Global welfare comparisons

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Abstract

To provide a normative foundation for transfers between different economies, one needs information on their "per capita welfare". This paper considers various methods for doing this and reaches the following main conclusions: (i) Such global welfare comparisons are more demanding than usually thought. (ii) The ranking of methods differs from that of local (e.g., over-time) comparisons, with real comprehensive per capita NNP being the least impractical method. The lesson is that global welfare comparisons should be performed with great care. The comparisons must be made in local real prices calculated according to "purchasing-power-parity", where non-traded environmental amenities play an important role.

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1 Introduction

Transfers between different economies are sometimes motivated by redistribution: the worse-off economies should receive transfers from the better-off economies. Such redistribution may occur between regions, states or provinces of a single country, between different countries of, e.g., the European Union, or between the rich and poor parts of the world. Also international negotiations on trade, debt relief and climate control are concerned about the implicit transfers that various forms of agreements will result in.

To provide a normative foundation for transfers between different economies, one needs information on the "per capita welfare" in economies that differ in many respects, including having different population sizes and technological constraints. This paper discusses how to do such *global welfare comparisons* by reviewing various methods based on the theory of national accounting in the tradition of Weitzman (1976).[1]

There is a dichotomy in the literature on welfare comparisons based on national accounting aggregates.

One line of literature is *not* primarily concerned with global welfare comparisons, but has developed and applied the theory of national accounting to the question of making over-time welfare comparison *within* economies (see, e.g., Aronsson and Löfgren, 1993; Arrow et al., 2003a,b; Asheim, 2004; Asheim and Weitzman, 2001; Dasgupta and Mäler, 2000; Kemp and Long, 1982; Pezzey, 2004; Sefton and Weale, 2006). The problems addressed include how to make accounting comprehensive by allowing for environmental degradation and natural resource depletion as well as technological progress and population growth. Measures include growth in comprehensive NNP and a positive *genuine savings indicator* (a term coined by Hamilton, 1994, p. 166) measuring the value of changes in capital stocks. Welfare improvement indicated by such measures has been associated with the concept of sustainable development, so that positive genuine savings mean that the current generation is managing its assets in a sustainable manner. By calculating the genuine savings indicator for different countries, one can compare to what extent they take care of their own descendants. However, this is not welfare comparisons between different countries, an issue that has been scarcely treated in this line of literature (with Weitzman, 2001, being an important exception).

On the other hand, several indices for global welfare comparisons have been sug-

[1] The term *global* is here used to signify that comparisons are made between economies that are not only marginally different. In contrast, a welfare comparison would be *local* if, e.g., we compare the welfare in an economy at one instance with the welfare of the *same* economy at the next instance.

gested, where national accounting aggregates enter as one component. Examples are UNDP's Human Development Index (HDI) and Osberg and Sharpe's (2002) Index of Economic Well-being. There are several problems with such indices. One is that the weights assigned to the different components in the indices lack welfare-economic foundation. Another is that the manner in which they mix measures of current well-being (consumption, health, education, security) with the potential for future development and growth (cf. Dasgupta, 2001, C1–C2). In particular, the only forward-looking component of the HDI is the gross investment part of GDP, which makes no allowance for capital depreciation and resource depletion. Fleurbaey and Gaulier (2009) present a more careful welfare-economic analysis of how do correct per capita GDP for labor, risk of unemployment, health, household demography, inequalities and sustainability. However, this line of literature appears not to discuss the welfare-economic basis for the suggestion that corrected per capita GDP can be used for global comparisons; e.g., Fleurbaey and Gaulier (2009) limit their analysis to the hypothetical case of a population permanently exposed to the current conditions.

In the present paper I show how the theory of national accounting can be used for global welfare comparisons, under given assumptions. I consider methods considered in the literature on over-time welfare comparisons and ask: Should we use

(a) real comprehensive per capita NNP,

(b) real comprehensive per capita wealth, or

(c) an integral of the value of comprehensive changes in per capita stocks?

Method (a) is a per capita variant of Weitzman's (1976) stationary welfare equivalent of future utility, while method (c) is an adaptation of the genuine savings indicator – the main indicator for doing local over-time welfare comparisons – to global welfare comparisons (cf. Dasgupta and Mäler, 2000, Proposition 6). Method (b) is included, as it has been suggested for valuing the relative well-being of different economies.

The main conclusions (Propositions 1–3) of the present paper are: (i) Global welfare comparisons are more demanding than usually thought, due to the fact that economies will differ in population size and technological constraints. (ii) Judging from the sets of assumptions that are sufficient to obtain positive results, the ranking of methods differs from that of local over-time comparisons, with method (a) – real comprehensive per capita NNP – being the least impractical method. Method (a) (the NNP measure) is based on the assumption that economies adhere to the same *discounted* *utilitarian* welfare function, method (b) (the wealth measure) entails that the assumption of *constant-returns-to-scale* is added, while method (c) (the savings measure) can be applied if, in addition, the two economies have the *same technological constraints*.

To show how the various methods invoke these strong assumptions, it is essential to choose a basic analytical framework that does not rely on them. Thereby, the analysis relating to the three measures conveys the role of discounted utilitarianism, constant-returns-to-scale, and identical technological constraints. For the basic framework, I assume that economies have the same instantaneous utility function over the vector of consumption flows. Moreover, I assume that economies have the same social preferences over paths of such flows. Finally, I assume that economies allocate resources competitively and compatible with a weak optimality property (suggested by Asheim and Buchholz, 2004, cf. their Property 2). Implementation of a discounted utilitarian optimum (given an identical discounted utilitarian welfare function in all economies) is a special case that satisfies these assumptions, but is not implied by them. The basic analytical framework imposes non-increasing (but not necessarily constant) returns-to-scale, and allow technological constraints to vary between economies.

A preview of the analysis underlying the main results, Propositions 1–3, provides some insights into why doing global welfare comparisons between economies that differ in population size and technological constraints is more demanding than doing local over-time welfare comparisons within a given economy with a constant population.

To use NNP for local over-time comparisons, it is sufficient to establish that NNP growth indicates welfare improvement. This requires neither measurement of utility (Asheim and Weitzman, 2001) nor an assumption of discounted utilitarianism (Asheim and Buchholz, 2004). To use NNP for *global* comparisons, one must establish that per capita NNP is positively related to per capita welfare. This is done in two steps: (i) Showing that per capita utility NNP (= utility plus value of investments) is positively related to per capita welfare. (ii) Arriving at an expression for per capita consumption that measures per capita utility. For step (i), discounted utilitarianism is sufficient (Weitzman, 1976) as it makes per capita utility NNP proportional to per capita welfare, while my basic assumptions (that economies allocate resources competitively and compatible with a weak optimality property) are not (Asheim and Buchholz, 2004, Proposition 3). For step (ii), the techniques of Weitzman (2001) are developed further.

To use savings for local over-time comparisons, it is sufficient to establish that positive genuine savings indicate welfare improvement. As for NNP growth, this requires neither measurement of utility nor an assumption of discounted utilitarianism. To use the value of changes in stocks for *global* comparisons, these changes in stocks must capture all technological differences between the economies, entailing that both economies must have same technology as a function of the vector of these stocks. In contrast, NNP does not depend on stocks that do not change over time. To use the value of changes in *per capita* stocks for global comparison of *per capita* welfare, the economy must exhibit constant-returns-to-scale. In contrast, per capita NNP does not depend on per capita stocks, only per capita investment flows. Finally, to integrate the value of changes in stocks, the value of changes in stocks must be in a fixed proportion to welfare changes, a requirement that is satisfied by discounted utilitarianism.

The wealth measure is not very useful for local over-time comparisons, as it requires constant-returns-to-scale to measure the present value of utilities in supporting utility discount factors, which in turn is a welfare measure under discounted utilitarianism, but not necessarily otherwise. The same assumptions of discounted utilitarianism and constant-returns-to-scale remain sufficient also for global welfare comparisons between economies that differ in population size and technological constraints.

My analysis extends the literature. As already mentioned, it generalizes the NNP measure of Weitzman (1976, 2001) to the case where the compared economies have different population sizes. In relation to the savings measure of Dasgupta and Mäler (2000, Proposition 6) and Arrow et al. (2003b, Theorem 3) it makes more explicit the assumptions that these results rely on and compare them to the assumptions invoked by the other measures. Compared to the taxonomy of assumptions and results presented in Asheim (2003, cf. the bottom line of Table 1), it drops the linear homogeneity of the utility function and the stationarity of the technology as prerequisites, and allows for the possibility that economies vary in population sizes and technological constraints.

It is a major restriction that I do not allow for and discuss differences in utility functions and social preferences between economies. I also abstract from income inequalities within economies, and differences in leisure (due to working hours or unemployment) between economies. Some of these issues are addressed in an insightful manner by Fleurbaey and Gaulier (2009). To the extent my analysis points to problems of doing global welfare comparisons, such abstractions make my results *stronger*: without them, the task of doing global welfare comparisons is even harder.

After presenting the general model in Section 2, the basic assumptions made on the functioning of the resource allocation mechanism are presented in Section 3. An extension to the case different population sizes of Weitzman's (2001) method for measuring utility is presented in Section 4. This enables a variant of Weitzman's (1976) result on the NNP measure as the stationary equivalent utility to be established in Section 5. The viability of the wealth and savings measures is considered in Sections 6 and 7. An example presented in Section 8 illustrates the analysis, indicating that welfare comparisons between different economies must be made in local real prices calculated according to "purchasing-power-parity", where non-traded environmental amenities may play an important role, rather than in international prices calculated according to exchange rates. Concluding remarks are included in Section 9.

2 Model

Consider a world divided into different economies. I assume that, for any economy, population is constant over time.[2] Use N to denote population, and let N^i represent the constant population of economy i, where $i = a$, b, etc.

Denote by $\mathbf{C} = (C_1, \ldots, C_m)$ the non-negative vector of goods that are consumed in a given economy. To concentrate on the issue of distribution between different economies, I assume that goods and services consumed at any time are distributed equally among the population at that time. Thereby the instantaneous well-being for each individual may be associated with the *utility* $u(\mathbf{c})$ that is derived from the per capita vector of consumption flows, $\mathbf{c} := \mathbf{C}/N$.[3] Assume that u is a time-invariant, increasing, concave, and differentiable function, which is identical for all economies. That u is time-invariant means that all variable determinants of current well-being are included in the vector of consumption flows, implying that an individual's instantaneous well-being is increased by moving from $\mathbf{c}'$ to $\mathbf{c}''$ if and only if $u(\mathbf{c}') < u(\mathbf{c}'')$. Labor supply is assumed to be constant and equal to population size.

Denote by $\mathbf{K} = (K_1, \ldots, K_n)$ the non-negative vector of *capital* goods. This vector includes not only the usual kinds of man-made capital stocks, but also stocks of natural resources, environmental assets, human capital (like education and knowledge capital accumulated from R&D-like activities), and other durable productive assets, in the spirit of so-called "green" or comprehensive accounting. Corresponding to the stock of

[2]Under the assumption of constant-returns-to-scale, the case of exponential population growth can easily be accommodated, provided that only per capita consumption matters. Contributions where population growth need not be exponential and where instantaneous well-being also depends on population size have appeared (cf., e.g., Arrow et al., 2003a; Asheim, 2004), but only for the purpose of doing local over-time comparisons within the same economy.

[3]This does not necessarily rule out (impure) collective goods. It is sufficient that per capita utility as a function of $\mathbf{C}$ and N, $\tilde{u}(\mathbf{C}, N)$, is homogenous of degree 0.

capital of type j, K_j, there is a net *investment* flow: $I_j := \dot{K}_j$. Hence, $\mathbf{I} = (I_1, \ldots, I_n) = \dot{\mathbf{K}}$ denotes the vector of net investments.

The quadruple $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N)$ is *attainable* in economy i at time t if $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N) \in \mathcal{C}^i(t)$, where $\mathcal{C}^i(t)$ is a convex and smooth set, with free disposal of consumption and net investment flows. The set $\mathcal{C}^i(t)$ describes economy i's technological constraints at time t. If i is an open economy in a competitive world economy, $\mathcal{C}^i(t)$ will also depend on the international prices that economy i faces.

The set of attainable quadruples, $\mathcal{C}^i(t)$, is allowed to depend directly on time. This reflects that the technological level and terms-of-trade may change over time. To make accounting comprehensive, the value of the passage of time will be added to the value of consumption and investments, so that formally all variable determinants of current productive capacity are included.[4]

The set of attainable quadruples, $\mathcal{C}^i(t)$, is also allowed to depend on the economy i. E.g., climate may influence the consumption and investment opportunities over and beyond the effect of the vector of capital stocks, $\mathbf{K}$, and time, t. If $\mathcal{C}^i(t)$ is a cone at each time t, then the technology exhibits constant-returns-to-scale. The assumption of constant-returns-to-scale will be imposed *only* in Sections 6 and 7.

Each economy i, with constant population N^i, makes decisions according to a *resource allocation mechanism* that assigns to any vector of capital stocks $\mathbf{K}$ and time t, a consumption-investment pair $(\mathbf{C}(\mathbf{K}, t; i), \mathbf{I}(\mathbf{K}, t; i))$ satisfying that $(\mathbf{C}(\mathbf{K}, t; i), \mathbf{I}(\mathbf{K}, t; i), \mathbf{K}, N^i)$ is attainable at time t.[5] I assume that there exists a unique solution $\{\mathbf{K}^i(t)\}_{t=0}^{\infty}$ to the differential equations $\dot{\mathbf{K}}^i(t) = \mathbf{I}(\mathbf{K}^i(t), t; i)$ that satisfies the initial condition $\mathbf{K}^i(0) = \mathbf{K}_0^i$, where $\mathbf{K}_0^i$ is given. Hence, $\{\mathbf{K}^i(t)\}$ is the capital path that the resource allocation mechanism implements in economy i. Write $\mathbf{C}^i(t) := \mathbf{C}(\mathbf{K}^i(t), t; i)$ and $\mathbf{I}^i(t) := \mathbf{I}(\mathbf{K}^i(t), t; i)$.

The program $\{\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t)\}_{t=0}^{\infty}$ is *competitive* if, at each t,

1. $(\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t), N^i)$ is attainable,

2. there exist present value prices of the flows of utility, consumption, labor input, and investment, $(\mu^i(t), \mathbf{p}^i(t), w^i(t), \mathbf{q}^i(t))$, with $\mu^i(t) > 0$ and $\mathbf{q}^i(t) \geq 0$, such that

    C1 $\mathbf{C}^i(t)$ maximizes $\mu^i(t) u(\mathbf{C}/N^i) - \mathbf{p}^i(t)\mathbf{C}/N^i$ over all $\mathbf{C}^i$,

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[4]Such accounting for the passage off time was introduced by Kemp and Long (1982) and has been applied by, e.g., Aronsson and Löfgren (1993), Vellinga and Withagen (1996) and Pezzey (2004).

[5]This is inspired by Dasgupta and Mäler (2000), Dasgupta (2001), and Arrow et al. (2003b).

C2 $(\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t), N^i)$ maximizes $\mathbf{p}^i(t)\mathbf{C} - w^i(t)N + \mathbf{q}^i(t)\mathbf{I} + \dot{\mathbf{q}}^i(t)\mathbf{K}$ over all $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N) \in \mathcal{C}^i(t)$.

Here C1 corresponds to instantaneous utility maximization, while C2 corresponds to instantaneous profit maximization.[6]

The term "present value" reflects that discounting is taken care of by the prices. In particular, if relative consumption prices and the real interest rate, R, are constant, then it holds that $\mathbf{p}^i(t) = \mathbf{p}^i(0) \cdot e^{-Rt}$. I will *not* assume that the relative consumption prices and the real interest rate are constant. The present value price of utility at time t, $\mu^i(t)$, is a supporting utility discount factor. When I assume in Sections 5–7 that the economy adheres to discounted utilitarianism and allocates its resources optimally, it follows that $\mu^i(t) = \mu^i(0) \cdot e^{-\rho t}$, where ρ is the utility discount rate.

3 Assumptions on the resource allocation mechanism

I make two basic assumptions on the functioning of the resource allocation mechanism. These assumptions are implied by, but do not imply, the set of assumptions invoked for the purpose establishing Propositions 1–3.

First, I assume that the implemented program in economy i, $\{\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t)\}_{t=0}^{\infty}$, is competitive with finite utility and consumption values,

$$\int_0^\infty \mu^i(t) N^i u(\mathbf{C}^i(t)/N^i) dt \ \text{ and } \int_0^\infty \mathbf{p}^i(t)\mathbf{C}^i(t) dt \text{ exist},$$

and that it satisfies a capital value transversality condition,

$$\lim_{t\to\infty} \mathbf{q}^i(t)\mathbf{K}^i(t) = 0\,. \tag{1}$$

It follows that the implemented program $\{\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t)\}_{t=0}^{\infty}$ maximizes

$$\int_0^\infty \mu^i(t) u(\mathbf{C}/N^i) dt$$

over all programs that are attainable at all times and satisfies the initial condition. Moreover, writing $\mathbf{c}^i(t) := \mathbf{C}^i(t)/N^i$, it follows from C1 and C2 that

$$\mathbf{p}^i(t) = \mu^i(t)\nabla u(\mathbf{c}^i(t))\,, \tag{2}$$

$$w^i(t) = \mathbf{p}^i(t)\frac{\partial \mathbf{C}(\mathbf{K}^i(t), N^i, t; i)}{\partial N} + \mathbf{q}^i(t)\frac{\partial \mathbf{I}(\mathbf{K}^i(t), N^i, t; i)}{\partial N}\,, \tag{3}$$

$$-\dot{\mathbf{q}}^i(t) = \mathbf{p}^i(t)\nabla_{\mathbf{K}}\mathbf{C}(\mathbf{K}^i(t), N^i, t; i) + \mathbf{q}^i(t)\nabla_{\mathbf{K}}\mathbf{I}(\mathbf{K}^i(t), N^i, t; i)\,, \tag{4}$$

[6] To see that $\mathbf{p}^i(t)\mathbf{C} - w^i(t)N + \mathbf{q}^i(t)\mathbf{I} + \dot{\mathbf{q}}^i(t)\mathbf{K}$ is instantaneous profit, note that $\mathbf{p}^i(t)\mathbf{C} + \mathbf{q}^i(t)\mathbf{I}$ is the value of production, $w^i(t)N$ is the cost of labor and $-\dot{\mathbf{q}}^i(t)\mathbf{K}$ is the cost of holding capital.

where ∇ denotes a vector of partial derivatives.

Denote by $\psi^i(t)$ the value of the passage of time measured in present value terms. Since $\psi^i(t)$ is measured in present value terms, the decrease of the value of the passage of time, $-\dot{\psi}^i(t)$, equals the marginal productivity of the passage of time:

$$-\dot{\psi}^i(t) = \mathbf{p}^i(t)\frac{\partial \mathbf{C}(\mathbf{K}^i(t), N^i, t; i)}{\partial t} + \mathbf{q}^i(t)\frac{\partial \mathbf{I}(\mathbf{K}^i(t), N^i, t; i)}{\partial t}\,. \tag{5}$$

For doing the welfare comparisons between economies, it will turn out to be a helpful intermediate result to value

$$\int_t^\infty \mu^i(s)\dot{u}^i(s)ds\,, \tag{6}$$

where, for all s, $u^i(s) = u(\mathbf{c}^i(s))$. By combining (2), (4), and (5), one obtains

$$\begin{aligned}\mu^i N^i \dot{u}^i &= \mu^i N^i \nabla u \cdot d(\mathbf{C}^i/N^i)/dt = \mu^i \nabla u \cdot \left(\nabla_{\mathbf{K}}\mathbf{C}\cdot \mathbf{I}^i + \tfrac{\partial \mathbf{C}}{\partial t}\right) \\ &= -\left(\dot{\mathbf{q}}^i\mathbf{I}^i + \mathbf{q}^i\dot{\mathbf{I}}^i + \dot{\psi}^i\right) = -\tfrac{d}{dt}\left(\mathbf{q}^i\mathbf{I}^i + \psi^i\right).\end{aligned} \tag{7}$$

Assuming that

$$\lim_{t\to\infty}\left(\mathbf{q}^i(t)\mathbf{I}^i(t) + \psi^i(t)\right) = 0$$

holds as an investment value value transversality condition, one arrives at the following result by integrating (7).

Lemma 1 *The present value of future changes in per capita utility is given by:*

$$\int_t^\infty \mu^i(s)\dot{u}^i(s)ds = (\mathbf{q}^i(t)\mathbf{I}^i(t) + \psi^i(t))/N^i\,.$$

Second, I let all economies have identical welfare judgements described by time-invariant complete and transitive social preferences on the set of per capita utility paths. Let $V^i(\mathbf{K}^i(t), N^i, t)$ be an index of per capita *dynamic welfare* in economy i at time t, given the path of per capita consumption flows that the resource allocation mechanism implements from time t. Even though preferences are identical, V^i depends on economy i since the technological constraints and the resource allocation mechanism depend on economy i. Following Asheim and Buchholz (2004, Property 2), assume that the resource allocation mechanism has a weak optimality property: at each time t, the resource allocation mechanism and the accompanying welfare index satisfy that welfare improvement is maximized subject to $(\mathbf{C}, \mathbf{I}, \mathbf{K}^i(t), N^i)$ being attainable and per capita utility being at least $u(\mathbf{C}^i(t)/N^i)$. This can be stated by the following property, where $\rho^i(\mathbf{K}^i(t), N^i, t)$ is the Lagrangian multiplier on the lower bound for per capita utility:

$$\begin{gathered}(\mathbf{C}^i(t), \mathbf{I}^i(t)) \text{ maximizes } \rho^i(\mathbf{K}^i(t), N^i, t)u(\mathbf{C}/N^i) + \nabla_{\mathbf{K}}V^i(\mathbf{K}^i(t), N^i, t)\mathbf{I} \\ \text{over all } (\mathbf{C}, \mathbf{I}, \mathbf{K}^i(t), N^i) \in \mathcal{C}^i(t)\,.\end{gathered} \tag{8}$$

Since u is concave and $\mathcal{C}^i(t)$ is convex and smooth, there is a unique n-dimensional hyperplane that supports the set of feasible $(n \times 1)$-dimensional utility-investment vectors. By comparing the competitiveness conditions, C1 and C2, with the no-waste-of-welfare-improvement property (8), the following conclusion is obtained.

Lemma 2 *The vector of partial derivatives of the welfare index, V^i, is given by:*

$$\nabla_{\mathbf{K}} V^i(\mathbf{K}^i(t), N^i, t) = \rho^i(\mathbf{K}^i(t), N^i, t)\frac{\mathbf{q}^i(t)}{\mu^i(t)} \cdot \frac{1}{N^i}\,.$$

The competitiveness conditions, C1 and C2, and the no-waste-of-welfare-improvement property (8) are satisfied in the special case where (i) economy i's resource allocation mechanism implements an optimal program and (ii) its per capita dynamic welfare is given by discounted utilitarianism, so that economy ranks programs according to the sum of per capita utilities discounted at a constant rate ρ. Hence, the implemented path of per capita consumption flows $\{\mathbf{c}^i(s)\}_{s=0}^{\infty}$ maximizes at each time t

$$\rho \int_t^{\infty} e^{-\rho(s-t)} u(\mathbf{c}(s)) ds\,.$$

over all feasible paths $\{\mathbf{c}(s)\}_{s=t}^{\infty}$ at time t. In this case, $\rho^i(\mathbf{K}^i(t), N^i, t) = \rho$ at each t (cf. Asheim and Buchholz, 2004, Section III), and $\{\mu^i(t)\}_{t=0}^{\infty} = \{\mu^i(0) \cdot e^{-\rho t}\}_{t=0}^{\infty}$, where $1/\mu^i(0)$ is the marginal utility of expenditures at time 0 measured in present value prices. Hence, the special case of implemented discounted utilitarianism insures that differences in per capita dynamic welfare can be measured by means of prices in terms of utility, since $\rho^i(\mathbf{K}^i(t), N^i, t)$ does not vary with i, $\mathbf{K}^i(t)$, N^i or t, but equals ρ.

4 Measuring utility

All methods discussed in this paper requires that current observable market behavior can be used to determine prices in terms of utility. Weitzman (2001) shows how, in principle, such measurement can be made by considering a consumer surplus term. The current section establishes how his analysis can be extended to the case where different economies have different population sizes, and provides a novel interpretation of the consumer surplus term in this context.

Let the consumption vector $\mathbf{c}^0$ satisfy $u(\mathbf{c}^0) = 0$. If the ranking of paths according to dynamic welfare is invariant to affine transformations of u, this may be considered just a normalization. In particular, if c is a scalar and there is constant relative inequality aversion, then $c^0 = 1$ by the choosing $u(c) = \left(c^{1-\eta} - 1\right)/(1-\eta)$ if $\eta \neq 1$ and $u(c) = \ln c$ if $\eta = 1$, as this normalization ensures $u(1) = 0$ (and $u'(1) = 1$).

However, with different population sizes, $\mathbf{c}^0$ may be given substantive significance: Let $\mathbf{c}$ be the economy's existing vector of per capita consumption flow. Suppose an additional individual were brought into the economy and offered $\mathbf{c}$. Endow $\mathbf{c}^0$ with the significance that this would increase the economy's total instantaneous well-being if and only if $\mathbf{c}$ is preferred to $\mathbf{c}^0$ (i.e., $u(\mathbf{c}) > u(\mathbf{c}^0) = 0$). I will appeal to this interpretation below, even though the assumption that population is fixed within each economy will be maintained throughout. Indeed, if population size were endogenous, then economies might choose to spread a given total amount of consumption on a larger population, invalidating the use of per capita dynamic welfare as a basis for distributional policies.

By using Weitzman's (2001, p. 15) "benchmark invariant ideal market-basket price index" and normalizing to 1 the marginal utility of expenditures in terms of current value prices, integration by parts along any path between $\mathbf{c}^0$ and $\mathbf{c}^i$ leads to the following expression for economy i's instantaneous per capita utility:[7]

$$u(\mathbf{c}^i) = u(\mathbf{c}^i) - u(\mathbf{c}^0) = \int_{\mathbf{c}^0}^{\mathbf{c}^i} \nabla u(\mathbf{c}) d\mathbf{c} = \mathbf{P}^i\mathbf{c}^i - \mathbf{P}^0\mathbf{c}^0 - \int_{\mathbf{c}^0}^{\mathbf{c}^i} \mathbf{c} d\nabla u(\mathbf{c})\,. \tag{9}$$

Here, $\mathbf{P}^i = \nabla u(\mathbf{c}^i)$ and $\mathbf{P}^0 = \nabla u(\mathbf{c}^0)$ are current value consumption prices, and the integral $\int_{\mathbf{c}^0}^{\mathbf{c}^i} \mathbf{c} d\nabla u(\mathbf{c})$ is a consumer surplus term. It follows from (9) that the difference between the instantaneous per capita utility in economies a and b is given by

$$u(\mathbf{c}^b) - u(\mathbf{c}^a) = \mathbf{P}^b\mathbf{c}^b - \mathbf{P}^a\mathbf{c}^a - \int_{\mathbf{c}^a}^{\mathbf{c}^b} \mathbf{c} d\nabla u(\mathbf{c})\,.$$

In the present case where the marginal utility of expenditures is normalized to 1, this corresponds to the result established by Weitzman (2001, (A11)).

It remains to interpret the terms $-\mathbf{P}^0\mathbf{c}^0 - \int_{\mathbf{c}^0}^{\mathbf{c}^i} \mathbf{c} d\nabla u(\mathbf{c})$ with which the per capita value of consumption $\mathbf{P}^i\mathbf{c}^i$ must be adjusted. To do so, note that

$$U(\mathbf{C}^i, N^i) := N^i u(\mathbf{C}^i/N^i)$$

is homogeneous of degree 1, with $\nabla_{\mathbf{C}} U(\mathbf{C}^i, N^i) = \nabla u(\mathbf{c}^i) = \mathbf{P}^i$ and

$$\frac{\partial U(\mathbf{C}^i, N^i)}{\partial N} = u(\mathbf{c}^i) - \nabla u(\mathbf{c}^i)\mathbf{c}^i = u(\mathbf{c}^i) - \mathbf{P}^i\mathbf{c}^i = -\mathbf{P}^0\mathbf{c}^0 - \int_{\mathbf{c}^0}^{\mathbf{c}^i} \mathbf{c} d\nabla u(\mathbf{c}) \tag{10}$$

[7] See Li and Löfgren (2002) for a similar expression that does not involve the $\mathbf{c}^0$ term. See also Li and Löfgren (2006). Moreover, Li and Löfgren (2007) suggest an interesting alternative way of making welfare comparison within a single economy. However, they do not treat comparisons between different economies with different characteristics, which is the main focus of the present paper.

by (9). Hence, given the significance suggested for $\mathbf{c}^0$ above, we obtain the following result by combining (9) and (10):

Lemma 3 *Economy i's instantaneous per capita utility is given by*

$$u(\mathbf{c}^i) = \mathbf{P}^i\mathbf{c}^i + P_N^i \,,$$

where $\mathbf{P}^i = \nabla u(\mathbf{c}^i)$ *and*

$$P_N^i := -\mathbf{P}^0\mathbf{c}^0 - \int_{\mathbf{c}^0}^{\mathbf{c}^i} \mathbf{c} d\nabla u(\mathbf{c}) \tag{11}$$

may be interpreted as the marginal value of consumption spread, measured in terms of current utility.

If $u(\cdot)$ is homogeneous of degree 1, so that $u(\mathbf{c}^i) = \nabla u(\mathbf{c}^i)\mathbf{c}^i$, then $P_N^i = 0$. In (11) this corresponds to $\mathbf{c}^0 = 0$ (since $u(\cdot)$ is homogeneous of degree 1) and $\mathbf{c}d\nabla u(\mathbf{c}) = 0$ along any path between $\mathbf{c}^0$ and $\mathbf{c}^i$ (since each element of $\nabla u(\cdot)$ is homogeneous of degree 0). Otherwise, P_N^i will in general be non-zero, and for sure negative if $u(\mathbf{c}^i) < 0 = u(\mathbf{c}^0)$, as then an additional individual's utility would be negative, and adding him/her would reduce the per capita consumption for all other individuals.

Under the general assumptions for $u(\cdot)$ (see Section 2), it follows that

$$\mathbf{C}d\mathbf{P} + N^i dP_N = \big(\mathbf{c}d\mathbf{P} + dP_N\big) \cdot N^i = \big(\mathbf{c}d\nabla u(\mathbf{c}) - \mathbf{c}d\nabla u(\mathbf{c})\big) \cdot N^i = 0$$

along any path between $\mathbf{c}^0$ and $\mathbf{c}^i$. This means that Weitzman's (2001, p. 15) "ideal market-basket price index" is a *Divisia* consumer *price index* when P_N is included. Furthermore, its "benchmark-invariance" (Weitzman, 2001, Lemma) corresponds to $U(\cdot,\cdot)$ as a function of $\mathbf{C}$ and N being homothetic, entailing that the Divisia price index is path independent, so that real prices can be determined globally.[8] Since, by normalization, $\mathbf{P}^i = \nabla u(\mathbf{c}^i)$, it follows from (2) that real (= current value in terms of utility) and present value consumption prices are related as follows, at each t:

$$\mathbf{P}^i(t) = \nabla u(\mathbf{c}^i(t)) = \frac{\mathbf{p}^i(t)}{\mu^i(t)} \,,$$

and likewise for other prices:

$$\mathbf{Q}^i(t) = \frac{\mathbf{q}^i(t)}{\mu^i(t)} \tag{12}$$

$$\Psi^i(t) = \frac{\psi^i(t)}{\mu^i(t)} \,. \tag{13}$$

[8]Cf. Hulten (1987) for a discussion of the properties of Divisia indices, and Asheim and Weitzman (2001) and Sefton and Weale (2006) for a demonstration of the importance of a *consumer* price index.

For later use, let $p_N^i(t) = \mu^i(t) P_N^i(t)$ denote the marginal value of consumption spread in terms of present value prices.

5 Stationary welfare equivalent

Under discounted utilitarianism, $\{\mu^i(s)\}_{s=0}^{\infty} = \{\mu^i(0) \cdot e^{-\rho s}\}_{s=0}^{\infty}$ and

$$\frac{d}{dt}\Big(\int_t^\infty e^{-\rho(s-t)} u^i(s) ds\Big) = -u^i(t) + \rho \int_t^\infty e^{-\rho(s-t)} u^i(s) ds = \int_t^\infty e^{-\rho(s-t)} \dot{u}^i(s) ds \,,$$

where the second equality follows by integrating by parts. Then, by Lemma 1:

$$\rho \int_t^\infty e^{-\rho(s-t)} u^i(s) ds = u^i(t) + \frac{1}{\mu^i(t)} \big(\mathbf{q}^i(t)\mathbf{I}^i(t) + \psi^i(t)\big)/N^i \,. \tag{14}$$

This is Weitzman's (1976) seminal result in the current setting, showing that utility NNP (the r.h.s. of eq. (14)) is a stationary per capita welfare equivalent of future utility. Furthermore, by invoking Lemma 3 and applying (12)–(13), the results of Weitzman (1976, 2001) are generalized to the case where different economies have different population sizes:

Proposition 1 *Under discounted utilitarianism, real comprehensive per capita NNP in money terms, including the marginal value of consumption spread, $P_N^i(t)$, and the per capita value of the passage of time, $\Psi^i(t)/N^i$,*

$$\big(\mathbf{P}^i(t)\mathbf{C}^i(t) + P_N^i(t)N^i + \mathbf{Q}^i(t)\mathbf{I}^i(t) + \Psi^i(t)\big)/N^i \,,$$

is the stationary per capita welfare equivalent of future utility.

Consider now the case with two economies, a and b. Economies a and b may, at any time t, have a different set of attainable quadruples. Hence, $\mathcal{C}^a(t)$ may differ from $\mathcal{C}^b(t)$ due, e.g., to different climatic conditions. However, I maintain the assumptions made in Sections 2 and 3 and add implemented discounted utilitarianism. In particular, both economies follow a competitive program that maximizes dynamic welfare. Moreover, the utility function is identical in economies a and b, and that both economies adhere to discounted utilitarianism with the same discount rate ρ. It now follows from Proposition 1 that per capita welfare is higher in economy a than in economy b if and only if

$$\begin{aligned}&\big(\mathbf{P}^a(t)\mathbf{C}^a(t) + P_N^a(t)N^a + \mathbf{Q}^a(t)\mathbf{I}^a(t) + \Psi^a(t)\big)/N^a > \\ &\qquad\qquad \big(\mathbf{P}^b(t)\mathbf{C}^b(t) + P_N^b(t)N^b + \mathbf{Q}^b(t)\mathbf{I}^b(t) + \Psi^b(t)\big)/N^b \,.\end{aligned}$$

The assumption of discounted utilitarianism is indispensable for this result, as (14) cannot be derived otherwise.

Application of the result requires that $P_N^a(t)$ and $P_N^b(t)$—the marginal value of consumption spread—be calculated in each country, using techniques developed by Weitzman (2001). Furthermore, $\Psi^a(t)$ and $\Psi^b(t)$—the value of passage of time—must be estimated in each country by integrating (5) (cf. footnote 4).

6 Real per capita wealth

An economy's per capita wealth is usually identified with the per capita value of its current vector of capital vectors plus the capitalized per capita value of labor. Such a notion of per capita wealth has welfare significance only if the technology exhibits constant-returns-to-scale, so that all flows of future earnings can be treated as currently existing capital.[9] Hence, add the assumption that economy i's set of attainable quadruples, $\mathcal{C}^i(t)$, is a cone. Then it follows directly from C2 that, at each t,

$$\mathbf{p}^i(t)\mathbf{C}^i(t) - w^i(t)N^i + \mathbf{q}^i(t)\mathbf{I}^i(t) + \dot{\mathbf{q}}^i(t)\mathbf{K}^i(t) = 0\,,$$

or, equivalently,

$$-\frac{d\big(\mathbf{q}^i(t)\mathbf{K}^i(t)\big)}{dt} = \mathbf{p}^i(t)\mathbf{C}^i(t) - w^i(t)N^i\,.$$

This means that the present value of future consumption equals the value of capital plus the present value of future wages,

$$\int_t^\infty \mathbf{p}^i(s)\mathbf{C}^i(s)ds = \mathbf{q}^i(t)\mathbf{K}^i(t) + \int_t^\infty w^i(t)N^i ds\,, \tag{15}$$

provided that the appropriate transversality condition holds.

In order for per capita wealth to be useful for global welfare comparisons, we must, however, also include the real capitalized value of consumption spread. Thus, write:

$$Q_N^i(t) = \frac{1}{\mu^i(t)} \cdot \int_t^\infty \big(p_N^i(t) + w^i(s)\big)\,ds\,,$$

where $Q_N^i(t)$ can be interpreted as the real capitalized value of adding an additional individual to economy i at time t. It consists of two parts:

[9] An alternative explored by Heal and Kriström (2005) is to identify an economy's wealth with the present value of its future consumption. This alternative does not require constant-returns-to-scale, but begs the question how to measure the present value of the future consumption.

(i) $\int_t^\infty p_N^i(t)ds/\mu^i(t)$ is the real capitalized value of consumption spread.

(ii) $\int_t^\infty w^i(s)ds/\mu^i(t)$ is the real capitalized value of adding an additional worker.

Then it follows from Lemma 3 that (15) can be rewritten as

$$\frac{1}{\mu^i(t)} \cdot \int_t^\infty \mu^i(s)u^i(s)ds = \mathbf{Q}^i(t)\mathbf{K}^i(t)/N^i + Q_N^i(t)\,. \tag{16}$$

In the special case of discounted utilitarianism, the l.h.s. of (16) is a welfare index.[10] Thus, the following result has been established.

Proposition 2 *Under discounted utilitarianism and constant-returns-to-scale, real per capita wealth, including the real capitalized value, $Q_N^i(t)$, of adding an individual to the economy,*

$$\mathbf{Q}^i(t)\mathbf{K}^i(t)/N^i + Q_N^i(t)\,,$$

equals per capita dynamic welfare.

The assumption of constant-returns-to-scale imposes strong informational demands in the sense that it entails that, not only variable determinants, but also fixed determinants of current productive capacity are included. Since thus all flows of future earnings are treated as currently existing capital, no term involving the effects of technological progress need be explicitly taken into account. However, in the spirit of Lindahl (1933, pp. 401–402) and (Samuelson, 1961, p. 53), the result requires that the real capitalized per capita value of labor is included. In addition, as a result of the analysis of Section 4, the real capitalized value of consumption spread must be included.

Note that, when comparing the welfare of two economies by means of Proposition 2, per capita wealth must be made comparable through the application of a Divisia consumer price index. This will be illustrated by the example of Section 8.

7 Value of changes in per capita stocks

As mentioned in the introduction, the main indicator for doing local over-time welfare comparisons is the genuine savings indicator, measuring the value of changes in capital stocks. The welfare significance of this indicator was first shown (although not emphasized) by Weitzman (1976) and it has figured prominently in many contributions, see,

[10] Note that the l.h.s. of (16) is not a welfare index for any social preferences. E.g., in the case of maximin, the left-hand side is a welfare index if the path of supporting utility discount factors $\{\mu^i(s)\}_{s=0}^\infty$ is an exponentially decreasing function, but not necessary otherwise.

e.g., Arrow et al. (2003b). The result derives from the property that the prices used to value the stock changes are the marginal derivatives of the welfare index as a state valuation function. Thus, they measure the welfare effects of marginal stock changes.

A corresponding result for global comparisons between two economies, say a and b, can be established only if economies a and b have the same welfare index $V(\mathbf{K}^i(t), N^i, t)$ of per capita dynamic welfare. In turn this requires that, at each time t:

(i) Both economies have the same set of attainable quadruples: $\mathcal{C}^a(t) = \mathcal{C}^b(t)$. Therefore, add to the earlier assumptions that economy i's set of attainable quadruples at time t, $\mathcal{C}^i(t)$, does not depend on economy's i characteristics, and hence, can be written without superscript i: $\mathcal{C}(t)$. This means that, e.g., the effects of a geographically differentiated climate on the consumption and investment opportunities are captured by the vector of capital stocks, $\mathbf{K}$.

(ii) Both economies have the same resource allocation mechanism, which I will assume satisfies the properties of Section 3.

Then, by Lemma 2, the common welfare index satisfies:

$$\nabla_{\mathbf{K}} V(\mathbf{K}^i(t), N^i, t) = \rho(\mathbf{K}^i(t), N^i, t)\frac{\mathbf{q}^i(t)}{\mu^i(t)} \cdot \frac{1}{N^i} = \rho(\mathbf{K}^i(t), N^i, t)\frac{\mathbf{Q}^i(t)}{N^i}\,.$$

However, unless $\rho(\mathbf{K}^i(t), N^i, t)$ is independent of $\mathbf{K}^i(t)$ and N^i, the value of changes in stocks cannot be integrated. Therefore, assume in addition that the common resource allocation mechanism implements a discounted utilitarian optimum. Then

$$V(\mathbf{K}^i(t), N^i, t) = \rho \int_t^\infty e^{-\rho(s-t)} N^i u(\mathbf{C}^i(s)/N^i)ds\,, \tag{17}$$

with $\rho(\mathbf{K}^i(t), N^i, t) = \rho$ for all $K^i(t)$, N^i and t.

By adding the assumption that the technology satisfies constant-returns-to-scale—i.e. that the set of attainable quadruples, $\mathcal{C}(t)$, is a cone—it follows that the index of per capita dynamic welfare, $V(\mathbf{K}^i(t), N^i, t)$, is homogeneous of degree 0 in $K^i(t)$ and N^i. Then we can write $v(\mathbf{k}^i(t), t) := V(\mathbf{K}^i(t), N^i, t)$, where $\mathbf{k} := \mathbf{K}/N$ denotes the per capita vector of capital stocks, and where

$$\nabla_{\mathbf{k}} v(\mathbf{k}^i(t), t) = \nabla_{\mathbf{K}} V(\mathbf{K}^i(t), N^i, t)N^i = \rho\mathbf{Q}^i(t)\,. \tag{18}$$

Consider now again the case with two economies, a and b. By (18), the difference in per capita dynamic welfare at time t between these economies can be written as:

$$v(\mathbf{k}^b(t), t) - v(\mathbf{k}^a(t), t) = \int_{\mathbf{k}^a(t)}^{\mathbf{k}^b(t)} \nabla_{\mathbf{k}} v(\mathbf{k}^i(t), t)d\mathbf{k}^i(t) = \rho \int_{\mathbf{k}^a(t)}^{\mathbf{k}^b(t)} \mathbf{Q}^i(t)d\mathbf{k}^i(t)\,,$$

where the integral is independent of the path, $\{\mathbf{k}^i(t)\}$, between $\mathbf{k}^a(t)$ and $\mathbf{k}^b(t)$. However, the path of investment prices in utility terms, $\{\mathbf{Q}^i(t)\}$, must be calculated in utility terms along the path of imaginary intermediate economies determined by $\{\mathbf{k}^i(t)\}$.

The following result has been shown.

Proposition 3 *Under discounted utilitarianism, constant-returns-to-scale, and economy-independent technology, economy b's per capita dynamic welfare exceeds that of economy a if and only if* $\int_{\mathbf{k}^a(t)}^{\mathbf{k}^b(t)} \mathbf{Q}^i(t) d\mathbf{k}^i(t) > 0$.

Hence, across space welfare comparisons can be made by means of an integral of the value of changes in per capita stocks. However, to establish this result, I have, in addition to discounted utilitarianism and constant-returns-to-scale, also invoked the assumptions that

- the sets of attainable quadruples in the two economies in question, $\mathcal{C}^a(t)$ and $\mathcal{C}^b(t)$, coincide at each point in time, and
- investment prices in utility terms, $\{\mathbf{Q}^i(t)\}$, can be calculated in utility terms along a path of imaginary economies that lie between economies a and b.

These assumptions appear to be very strong, and they are added to those used to establish Propositions 1 and 2.

8 An example

To illustrate how real comprehensive per capita NNP can be used to compare the per capita welfare in two economies (cf. Proposition 1), it is useful to consider the following example. The example is intended to highlight the following observations:

(1) The real prices in each economy depend on the domestic consumption vector.

(2) Non-traded environmental amenities are not only important to make NNP comprehensive, but also to calculate the real prices in each economy.

(3) Alternatives to Proposition 1, involving comparison of real per capita wealth (cf. Proposition 2) or the integral of the real value of per capita stock changes (cf. Proposition 3), are even more difficult to apply for the purpose of comparing the per capita welfare of different economies.

Consider a competitive world economy consisting of two economies, a and b, where $N^a = 1$ and $N^b = 2$ are the sizes of populations that cannot migrate between the economies. There are two capital goods: First, a reproducible capital good that can be used in either economy independent of ownership, where $K^a = 0$ and $K^b = 3$ (i.e., all capital-owners live in economy b). Second, an immobile environmental amenity good that can be thought of as space, where $E^a = 1$ and $E^b = 1$. Hence, there is less space per capita in economy b, but to compensate for this economy b has the ownership to the whole stock of reproducible capital. Assume that there is zero net investment in reproducible capital, and suppose that the available stocks of the environmental amenity good cannot change over time.

Production of a freely traded material consumption good is governed by a constant-returns-to-scale production function,

$$K^{0.4}L^{0.6}\,,$$

leading to a total production of 3, where the production in economy a equals 1 and production in economy b equals 2, but where the remuneration to the factor owners implies that 0.6 is allocated to economy a and 2.4 to economy b. The investment in reproducible capital is zero in both economies, entailing that material consumption is given by $C^a = 0.6$ and $C^b = 2.4$ and per capita consumption by $c^a = 0.6$ and $c^b = 1.2$.

Environmental amenities constitute the other consumption good and equal per capita space: $e^a = 1$ and $e^b = 0.5$. In each economy, the utility function is assumed to be homogeneous of degree 1 and given by[11]

$$u(c, e) = 4 \cdot 0.6^{0.5} \cdot c^{0.5} \cdot e^{0.5}\,,$$

leading to the following real prices in the two economies (using a Divisia consumer price index, which is path independent due to the homothetic utility function),

$$\begin{aligned} P^a_c &= 2 & \qquad\qquad P^b_c &= 1 \\ P^a_e &= 1.2 & P^b_e &= 2.4\,, \end{aligned}$$

entailing that material consumption and environmental amenities yield the same utility in both economies,

$$P^a_c c^a + P^a_e e^a = 2.4 \qquad\qquad P^b_c c^b + P^b_e e^b = 2.4\,,$$

[11]The utility function can be found by integrating from a demand system that is consistent with observed quantities and prices, noting that the expenditure share for each good in either country is 0.5. Since the utility function is here assumed to be homogenous of degree 1, it follows that the marginal value of consumption spread, P^a_N and P^b_N, equals zero and need not be considered.

something that can be checked directly from the utility function. Since there is zero investment in both economies, this is the real comprehensive per capita NNP in money terms, which by Proposition 1 is the stationary per capita welfare equivalent of future utility. Hence, per capita welfare is the same in both economies.

To complete the derivation of real prices, it follows from the value of marginal products that real wages are given by

$$W^a = 1.2 \qquad\qquad W^b = 0.6\,,$$

and that the real interest rate is given by

$$R = 0.4$$

in both economies. The latter result—combined with the observation that investment in reproducible capital is zero—means that, in either economy, the dynamic discounted utilitarian welfare of the implemented program at time t is

$$\rho \int_t^\infty e^{-0.4(s-t)} u(c,e) ds\,.$$

It is important to note that, although material consumption, c, is a freely traded good, the real price of material consumption for the purpose of comparative welfare analysis differs in the two economies. The comparative welfare analysis is *not* made in international prices calculated according to exchange rates. Rather, *the comparative welfare analysis is made in local real prices calculated according to "purchasing-power-parity", on the basis of the consumption goods c and e, and where not only material consumption c but also non-traded environmental amenities e play an important role.* It can be seen that, in international prices calculated according to exchange rates, the value of consumption in economy b is twice as big as the value of consumption in economy a; this does *not* reflect the fact that c and e yield the same utility in both economies.

Since the production function exhibits constant-returns-to-scale, per capita wealth can also be used for welfare comparisons. However, to be able to invoke Proposition 2 for such a comparison, per capita wealth in each economy must be calculated in local real prices and includes the present value of future wages. The capitalized real per unit value of reproducible capital, environmental amenities, and labor is given by

$$\begin{aligned} Q_K^a &= 2 & \qquad Q_K^b &= 1 \\ Q_E^a &= 3 & \qquad Q_E^b &= 6 \\ Q_N^a &= 3 & \qquad Q_N^b &= 1.5\,, \end{aligned}$$

entailing that real per capita wealth, including the present value of future wages are the same in both economies,

$$\big(Q_K^a K^a + Q_E^a E^a\big)/N^a + Q_N^a = 6 \qquad\qquad \big(Q_K^b K^b + Q_E^b E^b\big)/N^b + Q_N^b = 6\,.$$

Note, however, that a comparison of per capita wealth excluding the present value of future wages and measured in international prices calculated according to exchange rates gives a very different result. Such a comparison would produce the result that per capita wealth in economy b is three times the per capita wealth in economy a, a result that lacks welfare significance. This shows the importance of the Divisia consumer price index when calculating real prices for the purpose of per capita wealth comparison.

Since both economies have the same set of attainable quadruples, we may also consider integrating the value of per capita stock changes when going from economy a to economy b (cf. Proposition 3). The per capita stocks in the two economies are given by

$$\begin{aligned} K^a/N^a &= 0 \qquad\qquad & K^b/N^b &= 1.5 \\ E^a/N^a &= 1 \qquad\qquad & E^b/N^b &= 0.5\ , \end{aligned}$$

However, for such integration to have welfare significance, the relative price of the environmental amenity stock in terms of reproducible capital, Q_E/Q_K, must increase from $Q_E^a/Q_K^a = 1.5$ in economy a to $Q_E^b/Q_K^b = 6$ in economy b, in a manner that depends on the real stock prices in the imaginary intermediate economies that the integration passes through. This indicates that such a method is difficult to apply.

9 Conclusion

This paper has shown that global welfare comparisons between economies with different population sizes and technological constraints can be made according to an expanded measure of real comprehensive per capita NNP that includes a term which can be interpreted as the marginal value of consumption spread. The result assumes that the economies maximize the same discounted utilitarian welfare function. In the real world, economies may not allocate resources in accordance with discounted utilitarianism, and may have different social preferences over paths of per capita consumption flows.

Comparisons based on per capita wealth require in addition that the technologies exhibit constant-returns-to-scale. This assumption imposes strong informational demands in the sense that, not only variable determinants, but also fixed determinants of current productive capacity must be included.

Comparisons based on the value of changes in per capita stocks require even more assumptions: economy-independent technologies (entailing that, e.g., the effects of a geographically differentiated climate on the consumption and investment opportunities are captured by the vector of capital stocks) and an ability to determine real stock prices in the imaginary economies that lie between the economies that are compared.

The conclusion of the present analysis is therefore that global welfare comparisons between economies can be made by means of real comprehensive per capita NNP, a per capita variant of Weitzman's (1976) stationary welfare equivalent of future utility. However, real comprehensive per capita NNP has global welfare significance only under strong assumptions and alternatives to this notion appears to be even less practical.

References

Aronsson, T. and Löfgren, K.-G. (1993), Welfare consequences of technological and environmental externalities in the Ramsey growth model, *Natural Resource Modeling* **7**, 1–14.

Arrow, K., Dasgupta, P.S., and Mäler, K.-G. (2003a), The genuine savings criterion and the value of population, *Economic Theory* **21**, 217–225.

Arrow, K., Dasgupta, P.S., and Mäler, K.-G. (2003b), Evaluating projects and assessing sustainable development in imperfect economies, *Environmental and Resource Economics* **26**, 647–685.

Asheim, G.B. (2003), Green national accounting for welfare and sustainability: A taxonomy of assumptions and result, *Scottish Journal of Political Economy* **50**, 113–130.

Asheim, G.B. (2004), Green national accounting with a changing population, *Economic Theory* **23**, 601–619.

Asheim, G.B. and Buchholz, W. (2004), A general approach to welfare measurement through national income accounting, *Scandinavian Journal of Economics* **106**, 361–384.

Asheim, G.B. and Weitzman, M. (2001), Does NNP growth indicate welfare improvement?, *Economics Letters* **73**, 233–239.

Dasgupta, P.S. (2001), Valuing objects and evaluating policies in imperfect economies, *Economic Journal* **111**, C1–C29.

Dasgupta, P.S. and Mäler, K.-G. (2000), Net national product, wealth, and social well-being, *Environment and Development Economics* **5**, 69–93.

Fleurbaey, M. and Gaulier, G. (2009), International comparisons of living standards by equivalent incomes, *Scandinavian Journal of Economics* **111**, 597–624.

Hamilton, K. (1994), Green adjustments to GDP, *Resources Policy 20*, 155–168.

Heal, G. and Kriström, B. (2005a), National income and the environment, in K.-G. Mäler and J. Vincent (eds.), *Handbook of Environmental Economics 3*, North-Holland, Amsterdam, 1147–1217.

Hulten, C.R. (1987), Divisia index, in J. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave: A Dictionary of Economics*, Macmillan Press, London and Basingstoke, 899–901.

Kemp, M.C. and Long, N.V. (1982), On the evaluation of social income in a dynamic economy: Variations on a Samuelsonian theme, in G.R. Feiwel (ed.), *Samuelson and Neoclassical Economics*, Kluwer Academic Press, Dordrecht.

Li, C.-Z. and Löfgren, K.-G. (2002), On the choice of metrics in dynamic welfare analysis: Utility versus Money Measures, Umeå Economic Studies, No 590, University of Umeå.

Li, C.-Z. and Löfgren, K.-G. (2006), Comprehensive NNP, social welfare, and the rate of return on investment, *Economic Letters* **90**, 254–259.

Li, C.-Z. and Löfgren, K.-G. (2007), Properly Indexed Green NNP is a Perfect Welfare Indicator, mimeo, Department of Economics, University of Umeå.

Lindahl, E. (1933), The concept of income, in G. Bagge (ed.), *Economic Essays in Honor of Gustav Cassel*, George Allen & Unwin, London, 399–407.

Osberg, L. and Sharpe, A. (2002), An index of economic well-being for selected OECD countries, *Review of Income and Wealth* **48**, 291–316.

Pezzey, J. (2004), One-sided sustainability tests with amenities, and changes in technology, trade and population, *Journal of Environment Economics and Management* **48**, 613–631.

Samuelson, P.A. (1961), The evaluation of 'social income': Capital formation and wealth, in Lutz, F.A. and Hague, D.C. (eds.), *The Theory of Capital*, St. Martin's Press, New York, 32–57.

Sefton, J.A. and Weale, M.R. (2006), The concept of income in a general equilibrium, *Review of Economic Studies 73*, 219–249.

Vellinga, N. and Withagen, C. (1996), On the Concept of Green National Income, *Oxford Economic Papers* **48**, 499–514.

Weitzman, M.L. (1976), On the welfare significance of national product in a dynamic economy, *Quarterly Journal of Economics* **90**, pp. 156–162.

Weitzman, M.L. (2001), A contribution to the theory of welfare accounting, *Scandinavian Journal of Economics* **103**, pp. 1–23.