 0} f'_i(k) = \infty$  and  $\lim_{k \rightarrow \infty} f'_i(k) = 0$ . ‘Capital’ should be interpreted as a scalar that indicates the stocks of reproducible and natural capital available to the economy, while ‘consumption-NNP’ should be interpreted as a scalar that indicates its productive capacity.

A consumption path  $\{c(s)\}_{s=t}^{\infty}$  is feasible in economy  $i$  at time  $t$  if there exists a capital path  $\{k(s)\}_{s=t}^{\infty}$  satisfying  $k(t) = k_i(t)$  and, at each  $s \geq t$ ,  $f_i(k(s)) = c(s) + \dot{k}(s)$ . In the case where two economies,  $a$  and  $b$ , have the same technology ( $f_a(\cdot) = f_b(\cdot) = f(\cdot)$ ), the assumption that capital is one-dimensional means that sets of feasible consumption paths are nested: If  $k_a(t) \leq k_b(t)$ , and thus,  $f(k_a(t)) \leq f(k_b(t))$ , then the set of feasible consumption paths in economy  $a$  is a subset of the the set of feasible consumption paths in economy  $b$ . This property does not carry over to multiple capital good models, where the sets of a feasible consumption paths need not be nested according to their consumption-NNP.

Economy  $i$  is assumed to implement a discounted utilitarian optimum. This means

that, at each  $t$ ,  $\{c_i(s)\}_{s=t}^{\infty}$  maximizes

$$\int_0^{\infty} u_i(c(s))e^{-\rho_i(s-t)} ds$$

over all feasible consumption paths, where the *utility discount rate*  $\rho_i$  is positive and the instantaneous *utility function*  $u_i : \mathbb{R}_{++} \rightarrow \mathbb{R}$  is twice continuously differentiable, with  $u'_i(c) > 0$  and  $u''_i(c) < 0$  for all  $c > 0$ , and  $\lim_{c \rightarrow 0} u'_i(c) = \infty$  and  $\lim_{c \rightarrow \infty} u'_i(c) = 0$ .

The analysis assumes that economies allocate their resources perfectly according to their discounted utilitarian objective. Hence, the interesting issue of performing welfare comparisons in economies with imperfect resource allocation mechanisms (see Arrow et al., 2003b) is not addressed here. Furthermore, it assumes that economies do not take into account that high growth rates leading to increased future welfare may result in obligations for assisting economies that experience lower growth rates.

Under the assumption that  $k_i(0) > 0$ , standard analysis yields that  $\{c_i(s)\}_{s=0}^{\infty}$  and  $\{k_i(s)\}_{s=0}^{\infty}$  are interior paths satisfying, at each  $t$ ,

$$u'_i(c_i(t)) = Q_i(t), \tag{1}$$

$$f'_i(k_i(t)) = \rho - \frac{\dot{Q}_i(t)}{Q_i(t)}, \tag{2}$$

where  $Q_i(t)$  is the current value price of capital at time  $t$  in terms of utility. Refer to  $u_i(c_i(t)) + Q_i(t)\dot{k}_i(t)$  as the *utility-NNP* in economy  $i$  at time  $t$ .

This sets a simplified stage for asking the following question: What is the right concept for doing welfare comparisons between different economies? Should we use consumption (or, equivalently, utility, since the  $u$ -function is increasing)? Or should we use consumption-NNP or utility-NNP?

### 3 Different technological opportunities

Consider first the case where two economies,  $a$  and  $b$ , have the same social preferences, in the sense that  $u_a(\cdot) = u_b(\cdot) = u(\cdot)$  and  $\rho_a = \rho_b = \rho$ . Moreover, assume that the two economies have currently (at time 0) the same productive capacity, but different potential for growth. In particular, assume that  $f_a(k) = k^\alpha$  and  $f_b(k) = k^\beta$ , with  $\rho = \alpha < \beta$  and  $k_a(0) = k_b(0) = 1$ . This implies that

$$\begin{aligned} f_a(k_a(0)) &= 1 = f_b(k_b(0)), \\ \rho = \alpha &= f'_a(k_a(0)) < f'_b(k_b(0)) = \beta. \end{aligned}$$

It now follows from (1) and (2), combined with a transversality condition, that

$$\begin{aligned} c_a(0) = f_a(k_a(0)) = 1 \quad \text{and} \quad \dot{Q}_a(0) = 0, \\ c_b(0) < f_b(k_b(0)) = 1 \quad \text{and} \quad \dot{Q}_b(0) < 0. \end{aligned}$$

Since the rate of net productivity  $f'_a(k_a(0)) = \alpha$  in economy  $a$  equals the rate of utility discount  $\rho$ , the optimal path in economy  $a$  is to consume its consumption-NNP and keep capital constant. Hence, consumption and consumption-NNP will be kept equal to 1 at all times. The situation is different in economy  $b$  where the rate of net productivity  $f'_b(k_b(0)) = \beta$  exceeds the rate of utility discount  $\rho$ . Here, the optimal path entails that only a fraction of its consumption-NNP is consumed, while the rest is used for capital accumulation. This leads to consumption and consumption-NNP growing from, respectively,  $c_b(0) < 1$  and  $f_b(k_b(0)) = 1$  towards  $(\beta/\rho)^{\beta/(1-\beta)} > 1$ .

Which of these economies has higher welfare? First, note that the two economies are assumed to have identical social preferences, implying that they rank consumption paths in the same manner. Second, note that the optimal path in economy  $a$ , where consumption is kept constant at  $c^a(t) = 1$  is feasible in economy  $b$ , while the optimal path in economy  $b$ , where consumption grows from  $c^b(t) < 1$  initially to  $c^b(t) > 1$  eventually, is not feasible in economy  $a$  due to its lower rate of net productivity. It follows directly from these facts that economy  $a$  has lower welfare than economy  $b$ .

Two important conclusions follow from these observations.

- (i) They are sufficient to rule out the use of instantaneous well-being, either in the form of consumption or utility, as a correct welfare index. The reason is that

$$c_a(0) > c_b(0) \quad \text{and} \quad u(c_a(0)) > u(c_b(0));$$

hence, instantaneous well-being is higher in the economy with lower welfare.

- (ii) Consumption-NNP is not a correct welfare index, either. The reason is that

$$f_a(k_a(0)) = f_b(k_b(0));$$

hence, consumption-NNP does not reflect that welfare is higher in economy  $b$ .

How then should we do welfare comparisons between the two economies? Weitzman's (1976) seminal analysis implies that, for  $i = a, b$ ,

$$u(c_i(0)) + Q_i(0)\dot{k}_i(0) = \rho \int_0^\infty u(c_i(s))e^{-\rho s} ds.$$

Hence, utility-NNP, being the current value Hamiltonian for the economies' dynamic resource allocation problem, provides a correct measurement of welfare in the case where the two economies have identical social preferences.

It follows from (1) and the concavity of  $u$  that  $(c_i(0), \dot{k}_i(0))$  maximizes  $u(c(0)) + Q_i(0)\dot{k}(0)$  over all pairs  $(c(0), \dot{k}(0))$  satisfying  $c(0) + \dot{k}(0) \leq f_i(k_i(0))$ . Hence, if the optimal net investment at time 0,  $\dot{k}_i(0)$ , does not equal zero, then the strict concavity of  $u$  implies that utility-NNP and thus the welfare of economy  $i$  exceed the welfare associated with holding consumption constant at  $f_i(k_i(0))$ .

Above, we have considered an economy  $b$  which due to its high potential for growth (since  $\beta > \rho$ ) has positive optimal net investment at time 0 and therefore is better off than an economy  $a$  which has zero optimal net investment at time 0 (since  $\alpha = \rho$ ). In particular, with  $\beta > \rho = \alpha$ , economy  $b$  has a greater potential for turning current consumption into capital for a given gain of future productive capacity (as  $k^\alpha < k^\beta$  if  $\alpha < \beta$  and  $k > 1$ ).

However, the same conclusion would be obtained if economy  $b$  had a particularly low potential for growth (i.e.,  $\beta < \rho$ ), and thus negative optimal net investment at time 0. It would be still be better off than economy  $a$  given the shared discounted utilitarian social preferences. The reason that with  $\beta < \rho = \alpha$ , economy  $b$  would have a greater potential for turning capital into current consumption for a given loss of future productive capacity (as  $k^\alpha < k^\beta$  if  $\alpha > \beta$  and  $0 < k < 1$ ).

The results reported in this section beg the following question: How can utility  $u(c_i(0))$  be measured from observable quantities and prices, and other information that might be available? In fact, Arrow, Dasgupta and Mäler have in a series of paper (Dasgupta and Mäler, 2000; Dasgupta, 2001; Arrow et al., 2003b) argued against the use of utility-NNP on the grounds that is not linear in prices and quantities and therefore difficult to measure. In particular,  $u(c_i(0)) + Q_i(0)\dot{k}_i(0)$  need not equal  $Q_i(0)(c_i(0) + \dot{k}_i(0)) = Q_i(0)f_i(k_i(0)) = Q_i(0)y_i(0)$  due to the concavity of  $u$ . Weitzman (2001) has contributed to a solution of this problem by pointing out that  $u(c_i(0)) - Q_i(0)c_i(0)$  is a consumer surplus term which in principle is measurable.

In the case where economies  $a$  and  $b$  have different, but time-invariant, population sizes, the results of the present section can be given a per capita interpretation without further assumptions. Asheim (2009, Section 4) discusses the significance of the Weitzman's (2001) consumer surplus term in a context where economies have different population sizes.

## 4 Different social preferences for development

Consider next the case where two economies,  $a$  and  $b$ , have the same productive capacity and the same potential for growth, in the sense that  $k_a(0) = k_b(0) = k(0)$  and  $f_a(\cdot) = f_b(\cdot) = f(\cdot)$ . Assume also that economies  $a$  and  $b$  have the same instantaneous utility function:  $u_a(\cdot) = u_b(\cdot) = u(\cdot)$ . However, assume now that they have different social preferences in the sense that  $f'(k(0)) = \rho_a > \rho_b > 0$ . Hence,

$$\begin{aligned} f_a(k_a(0)) &= f(k(0)) = f_b(k_b(0)), \\ \rho_a = f'_a(k_a(0)) &= f'(k(0)) = f'_b(k_b(0)) > \rho_b. \end{aligned}$$

It now follows from (1) and (2), combined with a transversality condition, that

$$\begin{aligned} c_a(0) = f_a(k_a(0)) &= f(k(0)) \quad \text{and} \quad \dot{Q}_a(0) = 0, \\ c_b(0) < f_b(k_b(0)) &= f(k(0)) \quad \text{and} \quad \dot{Q}_b(0) < 0. \end{aligned}$$

Since the rate of net productivity  $f'_a(k_a(0)) = f'(k(0))$  in economy  $a$  equals its rate of utility discount  $\rho_a$ , the optimal path in economy  $a$  is to consume its consumption-NNP and keep capital constant. Hence, consumption and consumption-NNP will be kept equal to  $f(k(0))$  at all times. The situation is different in economy  $b$  where the rate of net productivity  $f'_b(k_b(0)) = f'(k(0))$  exceeds its rate of utility discount  $\rho_b$ . Here, the optimal path entails that only a fraction of its consumption-NNP is consumed, while the rest is used for capital accumulation. This leads to consumption and consumption-NNP growing from, respectively,  $c_b(0) < f(k(0))$  and  $f_b(k_b(0)) = f(k(0))$  towards some common level,  $f(k(\infty))$ , exceeding  $f(k(0))$ , where  $k(\infty)$  is determined by  $f'(k(\infty)) = \rho_b$ .

In this case, which economy has higher welfare? Since the two economies are assumed to have the same productive capacity and the same potential for growth, they have identical sets of feasible consumption paths. Moreover, since in each economy the optimal path is unique, economy  $a$  according to its social preferences strictly prefers  $\{c_a(s)\}_{s=0}^{\infty}$  to  $\{c_b(s)\}_{s=0}^{\infty}$ , while economy  $b$  has the opposite ranking.

One position is to conclude that the two economies are equally well off, given that their sets of feasible consumption paths are identical. This conclusion is reached by basing the welfare comparisons on consumption-NNP, as both economies' consumption-NNP at time 0 equals  $f(k(0))$ .

Building on this position, one can conclude as follows:

- (i) Instantaneous well-being, either in the form of consumption or utility, is not a correct welfare index, even in the present case where different growth rates are



caused by different social preferences for development. The reason is that

$$c_a(0) > c_b(0) \quad \text{and} \quad u(c_a(0)) > u(c_b(0));$$

hence, instantaneous well-being is higher in the economy with lower growth.

(ii) Utility-NNP is not a correct welfare index, either. The reason is that

$$a\text{'s utility-NNP} = u(c_a(0)) < u(c_b(0)) + Q_b(0)\dot{k}_b(0) = b\text{'s utility-NNP}$$

since  $(c_b(0), \dot{k}_b(0))$  uniquely maximizes  $u(c(0)) + Q_b(0)\dot{k}(0)$  over all pairs  $(c(0), \dot{k}(0))$  satisfying  $c(0) + \dot{k}(0) \leq f(k(0)) = c_a(0)$ . Hence, utility-NNP does not reflect that the two economies have the same set of feasible consumption paths.

In the case where economies  $a$  and  $b$  have different, but time-invariant, population sizes, the results of the present section can be given a per capita interpretation provided that  $f(k) := F(k, 1)$ , where  $F : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$  is a constant-returns-to-scale production function of total capital,  $K$ , and total labor,  $L$  (= population), and where  $k := K/L$  is the per capita capital stock. Under this assumption on  $F$ , per capita production possibilities will be independent of scale.

## 5 Concluding remarks

On the basis of the simplified analysis of the present paper, it follows that instantaneous well-being cannot be used for international comparisons of welfare between economies with different growth rates. Provided that one accepts the assumption that the compared economies maximize welfare, this conclusion holds independently of what specific conception on instantaneous well-being one adopts.

Furthermore, the analysis illustrates how utility-NNP as proposed by Weitzman (1976) is applicable if the different growth rates are due to different technological opportunities and not different social preferences for development.

Finally, it suggests that consumption-NNP yields a more reasonable conclusion if the situation is the opposite; when the different growth rates are due to different social preferences for development rather than different technological opportunities.

The analysis has only considered the polar cases, where either social preferences are identical, so that different sets of feasible consumption paths can be evaluated by their most preferred elements, or technological constraints are identical, so that different sets of feasible consumption paths are nested and can be evaluated by set inclusion.

The hybrid situation, where both technological constraints and social preferences are different, is of course more complicated and not covered by the present paper.

Real-world measurement of national product has many deficiencies. Per capita consumption, as measured, may for various reasons not correspond to instantaneous well-being. Moreover, the measurement of national product may not incorporate adequate allowance for capital depreciation and resource depletion, and may not account for technological progress.

It follows from the theory of national accounting in the tradition of Weitzman (1976) that utility-NNP, not consumption-NNP, has welfare significance when making welfare comparisons between different economies. As shown by Weitzman (2001), this means that national product has to be subject also to a consumer surplus adjustment. The analysis of the present paper suggests that this latter adjustment may not be appropriate if different growth rates are due to different social preferences rather than different technological opportunities.

It is an empirical question whether variation in growth rates between different economies are caused by the weight they place on future prospects rather than their ability to turn current savings into increased future productivity. If the process of globalization has disseminated technological knowledge without eliminating cultural differences, then the analysis of this paper suggests that a properly adjusted consumption-NNP may serve as a better indicator for welfare than the theory of national accounting has lead us to believe.

When moving beyond the simple framework in which the present analysis has been performed, it is, however, an open problem to investigate how to compare the welfare of growing economies in the case where divergence in growth rates is due to differences in social preferences for development. In particular, with multiple capital goods, it does not hold that set of feasible consumption paths in different economies are nested according to the value of their consumption-NNP. The complications are magnified within a multiple consumption-good setting where economies also differ in their preferences over consumption bundles, and within a setting where economies also differ in their technological constraints.

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