

The relationship between welfare measures and indicators of sustainable development

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May 12, 2008

Abstract

This chapter investigates whether measures of welfare improvement indicates sustainability. First it shows how the value of net investments and real NNP growth (appropriately extended if population growth is positive) can be used to measure welfare improvement, before turning to following two questions: (1) Does non-negative value of net investments imply sustainable development? (2) Does sustainable development imply non-negative value of net investments? The main conclusion is that welfare improvement is not a sufficient condition for sustainability, but under special conditions it is a necessary condition for sustainability.

Keywords and Phrases: National income accounting, dynamic welfare

JEL Classification Numbers: C43, D60, O47, Q01

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1 Introduction

What is the relationship between welfare measures and indicators of sustainable development? This chapter studies to what extent measures of welfare improvement can also be used as indicators of sustainability. It builds on (and borrows freely from) published papers by myself and co-authors (Asheim, 1994, 2003, 2004, 2007a; Asheim, Buchholz and Withagen, 2003; Asheim and Weitzman, 2001); most of these papers are included in Asheim (2007b).

The relationship between welfare measures and indicators of sustainable development is particularly interesting a setting where there is population growth, and thus, in large part of this chapter I will allow for positive population growth.

What constitutes welfare improvement when population is changing? The answer depends on whether a bigger future population for a given flow of per capita consumption leads to a higher welfare weights for people living at that time, or, alternatively, only per capita consumption matters. When applying, e.g., discounted utilitarianism to a situation where population changes exogenously through time, it seems reasonable to represent the instantaneous well-being of each generation by the product of population size and the utility derived from per capita consumption. This is the position of ‘total utilitarianism’, which has been endorsed to by, e.g., Meade (1955) and Mirrlees (1967), and which is the basic assumption in Arrow, Dasgupta and Mäler’s (2003b) study of savings criteria with a changing population. Within a utilitarian framework, the alternative position of ‘average utilitarianism’, where the instantaneous well-being of each generation depends only on per capita consumption, have been shown to yield implications that are not ethically defensible.¹

What does sustainability mean when population is changing? If the economy cares about sustainability (in the sense that current per capita utility should not exceed what is potentially sustainable), then it becomes important to compare the level of individual utility for different generations, irrespectively of how population size develops. Therefore, utility derived from per capita consumption seems more

¹See Dasgupta (2001b, Sect. 6.4) for a discussion of the deficiency of ‘average utilitarianism’.

relevant in a discussion of sustainability. Hence, with positive population growth, it is possible to have total utility increasing throughout so that welfare improves, while at the same time per capita utility is falling so that development is not sustainable. However, it turns out that the relationship between welfare improvement and sustainability is not straightforward even if population is constant.

Since the major results of this chapter concern the problems of associating measures of welfare improvement with sustainability, it is justified to make rather stringent assumptions concerning the working of the economy, since the problems of such association will be even more serious in an economy with a poorer performance. Hence, in the basic model presented in Section 2, I assume that the economy implements a competitive path. In Section 3 I show the welfare significance of the present value of future consumption changes even in the presence of population growth, while in Section 4 I report on how the present value of future consumption can be measured through national accounting aggregates (both the value of net investments and real NNP growth). On this basis, I present in the following four sections a discussion of whether measures of welfare improvement can serve as indicators of sustainability. The main conclusion (first made by Pezzey, 2004) is that welfare improvement is not a sufficient condition for sustainability, but under special conditions it is a necessary condition for sustainability.

2 Model

Following Arrow, Dasgupta and Mäler (2003b) and Asheim (2004), assume that population N develops exogenously over time. The population trajectory $\{N(t)\}_{t=0}^{\infty}$ is determined by the growth function

$$\dot{N} = \phi(N)$$

and the initial condition $N(0) = N^0$. Two special cases are exponential growth,

$$\phi(N) = \nu N,$$

where ν denotes the constant growth rate, and logistic growth,

$$\phi(N) = \bar{\nu}N\left(1 - \frac{N}{N^*}\right),$$

where $\bar{\nu}$ denotes the maximum growth rate, and N^* denotes the population size that is asymptotically approached. As mentioned by Arrow, Dasgupta and Mäler (2003b), the latter seems like the more acceptable formulation in a finite world. In general, denote by $\nu(N)$ the rate of growth of population as a function of N , where

$$\nu(N) = \phi(N)/N.$$

Let \mathbf{C} represent a m -dimensional *consumption* vector that includes also environmental amenities and other externalities. Let u be a given concave and non-decreasing *utility function* with continuous partial derivatives that associates the instantaneous well-being for each individual with the *utility* $u(\mathbf{c})$ that is derived from the per capita vector of consumption flows, $\mathbf{c} := \mathbf{C}/N$. Assume an idealized world where \mathbf{c} contains all variable determinants of current instantaneous well-being, implying that an individual's instantaneous well-being is increased by moving from \mathbf{c}' to \mathbf{c}'' if and only if $u(\mathbf{c}') < u(\mathbf{c}'')$. At any time, labor supply is assumed to be exogenously given and equal to the population size at that time.

Let \mathbf{K} denote a n -dimensional *capital* vector that includes not only the usual kinds of man-made capital stocks, but also stocks of natural resources, environmental assets, human capital (like education and knowledge capital accumulated from R&D-like activities), and other durable productive assets. Moreover, let $\mathbf{I} (= \dot{\mathbf{K}})$ stand for the corresponding n -vector of net *investments*. The net investment flow of a natural capital asset is negative if the overall extraction rate exceeds the replacement rate.

Assume again an idealized world where \mathbf{K} and N contain all variable determinants of current productive capacity, implying that the quadruple $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N)$ is *attainable* if $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N) \in \mathcal{C}$, where \mathcal{C} is a convex and smooth set, with free disposal of consumption and investment flows. Hence, the set of attainable quadruples does not depend directly on time. However, by letting time be one of the capital

components, this formulation encompasses the case where technology changes exogenously through time.² We thus make an assumption of “green” or *comprehensive* accounting, meaning that current productive capacity depends solely on the vector of capital stocks and the population size.

Society makes decisions according to a *resource allocation mechanism* that assigns to any vector of capital stocks \mathbf{K} and any population size N a consumption-investment pair $(\mathbf{C}(\mathbf{K}, N), \mathbf{I}(\mathbf{K}, N))$ satisfying that $(\mathbf{C}(\mathbf{K}, N), \mathbf{I}(\mathbf{K}, N), \mathbf{K}, N)$ is attainable.³ I assume that there exists a unique solution $\{\mathbf{K}^*(t)\}_{t=0}^{\infty}$ to the differential equations $\dot{\mathbf{K}}^*(t) = \mathbf{I}(\mathbf{K}^*(t), N(t))$ that satisfies the initial condition $\mathbf{K}^*(0) = \mathbf{K}^0$, where \mathbf{K}^0 is given. Hence, $\{\mathbf{K}^*(t)\}$ is the capital path that the resource allocation mechanism implements. Write $\mathbf{C}^*(t) := \mathbf{C}(\mathbf{K}^*(t), N(t))$ and $\mathbf{I}^*(t) := \mathbf{I}(\mathbf{K}^*(t), N(t))$.

Say that the program $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ is *competitive* if, at each t ,

1. $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t), N(t))$ is attainable,
2. there exist present value prices of the flows of utility, consumption, labor input, and investment, $(\mu(t), \mathbf{p}(t), w(t), \mathbf{q}(t))$, with $\mu(t) > 0$ and $\mathbf{q}(t) \geq 0$, such that

C1 $\mathbf{C}^*(t)$ maximizes $\mu(t)u(\mathbf{C}/N(t)) - \mathbf{p}(t)\mathbf{C}/N(t)$ over all \mathbf{C} ,

C2 $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t), N(t))$ maximizes $\mathbf{p}(t)\mathbf{C} - w(t)N + \mathbf{q}(t)\mathbf{I} + \dot{\mathbf{q}}(t)\mathbf{K}$ over all $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N) \in \mathcal{C}$.

Here C1 corresponds to utility maximization, while C2 corresponds to intertemporal profit maximization.⁴ The term “present value” reflects that discounting is taken care of by the prices. In particular, if relative consumption prices are constant throughout and there is constant real interest rate R , then it holds that $\mathbf{p}(t) =$

²This leads to the problem of measuring the “value of passage of time” using forward-looking terms. Methods for such measurement have been suggested by e.g. by Aronsson et al. (1997), Kemp and Long (1982), Pezzey (2004), Sefton and Weale (1996), and Vellinga and Withagen (1996).

³This is inspired by Dasgupta and Mäler (2000), Dasgupta (2001a, p. C20) and Arrow, Dasgupta and Mäler (2003a).

⁴To see that $\mathbf{p}(t)\mathbf{C} - w(t)N + \mathbf{q}(t)\mathbf{I} + \dot{\mathbf{q}}(t)\mathbf{K}$ is instantaneous profit, note that $\mathbf{p}(t)\mathbf{C} + \mathbf{q}(t)\mathbf{I}$ is the value of production, $w(t)N$ is the cost of labor and $-\dot{\mathbf{q}}(t)\mathbf{K}$ is the cost of holding capital.

$e^{-Rt}\mathbf{p}(0)$. However, I will allow for non-constant relative consumption prices and will return to the question of how to determine real interest rates from $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ in this more general case.

Assume that the implemented program $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ is competitive with finite utility and consumption values,

$$\int_0^{\infty} \mu(t)N(t)u(\mathbf{C}^*(t)/N(t))dt \text{ and } \int_0^{\infty} \mathbf{p}(t)\mathbf{C}^*(t)dt \text{ exist,}$$

and that it satisfies a capital value transversality condition,

$$\lim_{t \rightarrow \infty} \mathbf{q}(t)\mathbf{K}^*(t) = 0. \quad (1)$$

It follows that the implemented program $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ maximizes

$$\int_0^{\infty} \mu(t)N(t)u(\mathbf{C}/N(t))dt$$

over all programs that are attainable at all times and satisfies the initial condition.

Moreover, writing $\mathbf{c}^*(t) := \mathbf{C}^*(t)/N(t)$, it follows from C1 and C2 that

$$\mathbf{p}(t) = \mu(t)\nabla_{\mathbf{c}}u(\mathbf{c}^*(t)), \quad (2)$$

$$w(t) = \mathbf{p}(t)\frac{\partial \mathbf{C}(\mathbf{K}^*(t), N(t))}{\partial N} + \mathbf{q}(t)\frac{\partial \mathbf{I}(\mathbf{K}^*(t), N(t))}{\partial N}, \quad (3)$$

$$-\dot{\mathbf{q}}(t) = \mathbf{p}(t)\nabla_{\mathbf{K}}\mathbf{C}(\mathbf{K}^*(t), N(t)) + \mathbf{q}(t)\nabla_{\mathbf{K}}\mathbf{I}(\mathbf{K}^*(t), N(t)). \quad (4)$$

3 Welfare significance of the present value of future consumption changes

Write $U(K, N) := Nu(\mathbf{C}(K, N)/N)$ and $U^*(t) := U(K^*(t), N(t))$ for the flow of *total utility*. In line with the basic analysis of Arrow, Dasgupta and Mäler (2003b), assume that $U^*(t)$ measures the social level of instantaneous well-being at time t .

Assume that, at time t , economy's *dynamic welfare* is given by a Samuelson-Bergson welfare function defined over paths of total utility from time t to infinity, and that this welfare function is time-invariant (i.e., does not depend on t). Moreover, assume that, for a given initial condition, the optimal path is time-consistent,

and that economy's resource allocation mechanism implements the optimal path. If the welfare indifference surfaces in infinite-dimensional utility space are smooth, then, at time t , $\{\mu(s)\}_{s=t}^{\infty}$ are local welfare weights on total utility flows at different times.⁵ Following a standard argument in welfare economics, as suggested by Samuelson (1961, p. 52) in the current setting, one can conclude that dynamic welfare is increasing at time t if and only if

$$\int_t^{\infty} \mu(s) \dot{U}^*(s) ds > 0. \quad (5)$$

To show that this welfare analysis includes discounted total utilitarianism, assume for the rest of this paragraph only that economy through its implemented program maximizes the sum of total utilities discounted at a constant rate ρ . Hence, the dynamic welfare of the implemented program at time t is

$$\int_t^{\infty} e^{-\rho(s-t)} U^*(s) ds.$$

Then the change in dynamic welfare is given by

$$\begin{aligned} \frac{d}{dt} \left(\int_t^{\infty} e^{-\rho(s-t)} U^*(s) ds \right) &= -U^*(t) + \rho \int_t^{\infty} e^{-\rho(s-t)} U^*(s) ds \\ &= e^{\rho t} \int_t^{\infty} e^{-\rho s} \dot{U}^*(s) ds, \end{aligned} \quad (6)$$

where the second equality follows by integrating by parts. Hence, (5) follows by setting $\{\mu(t)\}_{t=0}^{\infty} = \{e^{-\rho t}\}_{t=0}^{\infty}$.

The following result provides a connection between welfare improvement and the present value of future changes in consumption.

Proposition 1 *Under the assumptions of Section 2,*

$$\int_t^{\infty} \mu(s) \dot{U}^*(s) ds = \int_t^{\infty} \mathbf{p}(s) \dot{\mathbf{C}}^*(s) ds + \int_t^{\infty} v(s) \phi(N(s)) ds.$$

⁵By identifying the social level of instantaneous well-being at time t with $U^*(t)$, we assume that there are stable welfare indifference surfaces in infinite-dimensional space when the well-being of each generation is measured by total utility, irrespectively of how consumption flows and population size develop. Discounted total utilitarianism leads to linear indifference surfaces in this space.

where $v(t) := \mu(t)(u(\mathbf{c}^*(t)) - \nabla_{\mathbf{c}}u(\mathbf{c}^*(t))\mathbf{c}^*(t))$ denotes the marginal value of consumption spread, measured in present value terms.⁶

Proof. Since $U^*(t) = N(t)u(\mathbf{C}^*(t)/N(t))$, we have that

$$\dot{U}^* = \nabla_{\mathbf{c}}u(\mathbf{c}^*)\dot{\mathbf{C}}^* + \phi(N)u(\mathbf{c}^*) - N\nabla_{\mathbf{c}}u(\mathbf{c}^*)\mathbf{c}^*\nu(N),$$

The result follows from (2) and the definition of $v(t)$, since $\nu(N)N = \phi(N)$. ■

Corollary 1 *The present value of future consumption changes, $\int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)ds$, indicates welfare improvement in each of the following two situations:*

- (1) *There is a constant population.*
- (2) *The utility function u is linearly homogeneous.*

Proof. Part (1) follows directly from (5) and Proposition 1. Part (2) follows from (5) and Proposition 1 through the application of Euler's theorem. ■

These results can be generalized to the case where the economy's resource allocation mechanism does not implement an optimal path. In particular, (6) depends solely on the properties of discounted utilitarianism, and does not rely on the resource allocation mechanism implementing an optimal or even an efficient path. Furthermore, if (2) is used to define consumption shadow prices, then Proposition 1 and Corollary 1 remain true. In the case without population growth, I have through Asheim (2007a, Proposition 2(b)) generalized the results of this section to the case of any time-invariant Samuelson-Bergson welfare function satisfying a condition of independent future (so that the ranking of two paths that coincide from the current time t to a future time t' is the same at any time between t and t'), without making any assumptions about the working of economy's resource allocation mechanism.

⁶That $v(t)$ is positive means that instantaneous well-being is increased if an additional individual is brought into economy even when the total consumption flows are kept fixed and must be spread on an additional person. See Asheim (2004, Sect. 4) for a discussion of the term $v(t)$.

4 Measuring the present value of future consumption changes through national accounting aggregates

To tie the current chapter to contributions on the theory of welfare accounting, it is worthwhile to recapitulate how the present value of future consumption changes, which welfare significance was investigated in Section 3, can be measured through national accounting aggregates.

Within the setting of the model of Section 2, there are two ways to measure the present value of future consumption changes, $\int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)ds$: Through (i) the value of net investments and through (ii) real NNP growth, where each measure has been extended to take care of population growth.

Proposition 2 *Under the assumptions of Section 2,*

$$\int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)ds = \mathbf{q}(t)\mathbf{I}(t) + \int_t^\infty w(s)\phi(N(s))ds.$$

Proof. By combining (3) and (4), one obtains

$$\begin{aligned} \mathbf{p}\dot{\mathbf{C}}^* &= \mathbf{p}(\nabla_{\mathbf{K}}\mathbf{C} \cdot \mathbf{I}^* + \frac{\partial \mathbf{C}}{\partial N} \cdot \phi(N)) \\ &= -(\dot{\mathbf{q}}\mathbf{I}^* + \mathbf{q}\dot{\mathbf{I}}^* + w\phi(N)) = -\frac{d}{dt}(\mathbf{q}\mathbf{I}^*) + w\phi(N). \end{aligned} \tag{7}$$

Assuming that

$$\lim_{t \rightarrow \infty} \mathbf{q}(t)\mathbf{I}^*(t) = 0$$

holds as an investment value transversality condition, and $\int_t^\infty w(s)\phi(N(s))ds$ exists, the result is obtained by integrating (7). ■

Turn next to the question of how extended real NNP growth can measure the present value of future consumption changes. For this purpose, follow Asheim and Weitzman (2001) and Sefton and Weale (2006) by using a Divisia consumer price index when expressing comprehensive NNP in real prices. The application of a price index $\{\pi(t)\}$ turns the present value prices $\{\mathbf{p}(t), \mathbf{q}(t)\}$ into *real* prices $\{\mathbf{P}(t), \mathbf{Q}(t)\}$,

$$\begin{aligned} \mathbf{P}(t) &= \mathbf{p}(t)/\pi(t) \\ \mathbf{Q}(t) &= \mathbf{q}(t)/\pi(t), \end{aligned}$$

implying that the *real* interest rate, $R(t)$, at time t is given by

$$R(t) = -\frac{\dot{\pi}(t)}{\pi(t)}.$$

A *Divisia* consumption price index satisfies

$$\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t)},$$

implying that $\dot{\mathbf{P}}\mathbf{C}^* = 0$:

$$\dot{\mathbf{P}}\mathbf{C}^* = \frac{d}{dt} \left(\frac{\mathbf{P}}{\pi} \right) \mathbf{C}^* = \frac{\pi \dot{\mathbf{P}}\mathbf{C}^* - \dot{\pi} \mathbf{P}\mathbf{C}^*}{\pi^2} = 0.$$

Define comprehensive *NNP in real Divisia prices*, $Y(t)$, as the sum of the real value of consumption and the real value of net investments:

$$Y(t) := \mathbf{P}(t)\mathbf{C}^*(t) + \mathbf{Q}(t)\mathbf{I}^*(t).$$

Proposition 3 *Under the assumptions of Section 2,*

$$R(t) \cdot \left(\int_t^\infty \frac{\mathbf{p}(s)}{\pi(s)} \dot{\mathbf{C}}^*(s) ds \right) = \dot{Y}(t) + \frac{d}{dt} \left(\int_t^\infty \frac{w(s)}{\pi(s)} \phi(N(s)) ds \right).$$

Proof. Since

$$\begin{aligned} \frac{d}{dt} (\mathbf{Q}(t)\mathbf{I}^*(t)) &= \frac{1}{\pi(t)} \frac{d}{dt} (\mathbf{q}(t)\mathbf{I}^*(t)) + R(t)\mathbf{Q}(t)\mathbf{I}^*(t) \\ \frac{d}{dt} \left(\int_t^\infty \frac{w(s)}{\pi(s)} \phi(N(s)) ds \right) &= -\frac{w(t)}{\pi(t)} \phi(N(t)) + R(t) \left(\int_t^\infty \frac{w(s)}{\pi(s)} \phi(N(s)) ds \right), \end{aligned}$$

it follows from $\dot{\mathbf{P}}\mathbf{C}^* = 0$ and expression (7) that

$$\begin{aligned} 0 &= \frac{1}{\pi(t)} \left(\mathbf{p}(t)\dot{\mathbf{C}}^*(t) + \frac{d}{dt} (\mathbf{q}(t)\mathbf{I}^*(t)) - w(t)\phi(N(t)) \right) \\ &= \frac{d}{dt} (\mathbf{P}(t)\mathbf{C}^*(t)) + \frac{d}{dt} (\mathbf{Q}(t)\mathbf{I}^*(t)) + \frac{d}{dt} \left(\int_t^\infty \frac{w(s)}{\pi(s)} \phi(N(s)) ds \right) \\ &\quad - R(t) \left(\mathbf{Q}(t)\mathbf{I}^*(t) + \int_t^\infty \frac{w(s)}{\pi(s)} \phi(N(s)) ds \right). \end{aligned} \tag{8}$$

Hence, the result is obtained by using Proposition 2 and the definitions above. ■

5 Sustained development implies welfare improvement

Proposition 1 shows that the present value of future consumption changes is an indicator of welfare improvement, also under exogenous population growth, while Propositions 2 and 3 show how the present value of future consumption changes can be measured by means of national accounting aggregates. The remaining four sections of this chapter consider the relationship between the present value of future consumption changes, and thus the national accounting aggregates of Propositions 2 and 3, on the one hand, and the sustainability of the path, on the other hand.

To concentrate attention on intergenerational issues, abstract throughout from intratemporal distribution by assuming that all individuals living at time t obtains the average utility level, $u(\mathbf{c}^*(t))$, where $\mathbf{c}^*(t) = \mathbf{C}^*(t)/N(t)$ is the per capita consumption along the implemented path. Following Pezzey (1997), one can distinguish between *sustainable* and *sustained* development. A path constitutes sustainable development if, at each time t , the per capita utility level at time t can potentially be shared by all individuals of future generations. A path constitutes sustained development if, at each time, $u(\mathbf{c}^*(t))$ is non-decreasing. Any sustained development is also sustainable. On the other hand, the converse does not hold, since a development can be sustainable even if a generation makes a sacrifice for the benefit of successors that lowers its own per capita utility below those of its predecessors.

If population growth is non-negative and development is sustained, then it is a straightforward conclusion that that the present value of future consumption changes is non-negative and that welfare improves.

Proposition 4 *If $\nu(N(s)) \geq 0$ and $du(\mathbf{c}^*(s))/dt \geq 0$ for all $s > t$, then*

$$\int_t^\infty \mathbf{p}(s) \dot{\mathbf{C}}^*(s) ds \geq 0.$$

If, in addition, $u(\mathbf{c}^(t)) \geq 0$,⁷ then*

$$\int_t^\infty \mu(s) \dot{U}^*(s) ds \geq 0.$$

⁷That $u(\mathbf{c}^*(t))$ is positive means that instantaneous well-being is increased if an additional individual is brought into economy and offered the existing per capita consumption flows.

Proof. Since $\mathbf{c}^* = \mathbf{C}^*/N$, the following holds at each $s > t$:

$$du(\mathbf{c}^*)/dt = \nabla_{\mathbf{c}}u(\mathbf{c}^*) \left(\dot{\mathbf{C}}^*/N - \mathbf{c}^*\nu(N) \right).$$

By (2) and the premisses of the proposition, $\mathbf{p}(s)\dot{\mathbf{C}}^*(s) \geq 0$ at each $s > t$, thereby establishing the first part of the proposition. Since $U^* = Nu(\mathbf{c}^*)$, we have

$$\dot{U}^* = Ndu(\mathbf{c}^*)/dt + \nu(N)Nu(\mathbf{c}^*),$$

thereby establishing the second part. ■

It is an equally obvious result that welfare improvement, measured by a positive present value of future consumption changes, or a positive present value of future changes in total utility, cannot serve as an indicator of sustainability if there is positive population growth. The reason is that declining per capita utility throughout (i.e., $du(\mathbf{c}^*(s))/dt < 0$ for all $s > t$) is consistent with a positive present value of future consumption changes and a positive present value of future changes in total utility if population growth is sufficiently large.

Therefore, an investigation of converse versions of Proposition 4 is of interest only in the case of constant population. In this case it follows from Propositions 1 and 2 that both the present value of future consumption changes, $\int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)ds$, and the value of net investments, $\mathbf{q}(t)\mathbf{I}^*(t)$, are exact indicators welfare improvement independently of the properties of the function u (cf. footnote 6). The next section reports on the negative result that a positive value of net investments at time t does not imply that development at time t is sustainable.

The subsequent Section 7 presents an investigation of the question of whether sustainable development, rather than the stronger premise of sustained development used in Proposition 4, is sufficient for non-negative value of net investments, when there is no population growth that facilitates an expansion of the economy. It follows from the analysis of Pezzey (2004) that it is indeed the case that development is sustainable only if the value of net investments is non-negative, in the special case where the economy implements a discounted utilitarian optimum. However this result does not hold in general.

In both the next two sections I adopt the rather stringent assumptions on the working of the economy imposed in Section 2 and follow the analysis presented in Asheim, Buchholz and Withagen (2003). The population is assumed to be constant and normalized to 1, implying that $\mathbf{c} = \mathbf{C}$. As the results on the relationship between sustainability and the value of net investments are negative, such stringent assumptions make the results stronger. In the concluding remark I discuss the reliability of welfare improvement, as measured by the value of net investments, as an indicator of sustainable development in an economy that works less perfectly.

6 Does non-negative value of net investments imply sustainable development?

Consider the following claim: If the value of net investments $\mathbf{q}(t)\mathbf{I}^*(t)$ is non-negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}^*(t))$ can be sustained forever given $\mathbf{K}^*(t)$. This claim is not true in the Dasgupta-Heal-Solow model (see, e.g., Dasgupta and Heal, 1974, and Solow, 1974). In this model there are two capital stocks: man-made capital, denoted by K_M , and a non-renewable natural resource, the stock of which is denoted by K_N . So, $\mathbf{K} = (K_M, K_N)$. The initial stocks are given by $\mathbf{K}^0 = (K_M^0, K_N^0)$. The technology is described by a Cobb-Douglas production function $F(K_M, -I_N) = K_M^a(-I_N)^b$ depending on two inputs, man-made capital K_M and the raw material $-I_N$ that can be extracted without cost from the non-renewable resource. The output from the production process is used for consumption and for investments in man-made capital I_M . Hence, $(c(t), \mathbf{I}(t), \mathbf{K}(t), 1)$ is attainable at time t if and only if

$$c(t) + I_M(t) \leq K_M(t)^a(-I_N(t))^b \text{ where } a > 0, b > 0 \text{ and } a + b \leq 1,$$

and $c(t) \geq 0$, $K_M(t) \geq 0$, $K_N(t) \geq 0$, and $-I_N(t) \geq 0$. With $r(t) := -I_N(t)$ denoting the flow of raw material, these assumptions entail

$$\int_0^\infty r(t)dt \leq K_N^0 \text{ and } r(t) \geq 0 \text{ for all } t \geq 0.$$

Writing $i(t) := I_M(t)$, the competitiveness condition C2 requires that

$$c^*(t) + i^*(t) = K_M^*(t)^a r^*(t)^b \quad (9)$$

$$p(t) = q_M(t) \quad (10)$$

$$q_M(t) \cdot b \cdot K_M^*(t)^a r^*(t)^{b-1} = 1 \quad (11)$$

$$q_M(t) \cdot a \cdot K_M^*(t)^{a-1} r^*(t)^b = -\dot{q}_M(t), \quad (12)$$

where (11) follows from $q_M(t) \cdot b \cdot K_M^*(t)^a r^*(t)^{b-1} = q_N(t)$ and $0 = \dot{q}_N(t)$ by choosing extracted raw material as numeraire: $q_N(t) \equiv 1$. Note that (11) and (12) entail that the growth rate of the marginal product of raw material equals the marginal product of man-made capital; thus, the Hotelling rule is satisfied.

Assume that $a > b > 0$. Then there is a strictly positive maximum constant rate of consumption \bar{c} that can be sustained forever given \mathbf{K}^0 (see, e.g., Dasgupta and Heal, 1974, p. 203). It is well known that this constant consumption level can be implemented along a competitive path where net investment in man-made capital is at a constant level $\bar{i} = b\bar{c}/(1-b)$. To give a counterexample to the claim above, fix a consumption level $c^* > \bar{c}$. Set $i^* = bc^*/(1-b)$ and define T by

$$\int_0^T (i^*/b)^{\frac{1}{b}} (K_M^0 + i^*t)^{-\frac{a}{b}} dt = K_N^0. \quad (13)$$

For $t \in (0, T)$, consider the path described by $\mathbf{K}^*(0) = \mathbf{K}^0$ and

$$\begin{aligned} c^*(t) &= c^* \\ i^*(t) &= i^* \\ r^*(t) &= (i^*/b)^{\frac{1}{b}} (K_M^0 + i^*t)^{-\frac{a}{b}}, \end{aligned}$$

which by (13) implies that the resource stock is exhausted at time T . This feasible path is competitive during $(0, T)$ at prices $p(t) = q_M(t) = r^*(t)/i^*$ and $q_N(t) = 1$, implying that the value of net investments $q_M(t)i^* - q_N(t)r^*(t)$ is zero. Hence, even though the competitiveness condition C2 is satisfied (while C1 does not apply) and the value of net investments is non-negative during the interval $(0, T)$, the constant rate of consumption during this interval is not sustainable forever.

The path described above for the Dasgupta-Heal-Solow model is in fact not efficient, since the capital value transversality condition (1) is not satisfied: At time T a certain stock of man-made capital, $K_M^*(T) = K_M^0 + i^*T$, has been accumulated. At the same time the flow of extracted raw material falls abruptly to zero due the exhaustion of the resource. With a Cobb-Douglas production function, the marginal productivity of r is a strictly decreasing function of the flow of raw material for a given positive stock of man-made capital. This implies that profitable arbitrage opportunities can be exploited by shifting resource extraction from right before T to right after T , implying that the Hotelling rule is not satisfied at that time.

As the path in this counterexample is inefficient, it might be possible that the value of net investments does not indicate sustainability in the example due to this lack of efficiency. However, this is not true either. The claim above does not become valid even if we consider paths for which competitiveness holds throughout and the capital value transversality condition is satisfied.

Again, counterexamples can be provided in the framework of the Dasgupta-Heal-Solow model. Asheim (1994) and Pezzey (1994) independently gave a counterexample by considering paths where the sum of utilities discounted at a constant utility discount rate is maximized. If, for some discount rate, the initial consumption level along such a discounted utilitarian optimum exactly equals the maximum sustainable consumption level given K_M^0 and K_N^0 , then there exists an initial interval during which the value of net investments is strictly positive, while consumption is unsustainable given the current capital stocks $K_M^*(t)$ and $K_N^*(t)$. It is not quite obvious, however, that the premise of this statement can be fulfilled, i.e., that there exists some discount rate such that initial consumption along the optimal path is barely sustainable. This was subsequently established for the Cobb-Douglas case by Pezzey and Withagen (1998). The fact that their proof is quite intricate indicates, however, that this is not a trivial exercise.

Consequently, another type of counterexample is provided here. This example is also within the Dasgupta-Heal-Solow model and resembles the one given above.

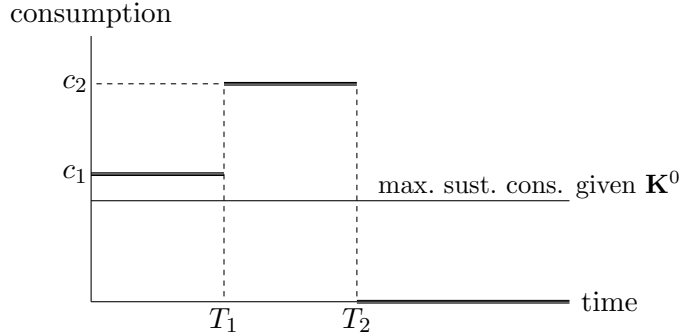


Figure 1: Non-negative value of net investments does not imply sustainability

In particular, a path identical to that described in the first counterexample during an initial phase can always be extended to an efficient path. Moreover, this second counterexample can be used to show that there exist regular paths with non-negative value of net investments during an initial phase even if $a \leq b$, entailing that a positive and constant rate of consumption cannot be sustained indefinitely.

The example, illustrated in Figure 1, consists of three separate phases with constant consumption, constructed so that there are no profitable arbitrage opportunities at any time, not even at the two points in time, T_1 and T_2 , where consumption is not continuous. Both capital stocks are exhausted at T_2 , implying that consumption equals zero for (T_2, ∞) .

In the construction of the example, K_M^0 is given, while K_N^0 is treated as a parameter. Fix some consumption level $c_1 > 0$ and some terminal time T_1 of the first phase of the path. In the interval $(0, T_1)$ the path is – as in the first example – described by $\mathbf{K}^*(0) = \mathbf{K}^0$ and

$$\begin{aligned} c^*(t) &= c_1 \\ i^*(t) &= i_1 \\ r^*(t) &= (i_1/b)^{\frac{1}{b}} (K_M^0 + i_1 t)^{-\frac{a}{b}}, \end{aligned}$$

where $i_1 = bc_1/(1-b)$, but with the difference that the resource stock will *not* be exhausted at time T_1 . As in the first example, the value of net investments equals zero during this phase.

The second phase starts at time T_1 . Consumption jumps upward discontinuously to $c_2 > c_1$, but we ensure that the flow of raw material is continuous to remove profitable arbitrage opportunities. Consumption is constant at the new and higher level c_2 , and, by the generalized Hartwick rule first established by Dixit, Hammond and Hoel (1980), the value of net investments measured in present value prices must be constant. I.e., there exists $\nu_2 < 0$ such that, for all $t \in (T_1, T_2)$, $q_M(t)i^*(t) = r^*(t) + \nu_2$. By (9) and (11), this equality may (for any c and ν) be written as

$$K_M(t)^a r(t)^b - c = b \cdot K_M(t)^a r(t)^{b-1} (r(t) + \nu). \quad (14)$$

As $K_M^a r^b - b \cdot K_M^a r^{b-1} r = (1-b) \cdot K_M^a r^b$, this implies

$$c = (1-b) \cdot K_M(t)^a r(t)^b \left(1 - \frac{b}{1-b} \cdot \frac{\nu}{r(t)} \right). \quad (15)$$

Since both $K_M^*(t)$ and $r^*(t)$ are continuous at time T_1 , we can now use (15) to determine ν_2 as follows:

$$c_2 = (1-b) \cdot K_M^*(T_1)^a r^*(T_1)^b \left(1 - \frac{b}{1-b} \cdot \frac{\nu_2}{r^*(T_1)} \right). \quad (16)$$

By choosing $c_2 > K_M^*(T_1)^a r^*(T_1)^b (> c_1)$ and fixing ν_2 according to (16), $q_M(t)i^*(t) = r^*(t) + \nu_2$ combined with (9) determines a competitive path along which investment in man-made capital becomes increasingly negative. Determine T_2 as the time at which the stock of man-made capital reaches 0, and determine K_N^0 such that the resource stock is exhausted simultaneously. With both stocks exhausted, consumption equals 0 during the third phase (T_2, ∞) .

The Hotelling rule holds for $(0, T_1)$ and (T_1, T_2) , and by the construction of ν_2 , a jump of the marginal productivity of the natural resource at T_1 is avoided such that the Hotelling rule obtains even at T_1 . Thus, the path is competitive throughout. By letting $u(c) = c$ and, for all $t \in (0, T_2)$, $\mu(t) = p(t)$, it follows that the path satisfies all assumptions of Section 2.

Note that the above construction is independent of whether $a > b$. If $a \leq b$, so that no positive and constant rate of consumption can be sustained indefinitely, we have thus shown that having non-negative value of net investments during an initial

phase of a regular path is compatible with consumption exceeding the sustainable level.

However, even if $a > b$, so that the production function allows for a positive level of sustainable consumption, a counterexample can be obtained. For this purpose, increase c_2 beyond all bounds so that ν_2 becomes more negative. Then T_2 decreases and converges to T_1 , and the aggregate input of raw material in the interval (T_1, T_2) – being bounded above by $r(T_1) \cdot (T_2 - T_1)$ since $r(t)$ is decreasing – converges to 0. This in turn means that, for large enough c_2 , c_1 cannot be sustained forever given the choice of K_N^0 needed to achieve exhaustion of the resource at time T_2 .

7 Does sustainable development imply non-negative value of net investments?

The counterexample of Figure 1 shows that non-negative value of net investments on an open interval is not a *sufficient* condition for having consumption be sustainable. Consider in this section whether this is a *necessary* condition: Does a negative value of net investments during a time interval imply that consumption exceeds the sustainable level? The following result due to Pezzey (2004) shows that such a converse implication holds under discounted utilitarianism.

Proposition 5 *Let $T > 0$ be given. Consider a path $\{\mathbf{c}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ satisfying the assumptions of Section 2 in a constant-population economy, with $\{\mu(t)\}_{t=0}^{\infty} = \{e^{-\rho t}\}_{t=0}^{\infty}$. If the value of net investments $\mathbf{q}(t)\mathbf{I}^*(t)$ is negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}^*(t))$ cannot be sustained forever given $\mathbf{K}^*(t)$.*

Proof. It follows from (2) and (7) that $\mu(t)du(\mathbf{c}^*(t))/dt + d(\mathbf{q}(t)\mathbf{I}^*(t))/dt = 0$, implying $d(\mu(t)u(\mathbf{c}^*(t)))/dt + d(\mathbf{q}(t)\mathbf{I}^*(t))/dt = \dot{\mu}(t)u(\mathbf{c}^*(t))$. By combining this with $\mu(t) = e^{-\rho t}$ and $\lim_{t \rightarrow \infty} \mathbf{q}(t)\mathbf{I}^*(t) = 0$, so that $\int_t^{\infty} \mu(s)ds = \mu(t)/\rho$ and

$$\mu(t)u(\mathbf{c}^*(t)) + \mathbf{q}(t)\mathbf{I}^*(t) = - \int_t^{\infty} \dot{\mu}(s)u(\mathbf{c}^*(s))ds = \rho \int_t^{\infty} \mu(s)u(\mathbf{c}^*(s))ds,$$

Weitzman's (1976) main result can be established:

$$\int_t^\infty \mu(s) \left(u(\mathbf{c}^*(t)) + \frac{\mathbf{q}(t)}{\mu(t)} \mathbf{I}^*(t) \right) ds = \int_t^\infty \mu(s) u(\mathbf{c}^*(s)) ds. \quad (17)$$

Since the path satisfies the condition of Section 2, it maximizes $\int_t^\infty \mu(s) u(\mathbf{c}(s)) ds$ over all feasible paths. This combined with (17) implies that the maximum sustainable utility level given $\mathbf{K}^*(t)$ cannot exceed $u(\mathbf{c}^*(t)) + \mathbf{q}(t)\mathbf{I}^*(t)/\mu(t)$. Suppose $\mathbf{q}(t)\mathbf{I}^*(t) < 0$ for $t \in (0, T)$. Then $u(\mathbf{c}^*(t)) > u(\mathbf{c}^*(t)) + \mathbf{q}(t)\mathbf{I}^*(t)/\mu(t)$. Hence, $u(\mathbf{c}^*(t))$ exceeds the maximum sustainable utility level and cannot be sustained forever given $\mathbf{K}^*(t)$. ■

It is *not*, however, a general result that sustainability implies non-negative value of net investments. This will be established next by showing that the following claim is not true, even under the conditions of Section 2: If the value of net investments $\mathbf{q}(t)\mathbf{I}^*(t)$ is negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}^*(t))$ cannot be sustained forever given $\mathbf{K}^*(t)$.

Also in this case a counterexample will be provided in the framework of the Dasgupta-Heal-Solow model. Assume that $a > b$ so that the production function allows for a positive level of sustainable consumption. Again, the example (which is illustrated in Figure 2) consists of three separate phases with constant consumption, constructed so that there are no profitable arbitrage opportunities at any time, not even at the two points in time, T_1 and T_2 , where consumption is not continuous.

As before, K_M^0 is given, while K_N^0 is treated as a parameter. Fix some consumption level $c_1 > 0$ and some terminal time T_1 of the first phase of the path. Construct a path that has constant consumption c_1 and obeys the generalized Hartwick rule by having a negative and constant value of net investment. I.e., $q_M(t)i^*(t) = r^*(t) + \nu_1$ with $\nu_1 < 0$ in the interval $(0, T_1)$, where T_1 is small enough to ensure that $K_M^*(T_1) > 0$. Let the path have, as its second phase, constant consumption $c_2 > 0$ and obeying the generalized Hartwick rule with $\nu_2 > 0$ in the interval (T_1, T_2) . To satisfy the Hotelling rule at time T_1 , c_2 and ν_2 must fulfill (16); hence, by choosing $c_2 < (1 - b) \cdot K_M^*(T_1)^a r^*(T_1)^b$ it follows that $\nu_2 > 0$. Let

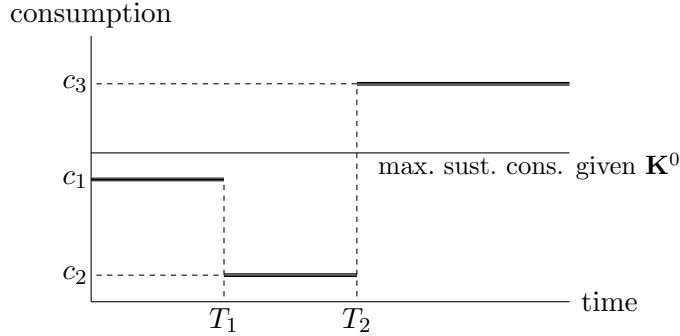


Figure 2: Sustainability does not imply non-negative value of net investments

$K_M^*(T_2)$ and $r^*(T_2)$ be the stock of man-made capital and the flow of raw material, respectively, at time T_2 . At this point in time the path switches over to the third phase with zero value of net investments, where the constant level of consumption is determined by $c_3 = (1 - b) \cdot K_M^*(T_2)^a r^*(T_2)^b$.

Since $a > b$, the production function allows for a positive level of sustainable consumption, and there exists an appropriate choice of K_N^0 that ensures resource exhaustion as $t \rightarrow \infty$ so that the capital value transversality condition (1) is satisfied. This initial resource stock depends on T_1 and T_2 , but it is finite in any case. Keep T_1 fixed and increase T_2 . As T_2 goes to infinity, then the stock K_N^0 needed will also tend to infinity.⁸ The same holds true for the maximum sustainable consumption level c^* that is feasible given K_M^0 and the initial resource stock K_N^0 determined in this way. Hence, by shifting T_2 far enough into the future, it follows that $c_1 < c^*$. Thus, a regular path can be constructed which has a first phase with a negative value of net investments even though the rate of consumption during this phase is sustainable given the initial stocks.

Both our counterexamples are consistent with the result of Proposition 2 (in the case with no population growth) that the value of net investments measures the present value of all future changes in utility. It follows directly from that result that if along an efficient path utility is monotonically decreasing/increasing

⁸It follows from (15) and $c_2 > 0$ that $r^*(t) > bv_2/(1 - b) (> 0)$ for all $t \in (T_1, T_2)$.

indefinitely, then the value of net investments will be negative/positive, while utility will exceed/fall short of the sustainable level. The value of net investments thus indicates sustainability correctly along such monotone utility paths. Hence, the counterexamples of Figures 1 and 2 are minimal by having consumption (and thus utility) constant except at two points in time.

It is worth emphasizing the point made in Asheim (1994) and elsewhere that the relative equilibrium prices of different capital stocks *today* depend on the properties of the whole future path. The counterexamples of Figures 1 and 2 show how the relative price of natural capital depends positively on the consumption level of the generations in the distant future. Thus, the future development – in particular, the distribution of consumption between the intermediate and the distant future – affects the value of net investments today and, thereby, the usefulness of this measure as an indicator of sustainability today.

8 Concluding remark

As shown in Sections 3 and 4, the value of net investments measures welfare improvement in an economy with no population growth, given that the assumptions of Section 2 are satisfied and the economy's dynamic welfare is given by a time-invariant Samuelson-Bergson welfare function that leads to a time-consistent optimal path. In Sections 6 and 7 we have shown that the value of net investments cannot serve as a reliable indicator of sustainability, even in a constant-population economy that satisfies the assumptions of Section 2.

It is worth to notice that the reliability of the value of net investments as an indicator of sustainability is further undermined if the resource allocation mechanism implements neither an optimal nor an efficient path. Consider, e.g., an economy where traditional growth is promoted through high investment in reproducible capital goods, but where incorrect (or lack of) pricing of natural capital leads to depletion of natural and environmental resources that is excessive both from the perspective of short-run efficiency and long-run sustainability. Then utility growth in the short

to intermediate run will, if the utility discount rate ρ is large enough, lead to current growth in discounted utilitarian dynamic welfare. Hence, both the value of net investments and real NNP growth will be positive.⁹ At the same time, the resource depletion may seriously undermine the long-run livelihood of future generations, so that current utility far exceeds the level that can be sustained forever.

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⁹It follows from Asheim (2007a, Propositions 1(b) and 2(b)) that the value of net investments and real NNP growth measure welfare improvement even if the resource allocation mechanism is imperfect, provided that appropriate shadow prices are applied.

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