The Norwegian Pension System

The Economic Effects of Funded Pension Benefits

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Foreword

I wish to thank my thesis supervisor, Kaiji Chen, for the wise comments and friendly advice I have received from him throughout the process of writing the thesis. This thesis is written with financial support from the ESOP centre at the University of Oslo and I wish to thank the centre for providing this assistance.

Abstract

This thesis has a twofold objective. The first objective is to characterize the Norwegian pension system before and after the pension reform scheduled to be implemented from 2010. The second and main objective is to research the long run quantitative macroeconomic and welfare effects of two counterfactual pension reforms. We focus on studying the effects of pension funding. The model frameworks we use are general and partial equilibrium overlapping generations models calibrated to Norwegian data. Consistent with the literature we find large quantitative increases in welfare and the capital stock in the new steady state as a result of pension funding under the general equilibrium model. We find marginal changes in aggregate labor supply. The partial equilibrium model is used to investigate the sensitivity of the results to the assumption of a closed economy in the general equilibrium model. The partial equilibrium model strengthens the quantitative increase in welfare and aggregate saving as a result of pension funding. Aggregate labor supply decreases strongly as a result of pension funding in the partial equilibrium model.

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1 Introduction

On March 21st 2007 all the parties in the sitting (2005-2009) Storting, except the Progress Party, agreed on the broad outline of how a future reformed National Insurance Scheme (NIS) will look like after several years of research and debate. The exact details of the reform and its implementation are still not clear, but the big picture has to a large extent emerged concerning issues relating to old age pensions, which is the focus of this thesis. According to the settlement reached between the parties; the main lines of reform will largely be based on Report no. 5 (2006-2007) to the Storting (Stortingsmelding 5, 2006-2007).

This thesis has a twofold objective. The first objective is to provide a big picture overview of the Norwegian system for old age pensions[1] within the National Insurance Scheme before the pension reform and to outline what the system for old age pensions will look like after the reform. Furthermore, the thesis will attempt to relate the actual pension reform to themes in the theoretical literature on the economics of pension systems. The second and main objective of the thesis is to research the long run macroeconomic and welfare effects of two counterfactual pension reforms in a model economy calibrated to Norwegian historical data. In particular we research the effects of pension funding.

To achieve the first objective, the first part of the thesis is mainly descriptive and structured in the following way. I begin by providing a selective overview of the literature concerning the economics of pension systems. Thereafter I provide an overview of the taxonomy of pension systems introduced in Lindbeck & Persson (2003). Following this is a short description of the main features of the Norwegian system for old age pensions. Then I highlight important features of the pension reform for old age pensions as agreed upon in the Pension Settlement. Finally I use the taxonomy of pension systems introduced earlier to classify the Norwegian pension system before and after the reform.

To achieve the second objective, the second part of the thesis follows the line of quantitative research pioneered by Auerbach and Kotlikoff (1987) and is structured in the following way. I begin by constructing an overlapping generations (OLG) general equilibrium model with labor augmenting technology growth. The model exhibits population growth. The pension system in the model is of a Pay-As-You-Go (PAYGo), defined benefit, and quasi-actuarial

[1] I will exclude the consideration of disability pensions, AFP ("avtalefestet pensjon") pensions and employer provided pension schemes from the thesis.

type. This pension system is intended as a stylized version of the system for old age pensions within the National Insurance Scheme. A competitive equilibrium is defined for the model. The model parameters are calibrated to Norwegian data. The model is detrended and the competitive equilibrium along the detrended balanced growth path is characterized. The model is simulated and the results are then compared to the actual data.

Following this, I introduce an almost identical OLG model as was investigated earlier, but this time with a fully funded, defined contribution, actuarially fair pension system to conduct the counterfactual analysis. I argue in the first part of the thesis that the actual pension reform pulls the pension system more towards a defined contribution, funded pension system than it was before the pension reform and I choose to sharpen the main counterfactual analysis in this direction. This model is detrended and the competitive equilibrium along the detrended balanced growth path is characterized. The steady state is simulated numerically and the results are compared to results obtained under the PAYGo pension system.

Key findings regarding the long run effects of introducing a fully funded pension system relative to leaving the PAYGo system unreformed include:

- The quantitative increase in welfare is approximately 8.9 %.
- The increase in per capita aggregate labor supply is approximately 0.3 %.
- The capital to output ratio increases by approximately 20 %.
- Pension benefits are approximately 10.2 % lower due to the general equilibrium effect of a lower interest rate.

As an alternative pension reform, I eliminate the public pension system and compare the results to the results obtained in the benchmark model and the main counterfactual model. The results obtained under the eliminated pension system are identical to the result obtained under the main counterfactual pension reform for the capital to output ratio and the aggregate labor supply in addition to the households' lifecycle profiles of consumption and hours.

A caveat with the models discussed so far is the assumption of a closed economy where factor prices are determined solely on the basis of supply and demand in domestic factor markets. The Norwegian economy is a small open economy in which we would expect the interest rate to be determined on the international capital market and insensitive to the size of the domestic capital stock.

To investigate how sensitive the results I obtained earlier are to the assumption of a closed economy, I construct a partial equilibrium OLG model with the same population structure as in the previous models and the same pension system as in the previous fully funded model. The production sector is abstracted from and no attempt is made to model the current account or the aggregate resource constraint. Households face given factor and good prices. At these prices, factor demands and the supply of the macro good are perfectly elastic also at the aggregate level. The factor and good prices are set equal to the ones obtained under the general equilibrium model with the PAYGo pension system. The partial equilibrium is characterized, the model is simulated numerically and the numerical results are compared to the results obtained earlier.

Four results stand out as particularly interesting:

1. The quantitative increase in welfare is approximately two percentage points larger under the partial equilibrium model than under the general equilibrium model.
2. Aggregate hours decrease by 10.8 % under the partial equilibrium model while aggregate hours increase by 0.3 % under the general equilibrium fully funded model.
3. The quantitative increase in aggregate saving is approximately 17 percentage points larger under the partial equilibrium model than under the general equilibrium fully funded model.
4. The pension benefits increase by approximately 1 % under the partial equilibrium model.

The numerical experiments in this thesis are performed with MATLAB's optimization toolbox.

2 Overview of the Literature

The literature on the economics of pension reform is vast and a comprehensive analytical review would require a thesis of its own to do justice to the field. To narrow the scope of the task I will concentrate on two strands of the literature; the general literature on theoretical and quantitative analysis of economic policy, particularly concerning pension issues, within the framework of overlapping generations (OLG) models, and the literature on the economics of pension reform in Norway specifically.

The canonical OLG model is found in Diamond (1965), an extension of work by Samuelson (1958). Kotlikoff (2000) gives two early examples of studies that used life-cycle models to numerically solve for long run steady state effects of policy reform: Kotlikoff (1979) examined the long run effects of an unfunded social security system, while Summers (1981) researched the long run effects of tax reform. Both papers used a 55 period life-cycle model, where agents' economic lives range from age 20 through age 75 calibrated to US data. Kotlikoff (1979) is most relevant with regard to this thesis; his findings were that an unfunded social security system had a large negative impact on long run steady state aggregate capital stock and living standards.

These early models did not provide a way to study transition paths between steady states as a result of changes in government policy or (assumed) exogenous factors such as the demographic environment. The breakthrough in the dynamic analysis of life-cycle models came with Auerbach and Kotlikoff (1987) which studied various dynamic public finance issues and not only solved for long run steady states, but also for the transitions paths of the economy between steady states. De Nardi et al. (1999) is a particularly interesting paper in this tradition from our perspective because it studies the economic effects of various changes in pension policies from both a steady state and transition path perspective under an expected future demographic transition (i.e. population ageing) in the US. They found significantly higher long run costs associated with leaving the US social security system unreformed than official estimates. They also found that, with only one exception, all of the pension policies they proposed to alleviate the strain on future public finances significantly lowered the welfare of most transitional generations, even though the expected lifecycle welfare of agents born into the new steady state were higher. The only policy experiment that increased welfare of all current and future generations was to switch to a defined contribution unfunded system.

The issues of intra-cohort heterogeneity and the insurance perspective of pension systems have also been studied in recent years. The paper by de Nardi et al. (1999) investigates issues also along this dimension. Conesa and Krueger (1999) is another paper in this tradition. They find that the partial insurance for idiosyncratic risk provided by PAYGo system tends to strengthen the appeal of the status quo for most transitional agents when contemplating a change in pension systems from PAYGo to Fully Funded, even though they find that an individual born into a steady state under a fully funded system will have higher expected lifecycle welfare than an individual born into a steady state under a PAYGo system resembling the US pension system.

Two special papers in the literature worth mentioning are Imrohoroglu et al. (1999A), which focus on the computation of social security models, and Lindbeck & Persson (2003), which provide a condensed survey of general issues in the pension literature. In particular, Lindbeck & Persson (2003) contains a detailed discussion of dynamic inefficiency. An economy is said to be dynamically inefficient when the implicit return on a PAYGo pension system, i.e. the growth rate in the tax base, denoted as G, is at least as high as the market rate of interest, denoted as r. When $G \geq r$, one can introduce a PAYGo pension system that results in a Pareto improvement in welfare; where at least one generation is better off while no generations are left worse off.

Two relevant quantitative studies on the effects of pension reform in Norway are Fredrikssen et al. (2003) and Stensnes et al. (2007). Fredriksen et al. (2003) discuss the macroeconomic effects of pension reform in Norway. They consider three main alternatives for pension reform:

1. A modernized National Insurance Scheme (NIS), based on a fully funded system.
2. A modernized NIS, based on a continued PAYGo system, but with increased actuarial fairness.
3. A flat basic pension paid out to all old age pensioners, independent of lifecycle earnings, financed over the central government budget.

In alternatives 1 and 2; the average pension benefits are kept at today's level. Only the method of financing separates the two alternatives. In alternative 3 all old-age pensioners receive the same pension benefit from the NIS, this benefit is at the level of today's minimum pension, and any supplementation of pension benefits must be done through employer organized pension schemes and/or individual saving in the financial market.

Fredriksen et al. (2003) use a dynamic microsimulation model (MOSART) and an applied general equilibrium model for the Norwegian economy (MSG6) rather than an overlapping generations general equilibrium model. Important features that their model framework captures are:

- The endogenous retirement decision.
- The introduction of life expectancy indexation and the adjustment of pension benefits as a result of flexible retirement.
- The balance of trade.
- Disability and AFP pensions.
- The transition dynamics associated with phasing in new pension systems.

Two important general findings in Fredriksen et al. (2003) are, compared to a simulation under a continuation of the present pension system; all the three reform alternatives contribute to increasing GDP and employment and all the three alternatives alleviate the strain on public finances. Alternative 1 and 2 are closely related to my analysis in section 2 so I summarize some of the key findings in Fredriksen et al. (2003) for these alternatives below:

- Long term mainland GDP is approximately 5 % higher under both these alternatives compared to under the reference scenario. The growth in mainland GDP is largely a result of households' increase in labor supply.
- The average retirement age is higher under both reform alternatives than under the reference scenario.
- Real funding of the pension system is contingent on reduced consumption and increased labor supply during the build up of funds. This does however lead to increased consumption during retirement, financed by the return on the fund.
- Building up real funding would require large adjustments in the Norwegian economy. The building up of funds will mainly be realized by increased net export surpluses which means economic resources would have to be transferred from the domestic oriented sector to the export oriented sector.

Fredriksen et al. (2003) does not include an analysis of possible changes in inequality as a result of pension reform.

Stensnes et al. (2007) takes the outline of pension reform found in Report No. 5 to the Storting (2006-2007) (Stortingsmelding 5, 2006-2007) as a starting point for investigating the efficiency gains, the impact on public finances and equality as a result of this reform. They use the same model framework as in Fredriksen et al. (2003) and find that the government's reform proposal does entail efficiency gains and stimulates to increased labor supply. The increase in labor supply is mainly driven by the introduction of the life expectancy adjustment ratio and the introduction of a flexible retirement age. However, they also find that, if the AFP-arrangement, the system for disability pensions and the public sector pension system are left largely unreformed, the effect on labor supply as a result of the pension reform might be neutralized. Furthermore, they find that the reform proposal does alleviate the strain on public finances, the tax rate on wage and pension earnings required to balance the pension budget in each budget period changes from 22 % in an unreformed system in 2050 to 17.5 % in the reformed system in 2050. They also find that the reform does entail an increase in inequality in pension benefits as measured by a higher Gini-coefficient than under the current pension system.

This thesis is closely related to the early papers in the dynamic public finance literature, in the tradition of Kotlikoff (1979). Unlike the other quantitative studies of the Norwegian pension system I discussed above, this paper uses an OLG model. The model framework incorporates several realistic features such as technology growth, population growth, endogenous labor supply and stochastic lifetimes, but excludes from consideration other important issues such as intra-cohort heterogeneity and endogenous retirement decisions. Due to the model's relative richness and time and computational constraints, I focus exclusively on analyzing the detrended balanced growth path. I also specify a partial equilibrium model to try to investigate how sensitive my results are to the closed economy assumption I use in the main analysis.

3 The Norwegian Pension System

Chapter 3 discusses the Norwegian pension system and evaluates the proposed pension reform in light of economic research. The chapter is structured in the following way:

- Section 3.1 outlines a taxonomy of mandatory government run systems for old age pensions.
- Section 3.2 describes the Norwegian system for old age pensions within the National Insurance Scheme.
- Section 3.3 outlines the motivation for a comprehensive pension reform in Norway.
- Section 3.4 describes the government proposal for pension reform and the Pension settlement.
- Section 3.5 explains the funded element, i.e. the Government Pension Fund, within the Norwegian pension system.
- Section 3.6 uses the taxonomy of pension systems outlined in section 3.1 to relate the pension reform to themes in the literature on the economics of pension reform.

3.1 A Taxonomy of Pension Systems

We can use the paper by Lindbeck & Persson (2003) to provide a taxonomy of mandatory, government run, old age pension systems and to clarify terms and expressions that recur frequently in this thesis.

Lindbeck & Persson (2003) classify pension systems along three dimensions:

1. Defined Benefit vs. Defined Contribution pension systems.
2. Fully Funded vs. Pay-As-You-Go (PAYGo) pension systems.
3. Actuarial vs. non-actuarial pension systems.

Defined benefit vs. defined contribution

In a defined contribution scheme the contribution rate is exogenous while the benefits are endogenous. In a defined benefit scheme the benefits are exogenous while the contribution rate is endogenous in order to make the pension budget balance.

Fully funded vs. PAYGo

In a fully funded system the benefits are financed by the return on accumulated pension funds. In a PAYGo system the aggregate benefits are financed by taxing the working share of the population.

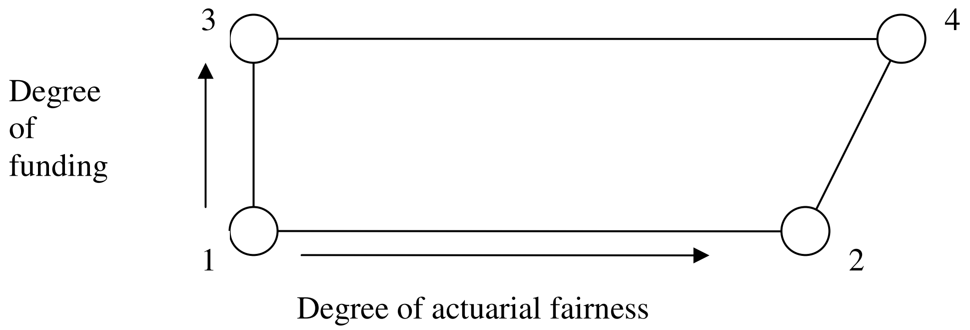
Actuarial vs. non-actuarial

Along this dimension Lindbeck & Persson (2003) discuss two different meanings of the term actuarial in the insurance literature. The first meaning of the term is macroeconomic in nature, a pension system is said to be in "actuarial balance" if and only if it is financially viable in the long run. The second meaning of the term is microeconomic in nature, and refers to the link between contribution and benefits at the individual level. Lindbeck & Persson (2003) refer to this feature as "actuarial fairness". Here fairness is used in the sense that a pension system has a high degree of actuarial fairness if there is a high degree of correspondence between an individual's contribution to the pension system through his work life and his pension benefits. Lindbeck & Persson (2003) assume, as do I in the remainder of this thesis, that any viable pension system must be in "actuarial balance", but that different degrees of "actuarial fairness" may be chosen within the class of viable pension systems.

In practice, government run pension systems are not clear-cut along any of the three dimensions, but the dimensions serve as useful tools to help organize the ideas on the economics of pension reform that are used in the remainder of this thesis.

Using the funded/unfunded and actuarial fairness dimensions gives four generic pension systems. This is represented as a trapezoid in figure 1, taken from Lindbeck & Persson (2003). We see that unfunded systems can be completely non-actuarial (position 1) or have a strong degree of actuarial fairness, i.e. be so called quasi-actuarial (position 2). Fully funded systems can likewise be completely non-actuarial (position 3) or entirely actuarial fair (position 4).

Figure 1: A taxonomy of pension systems



The return on the agent's contribution to pension system is equal to the market rate of interest in a completely actuarially fair fully funded system, while it is equal to the growth rate in the tax base in a quasi-actuarial unfunded system. Because the market rate of interest is higher than the growth rate of the tax base in most cases, Lindbeck & Persson (2003) chose a trapezoid figure rather than a square to indicate that position 2 is slightly less actuarially fair than position 4.

In theory, Lindbeck & Persson (2003) hold that it is possible to design both defined benefit and defined contribution schemes in all four corners of figure 1, though they state that it is very difficult in practice to construct defined benefit schemes in position 2 and 4. Therefore, pension schemes in position 2 and 4 will likely be defined contribution schemes. In position 1 and 3 it is, according to Lindbeck & Persson (2003), unproblematic to construct both defined benefit and defined contribution schemes.

3.2 The Norwegian Pension System before the Reform

The formal retirement age in Norway is 67 years, but various private and public arrangements have contributed to lowering the average age of exit from the labor market significantly. Gjedrem (2005) states that the average age of retirement is 60 if disability pensioners are included in the calculation and that the expected age of retirement for a 40 year old worker is 63.4 years.

The pension system is financed by taxing the working share of the population. I.e. it is mainly a PAYGo pension system. The pension system is based on a defined benefit scheme. The old age pension benefits in the NIS are based on a two-tier system: a basic pension and a supplementary pension.

A person's basic pension is only dependent on the length of membership in the NIS and, if married or living as a cohabitant, the income status of the person's partner. It is approximately equal to the basic amount (b.a.) in the National Insurance Scheme every year from retirement.

The supplementary pension is dependent on earnings over the lifecycle. Persons with very low earnings over the lifecycle get a special supplementary pension. This means that the minimum pension is calculated as: Minimum pension = Basic pension + Special supplementary pension.

The general supplementary pension is calculated in the following way; a point score is calculated based on the income of the person in each year of his working life relative to the b.a. in that year. The pensioner earns full supplementary pension after 40 years, this rule is known as the "40 years" rule. An adjusted average point score is calculated on the basis of the pensioner's 20 best years, i.e. the 20 years where income is highest relative to the b.a., and this adjusted average point score determines the pension benefits of the person. This rule is known as the "best years" rule. Income below one times the b.a. does not accrue any general supplementary pension, income between one and 6 times the b.a. accrues full supplementary pension entitlements, one accrues additional pension entitlements on one third of the income between 6 and 12 b.a. and one does not earn any pension entitlements on income above 12 b.a.

3.3 The Motivation for Pension Reform

The systematic case for a comprehensive pension reform in Norway is found in the report by the "Pension Commission" (NOU 2004:1, 2004). The main case for reform of the old age pension system is related to concerns about the actuarial balance of the National Insurance Scheme, i.e. the long run financial viability of the NIS. A large part of the concern over the long run financial viability of the NIS is related to the effects of an expected demographic change over the next half century in Norway. The following estimates serve to illustrate this point:

- For a 67 year old person the expected years remaining of life is estimated to grow from 17 years in 2003 to 22 years in 2050 (NOU 2004:1, 2004).
- The share of old age pension benefits as a percentage of Mainland-GDP without a reform of the NIS is expected to increase from approximately 6 % in 2001 to approximately 15 % in 2050 (NOU 2004:1, 2004).
- The ratio of workers to pensioners is 4.6 in 2006, while this ratio is expected to be 2.7 in 2050 under the baseline demographic scenario of Statistics Norway (Stensnes et al., 2007).

Another case for reform of the old age pension system is related to the perceived low degree of "actuarial fairness" in the old age pension system within the NIS. Two rules for calculation of old age pensions are cited as especially worrisome in this respect (NOU 2004:1, 2004):

- The "best years" rule.
- The "40 years" rule.

Two other rules must also be said to lower the degree of actuarial fairness in the NIS:

- The fact that one does not earn full supplementary pension entitlements on income above 6 times the b.a. This might contribute to reducing the incentive to work for middle to high income workers.
- The fact that one does not earn supplementary pension entitlements on income below one times the b.a. and the fact that the special (earnings-independent) supplementary pension is reduced "krone for krone" against the general (earnings-dependent) pension. This might contribute to reducing the incentive to work for low income workers.

According to NOU 2004:1 (2004), the current pension system does not benefit individuals with a long career in the workforce and an even lifetime earnings profile, but does benefit individuals with shorter careers and an uneven or increasing lifetime earnings profile. Furthermore, individuals with long careers, but low income, can be subject to the "minimum pension trap", whereby they get the same minimum old age pension benefits as individuals with very little or no income through their work life.

The problems of a perceived low degree of actuarial fairness and concerns about poor actuarial balance are also closely linked. Because the burden on the working share of the population to finance the NIS is estimated to be larger in 2050 than it is today, it is deemed important to provide incentives for workers to increase their labor supply; to work more during the span of their existing work life and/or to remain longer in the labor force.

A way to provide incentives for increased labor supply is to increase the degree of actuarial fairness in the NIS. By accumulating pension benefits for all the years a person is active in the labor force, the pension system might stimulate to a longer work life. By accumulating full pension entitlements on all income, including income in excess of 6 times b.a., the pension system could stimulate to increased labor supply during the existing work life. Both of these ways of increasing the actuarial fairness of the pension system might contribute to alleviating the strain on public finances related to old age pensions.

3.4 The Reform Settlement

Particularly important features of the new NIS system for old age pensions in Report No. 5 (2006-2007) to the Storting (Stortingsmelding 5, 2006-2007) and the Pension Settlement are:

- The abolishment of the "40 year" rule, a person will accrue pension benefits for all the years he works up until age 75.
- The abolishment of the "best year" rule, all years with income shall count equally with regards to the calculation of pension benefits.
- The introduction of a life expectancy adjustment ratio, whereby earned pension entitlement is distributed over a larger number of years to adjust for possible future increases in life expectancy in order to make the National Insurance Scheme's spending on old age pensions relatively robust to increased longevity.
- There will be a guarantee pension at the level of today's minimum pension.
- The guarantee pension will be reduced by 80 % against the earnings dependent pension benefits.
- All income up to 7.1 times the b.a. accrues full pension benefits, while income above 7.1 times the b.a. does not accrue any pension benefits.
- The introduction of a new indexation system whereby earned pension entitlements are adjusted in line with wage increases up to the time of retirement, while paid out pensions are adjusted in line with the average of the growth in prices and wages.

- Furthermore, the drawing of old-age pensions should be made more flexible. The retirement age shall be made flexible from age 62 and upwards, and it shall be easier to combine work and the drawing of pensions. However, early retirement will entail spreading the accrued pension benefits over more years, thereby decreasing the yearly pension benefits.

3.5 The Government Pension Fund

The Government Pension Fund was created on January 1st 2006 and this reorganization of existing government funds can be considered to be part of the broad pension reform package. The stated aim of Government Pension Fund is to support central government saving to finance the expenditures of the National Insurance Scheme on pensions and to strengthen the long term considerations of applying government petroleum revenues in the economy (Lov om Statens pensjonsfond, 2005). However, it is not a requirement that the capital in the Government Pension Fund shall correspond to a certain share of the pension liabilities of the Norwegian state under the National Insurance Scheme at all times (Stortingsmelding nr. 1, 2007-2008). The Government Pension Fund is managed by the ministry of finance and consists of two parts: The Government Pension Fund-Global and The Government Pension Fund-Norway.

The Government Pension Fund - Norway (GPF-N) was previously known as the National Insurance Scheme Fund. The operational management of the GPF-N is delegated to Folketrygdfondet. Since 1979, no government transfer to or from the GPF-N have been made. As such the income of the GPF-N consists of the return on the capital in the fund. This is the model for saving in the GPF-N that is proposed also for the future (Odelstingsproposisjon nr. 2, 2005-2006). The GPF-N invests mainly in domestic assets, but can also allocate some of its capital to assets in the broader Nordic region.

The market value of assets in the Government Pension Fund - Global (GPF-G) dwarfs the assets in the GPF-N. As of June 30 2007, the total market value of the Government Pension Fund was Norwegian Kroner 2057.1 billion. The market value of the GPF-G was 94.3 % of the total fund market value, while the market value of the GPF-N was 5.7 % of the total fund market value. Given this relative distribution of assets between the two parts of the fund and the more active role that the GPF-G plays in the public finances, as will be explained below, I will focus on the GPF-G in the remainder of this thesis.

The GPF-G is a continuation of the Norwegian Petroleum fund. The operational management of the GPF-G is delegated to Norges Bank Investment Management. The income of the GPF-G consists of the net cash flows that the Norwegian government earns from petroleum activities and the net results of financial transactions associated with petroleum activities in addition to the return on the assets in the GPF-G (Lov om Statens pensjonsfond, 2005). The GPF-G can only invest in foreign assets.

The GPF-G is fully integrated within the Norwegian fiscal budget and funds are transferred annually to the budget on resolution of the Storting. The funds are then used to finance the structural non-oil central government budget deficit. Over time the structural non-oil central government budget deficit should equal the expected long run real return on the GPF-G; this is the known as Norwegian fiscal rule ("Handlingsregelen"). The long run real return on the GPF-G is estimated to be 4 % annually (Stortingsmelding nr. 1, 2007-2008).

3.6 Classification of the Pension System before and after the Reform

PAYGo versus Fully Funded

The Norwegian pension system has been based on an intergenerational solidarity contract where the occupationally active are the main contributors to the financing of the old age pensions for the elderly (Stortingsmelding nr. 5, 2006-2007). Therefore the pension system before the reform must be classified as largely being a PAYGo system.

It is also clear that the occupationally active will remain the main contributors to the financing of old age pensions for the elderly also after the pension reform (Stortingsmelding nr. 5, 2006-2007). Therefore the pension system after the reform must also be classified as largely being a PAYGo system.

However, the renaming of the Norwegian Petroleum Fund to GPF-G makes the link between saved petroleum wealth and the financing of future pension benefits clearer. Grønvik (2006) states that this name change implies "real, but not legal funding", an apt description of what we might consider as a clearer element of funding in the government run system for old age pensions.

Defined Benefit versus Defined Contribution

The system outlined in Report no. 5 (2006-2007) to the Storting (Stortingsmelding nr. 5, 2006-2007) entails a model for the drawing of pensions after the pension reform that will to a large extent be based on a defined benefit system, as before the reform. However the introduction of a life expectancy adjustment ratio, as explained above, can be said to pull the NIS more towards a defined contribution system than it was before the reform.

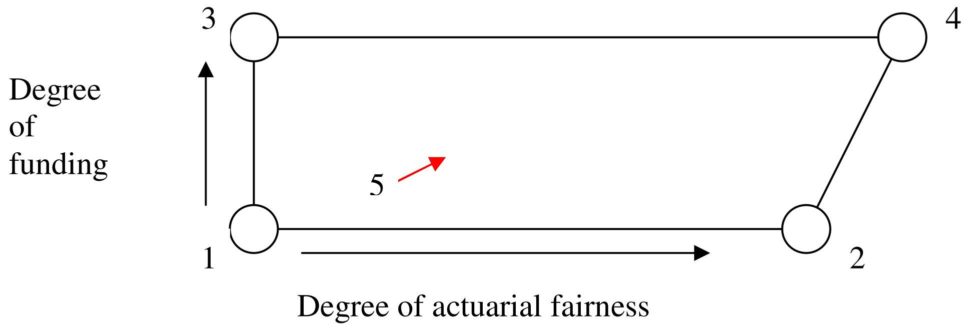
The degree of actuarial fairness

Concern over the degree of actuarial fairness is, as explained above, cited as a problem in its own right by Report No. 5 (2006-2007) to the Storting (Stortingsmelding nr. 5, 2006-2007) and also, perhaps more importantly, as a problem closely related to the concern over the actuarial balance of the NIS. The reform attempts to improve the actuarial fairness of the NIS system for old age pensions. The system for a flexible retirement age means that the pensioner bears the burden of early retirement. That pension benefits are accrued for all working years, that all income up to 7.1 times the b.a. accrues full pension benefits and that the guarantee pension is only reduced by 80 % against the earnings dependent pension means that the correspondence between what is paid into the NIS during the course of the work life and the pension benefits one receives as a retiree might be higher than in the pension system before the reform. Stensnes et al. (2007) estimates that earning one extra NOK during the work life will in fact on average lead to higher pension benefits in the pension system outlined in the reform proposal set forward by the government than in the unreformed NIS. As such, the reform must be said to achieve its objective of increasing the actuarial fairness of the system for old age pensions.

3.6.1 Conclusion

A reasonable conclusion of the outcome of a reform of old age pension system based on the "Pensjonsforliket" is a modified pay as you go pension system with a clearer funded element. The reformed pension system will be moderately more actuarially fair, but will still have a large redistributive element. The reformed pension system will incorporate more defined contribution ideas, most notably the introduction of the life expectancy adjustment ratio. We can use figure 2 to summarize the pension systems movement along the funding and actuarial fairness dimensions as a result of the pension reform. In figure 2, position 5 indicates the position of the Norwegian pension system before the reform. The red arrow indicates the direction of change that the pension reform causes if implemented based on Report No. 5 (2006-2007) to the Storting (Stortingsmelding nr. 5, 2006-2007) and the pension settlement.

Figure 2: Illustrating the pension reform within the taxonomy of pension systems.



4 The Benchmark Model

The remainder of the thesis is devoted to achieving the main objective of the thesis: To research the long run macroeconomic and welfare effects of two counterfactual pension reforms in a model economy calibrated to Norwegian historical data. Chapter 4 describes and analyzes the benchmark model with an unfunded pension system and is structured in the following way:

- Section 4.1 introduces the model economy with an unfunded pension system, i.e. the benchmark model. This model is calibrated to reproduce certain features of the Norwegian economy such as the long run capital to output ratio and the average working week for the occupationally active. The pension system outlined in the model is intended as a stylized version of the Norwegian pension system for old age pensions within the National Insurance Scheme before the pension reform. The stylized pension system is a pure PAYGo, defined benefit and quasi-actuarial pension scheme.
- Section 4.2 defines the competitive equilibrium along the balanced growth path in this model.
- Section 4.3 calibrates the model parameters to Norwegian historical data.
- Section 4.4 explains how the model's detrended balanced growth path is simulated numerically.
- Section 4.5 discusses how good the model predictions are relative to the historical data.

4.1 Environment

Demographics

At each period t, a new generation of individuals is born. The population grows at rate η per period. The individuals have stochastic lifetime. The stochastic lifetime is governed by a set of conditional survival probabilities $\{p_i\}_{i=1}^{3}$. The conditional survival probabilities have the following interpretation: Given that the agent is alive at period i, the agent will be alive in period $i+1$ with probability p_i. Individuals can live for a maximum of three periods, which implies that $p_3 = 0$. Total population is given by

$$N_t = P_t\left(1+\frac{p_1}{(1+\eta)}+\frac{p_1 p_2}{(1+\eta)^2}\right),$$

where P_t is the number of individuals born in period t and N_t is the size of the total population in period t.

Given time invariant conditional survival probabilities and population growth, the demographic environment in the model is stationary. Therefore, the cohort shares $\{\mu_i\}_{i=1}^{3}$ are constant and given as

$$\mu_i = \frac{p_{i-1}}{(1+\eta)} \mu_{i-1} \text{ for } i = 2,3, \tag{1}$$

where

$$\sum_{i=1}^{3} \mu_i = 1.$$

Technology

A representative firm uses a Cobb-Douglas production function with constant returns to scale and labor augmenting technological growth to produce output, a macro good, with labor and capital as inputs. Mathematically this is represented as:

$$Y_t = K_t^{\alpha} (A_t H_t)^{1-\alpha}, \tag{2}$$

where K_t is aggregate capital stock, A_t is the labor augmenting technology factor and H_t is aggregate labor input, i.e. aggregate efficient hours. This production function satisfies the well known Inada conditions.

The labor augmenting technology factor is governed by the deterministic law of motion:

$$A_{t+1} = A_t (1+g), \tag{3}$$

where the growth rate g is constant.

The aggregate capital stock evolves according to the law of motion:

$$K_{t+1} = X_t + (1-\delta) K_t, \tag{4}$$

where X_t denotes aggregate gross investment and δ is the constant depreciation rate.

The representative firm is a price taker in both the macro good and factor markets. Since only relative prices are determined in general equilibrium, the price of the macro good is normalized to unity. In other words, the macro good serves as numéraire in this economy. This implies the following period maximization problem for the representative firm:

$$\max_{K_t, H_t} \left\{ K_t^{\alpha} (A_t H_t)^{1-\alpha} - (R_t - 1 + \delta) K_t - w_t H_t \right\}, \tag{5}$$

where R_t is the interest rate factor net of depreciation and w_t is the wage rate.

Households' problem

Individuals derive utility from consumption of the macro-good and leisure. They do not have a bequest motive. Individuals maximize expected lifecycle utility. An individual born at time t solves a problem of the type:

$$\max_{\left\{\begin{smallmatrix} c_{1,t}, c_{2,t+1}, a_{2,t+1}, \\ a_{3,t+2}, h_{1,t}, h_{2,t+1} \end{smallmatrix}\right\}} \left\{ \begin{array}{l} \log c_{1,t} + \psi \log(1 - h_{1,t}) \\ + \beta p_1 (\log c_{2,t+1} + \psi \log(1 - h_{2,t+1})) + \beta^2 p_1 p_2 \log c_{3,t+2} \end{array} \right\}, \tag{6}$$

subject to a set of budget constraints:

$$c_{1,t} + a_{2,t+1} \leq R_t b_t + (1 - \tau_t) w_t \varepsilon_1 h_{1,t} \tag{7}$$

$$c_{2,t+1} + a_{3,t+2} \leq R_{t+1} (b_{t+1} + a_{2,t+1}) + (1 - \tau_{t+1}) w_{t+1} \varepsilon_2 h_{2,t+1} \tag{8}$$

$$c_{3,t+2} \leq R_{t+2} (b_{t+2} + a_{3,t+2}) + S_{t+2}. \tag{9}$$

Here β is the subjective discount factor, $c_{i,t+i-1}$ is the consumption of an age i individual in period $t+i-1$ for $i \in \{1,2,3\}$, $a_{2,t+1}$ and $a_{3,t+2}$ are the age 2 and age 3 agents' intended asset holdings in period $t+1$ and $t+2$ respectively, b_s are the lump sum transfers the agent receives in period $s \in \{t, t+1, t+2\}$, $h_{1,t}$ and $h_{2,t+1}$ are the age 1 and age 2 agent's supply of hours to the labor market in period t and $t+1$ respectively, ε_1 and ε_2 are the efficiency units of an age 1 and age 2 agent respectively, τ_s is the social security payroll tax rate in period $s \in \{t, t+1\}$, S_{t+2} denotes the pension benefits that the agent receives as a retiree in period $t+2$.

Agents are borrowing constrained, which implies that we require:

$$a_{2,t+1} \geq 0 \tag{10}$$

$$a_{3,t+2} \geq 0 \tag{11}$$

Furthermore, consumption and hours must satisfy the following conditions:

$$h_{1,t} \in [0,1] \tag{12}$$

$$h_{2,t+1} \in [0,1] \tag{13}$$

$$c_{1,t} \geq 0 \tag{14}$$

$$c_{2,t+1} \geq 0 \tag{15}$$

$$c_{3,t+2} \geq 0 \tag{16}$$

I.e. total time available to be supplied to the labor market is normalized to unity per working age period, and clearly negative hours or consumption cannot be possible.

The logarithmic utility function satisfies the well known Inada conditions. Therefore it is clear that $h_{1,t} \neq 1$, $h_{2,t+1} \neq 1$ and $c_{i,t+i-1} \neq 0$ for $i \in \{1,2,3\}$ at optimum.

Social security

An individual who is retired in period s, receives pension benefits, S_s, calculated as a fraction, $\theta \in [0,1]$, of average earnings net of taxes over the life cycle, indexed by technology growth:

$$S_s = \theta \frac{1}{2} \begin{pmatrix} (1-\tau_{s-2})(1+g)^2 w_{s-2}\varepsilon_1 h_{1,s-2} \\ +(1-\tau_{s-1})(1+g) w_{s-1}\varepsilon_2 h_{2,s-1} \end{pmatrix} \tag{17}$$

The pension system is a pure PAYGo system; it is run at zero cost and must be in actuarial balance in every period, which implies that the payroll tax rate is determined endogenously as:

$$\tau_s = \frac{\mu_3 S_s}{w_s(\mu_1\varepsilon_1 h_{1,s} + \mu_2\varepsilon_2 h_{2,s})}. \tag{18}$$

Lump sum transfers

Since agents face stochastic lifetime, they may leave unintended bequests when they die before their third period of life. All accidental bequests are immediately distributed as lump sum transfers to agents alive at no cost. In every period s, the following relation must then hold:

$$b_s = \frac{1}{1+\eta}(\mu_1(1-p_1)a_{2,s} + \mu_2(1-p_2)a_{3,s}). \tag{19}$$

4.2 Competitive equilibrium

A sequential competitive equilibrium for this economy with a stationary demographic structure consists of sequences of the social security tax rates and benefits $\{\tau_t, S_t\}_{t=0}^{\infty}$, lump sum transfers $\{b_t\}_{t=0}^{\infty}$, household allocations $\{c_{1,t}, c_{2,t+1}, c_{3,t+2}, a_{2,t+1}, a_{3,t+2}, h_{1,t}, h_{2,t+1}\}_{t=0}^{\infty}$, factor demands for the representative firm $\{K_t, H_t\}_{t=0}^{\infty}$ and factor prices $\{w_t, R_t\}_{t=0}^{\infty}$ such that:

1. Given factor prices, payroll tax rates and lump sum transfers, the household allocations solve the households' optimization problem.
2. Given factor prices, the factor demands solve the representative firm's optimization problem.
3. Factor prices are such that all markets clear, i.e. such that aggregate and individual behavior is consistent:
    - Capital market is in equilibrium:
      $$K_t = N_t\left(\mu_1 b_t + \mu_2\left(a_{2,t} + b_t\right) + \mu_3\left(a_{3,t} + b_t\right)\right) \qquad (20)$$
    - Labor market is in equilibrium:
      $$H_t = N_t\left(\mu_1 \varepsilon_1 h_{1,t} + \mu_2 \varepsilon_2 h_{2,t}\right) \qquad (21)$$
    - Commodity market is in equilibrium:
      $$C_t + X_t = Y_t \qquad (22)$$
    - Where the following relations hold:
      $$C_t = N_t\left(\mu_1 c_{1,t} + \mu_2 c_{2,t} + \mu_3 c_{3,t}\right) \qquad (23)$$
      $$X_t = K_{t+1} - (1-\delta)K_t \qquad (4)$$
      $$Y_t = K_t^{\alpha}\left(A_t H_t\right)^{1-\alpha} \qquad (2)$$
4. The payroll tax rate balances the pension system budget in every period, i.e. equations (17) and (18) hold in every period.
5. Lump sum transfers equal accidental bequests in every period, i.e. equation (19) holds in every period.

4.3 Calibration

The model requires calibration of the parameters in table 1 to allow for a numerical solution of the steady state under the PAYGo pension system:

Table 1: Parameters requiring calibration						
α	β	δ	ψ	θ	ε_1	ε_2
g	η	p_1	p_2	μ_1	μ_2	μ_3

The period length in the model is 23 years. Each agent begins his economic life at his 18th birthday. In this sense, 18 year old agents are frequently referred to as newborn in this model. The periods in the model are shown in table 2.

Table 2: Model periods	
Period #	Age
1	$[18,40]$
2	$[41,63]$
3	$[64,86]$

Survival probabilities

I use data on survival probabilities published by Statistics Norway for the year 2007. In the data, the conditional survival probabilities are given by yearly periods. To convert the conditional probabilities to a period length of 23 years, the following calculations are performed:

$$p_1 = \prod_{j=40}^{62} q_j \approx 0.93$$

$$p_2 = \prod_{j=63}^{85} q_j \approx 0.44\,,$$

where q_j is the yearly conditional survival probability.

Labor's share

Given the Cobb-Douglas specification of the production function, α is capital's share of production in the economy while $(1-\alpha)$ is labor's share of production. Saleemi (2007) reports

$$\alpha \approx 0.39$$

for the Norwegian mainland economy over the years 1986-2004. I use this value of α in the thesis.

Population growth rate

η is defined as the rate of growth in the population over a model period. To calibrate η, I first estimate the average yearly population growth in Norway during the years 1970 to 2006 and define this parameter as n. In the data

$$n \approx 0.0051.$$

The following relation between η and n must hold:

$$\sqrt[23]{(1+\eta)} = 1 + n$$
$$\Rightarrow \eta \approx 0.124$$

Labor augmenting technology

g is defined as the rate of growth in the labor augmenting productivity factor over a model period. As I show in the appendix, under the stationary population structure of the model, GDP per capita grows at a constant rate equal to the rate of growth in the labor augmenting technology factor along the balanced growth path. To calibrate g, I calculate the average yearly growth rate in mainland real GDP per capita during the years 1970 to 2005 and define this ratio as ξ. In the data

$$\xi \approx 0.0222.$$

The following relation between g and ξ must then hold:

$$\sqrt[23]{(1+g)} = 1 + \xi$$
$$\Rightarrow g \approx 0.657.$$

Depreciation rate

The law of motion for aggregate capital stock is described by equation (4). Assuming we are along the balanced growth path in period t and given the fact that the aggregate capital stock increases at a rate of $(g + \eta + g\eta)$ along the balanced growth path, (4) can be written as:

$$(1+g)(1+\eta)K_t = X_t + (1-\delta)K_t.$$

This expression is easily manipulated to yield:

$$\delta = \frac{X}{K} - \eta - g - \eta g \,,$$

where time subscript is suppressed. The average yearly fraction of mainland gross fixed capital formation over mainland fixed assets, defined as χ, is calculated to approximate the yearly investment to capital stock ratio during the years 1970 to 2007. In the data

$$\chi \approx 0.0738 \,.$$

Reflecting the fact that capital is a stock variable, and as such its measurement along the detrended balanced growth path is independent of time periods, while investment is a flow variable, and as such its measurement along the detrended balanced growth path is dependent on time periods, the investment to capital stock ratio must be adjusted for time periods. Along the detrended balanced growth path, aggregate investment over a model period is 23 times the aggregate investment over a year. δ is then calibrated as:

$$\delta = 23\chi - \eta - g - \eta g$$
$$\delta \approx 0.835$$

Efficiency units

The efficiency units ε_1 and ε_2 measure the agent's earning power on labor supply during respectively the first and the second period of life. A priori, a reasonable assumption might be that $\varepsilon_1 \leq \varepsilon_2$. However, to take the model to the data properly, it is necessary to find a numerical estimate for these efficiency units. Using data from the Labor Force Survey over the years 2000 to 2006, we obtain[2] an approximation for the ratio $\frac{\varepsilon_2}{\varepsilon_1} \approx 1.14$. I perform the normalization

$$\varepsilon_1 = 1 \,,$$

which then determines

$$\varepsilon_2 \approx 1.14 \,.$$

We can justify this normalization by studying the following expression:

$$\frac{(1-h_2)}{(1-h_1)} = \beta p_1 \frac{\varepsilon_1}{\varepsilon_2}\left(\frac{2R^2 + \theta(1+g)^2}{(1+g)(2R + \theta(1+g))}\right). \tag{NR2}$$

(NR2) is derived in the appendix and discussed further in section 5.4. Here it is important to see that, when determining the relative supply of labor over the lifecycle, the agent cares

[2] I explain the procedure in the appendix.

about the earnings potential on labor supply in period 1 relative to period 2, i.e. the ratio $\frac{\varepsilon_1}{\varepsilon_2}$, not about the values of these parameters separately.

Social security replacement rate

I set $\theta = 0.6$ to match the average replacement ratio of pension benefits to lifetime earnings net of taxes indexed by wage growth for a pensioner with an average salary during his working life. This replacement ratio is found in NOU 2004:1 (2004).

Subjective discount factor and disutility of labor

The remaining parameters, β and ψ, are chosen to target the capital to output ratio and the average working time in the data.

The capital to output ratio in the data is calculated as the average yearly ratio of mainland fixed assets to mainland GDP during the years 1970-2007. This average yearly ratio is approximately equal to 2.92. Since capital is a stock variable, while output is a flow variable, output must be adjusted for the period length in the model. The estimate for the capital to output ratio over a model period is

$$\frac{K}{Y} = \frac{1}{23} 2.92 \approx 0.1269 .$$

The average working week for all employed persons in 2006 in the data was approximately 34.5 hours[3]. Assuming an average person has 15 hours available for work each day, net of sleep and personal care, the person can work $15 * 7 = 105$ hours a week. The fraction of working time to total available time is then

$$\frac{34.5}{105} \approx 0.3286 .$$

It is not possible to derive closed form expressions for the parameters β and ψ as functions of aggregate moments. We pick β and ψ to match the capital to output ratio and hours worked specified above.

$$\beta = 0.964$$

[3] Source: Statistics Norway.

and

$$\psi = 1.997$$

allow us to hit these targets.

Cohort shares

Total population in period t is given by

$$N_t = P_t\left(1 + \frac{p_1}{(1+\eta)} + \frac{p_1 p_2}{(1+\eta)^2}\right).$$

I perform the normalization

$$P_0 = 1 .$$

Total population in period zero is then

$$N_0 = \left(1 + \frac{p_1}{(1+\eta)} + \frac{p_1 p_2}{(1+\eta)^2}\right).$$

The constant cohort shares are given as:

$$\mu_1 = \frac{1}{\left(1 + \frac{p_1}{(1+\eta)} + \frac{p_1 p_2}{(1+\eta)^2}\right)} \approx 0.4648$$

$$\mu_2 = \frac{p_1}{(1+\eta)}\mu_1 \approx 0.3846$$

$$\mu_3 = \frac{p_2}{(1+\eta)}\mu_2 \approx 0.1506$$

Table 3 summarizes the parameter values we have calibrated in this section of the thesis.

Table 3: Calibration results

α	0.39	η	0.124
β	0.964	g	0.657
δ	0.835	μ_1	0.4648
ψ	1.997	μ_2	0.3846
θ	0.6	μ_3	0.1506
ε_1	1	p_1	0.93
ε_2	1.14	p_2	0.44

4.4 Numerical Simulation

4.4.1 The Detrended Balanced Growth Path Equilibrium

As I show in the appendix, along the balanced growth path, H_t grows at a factor of $(1+\eta)$ per period, the aggregate variables Y_t, X_t, K_t, C_t grow at a factor of $(1+g)(1+\eta)$ per period. The per capita variables $b_t, S_t, c_{i,t}$ and $a_{i,t}$ for all $i \in \{1,2,3\}$, and the wage rate w_t grow at a factor of $(1+g)$ per period. To detrend the model, I define $\tilde{H}_t \equiv \frac{H_t}{N_t}$, $\hat{Z}_t \equiv \frac{Z_t}{A_t N_t}$ for the other growing aggregate variables and $\hat{z}_t \equiv \frac{z_t}{A_t}$ for all growing per capita variables and the wage rate. $\tilde{H}_t$, $\hat{Z}_t$ and $\hat{z}_t$ are constant along the balanced growth path.

The detrended competitive equilibrium is defined as the competitive equilibrium in section 4.2; by replacing the growing variables in the definition with the detrended variables. Along the detrended balanced growth path all variables in the model are constant. This implies that the time subscript can be dropped. A period equilibrium is characterized by a set of equations describing:

- The optimal behavior of households.
- The budget constraints of the households.
- The pension system budget balance.
- The optimal behavior of the representative firm.
- The market clearing conditions.

Since all the quantities and prices along the detrended balanced growth path are time invariant; solving for the one period equilibrium implies that we have recovered the whole equilibrium sequence.

Optimality conditions

The households' optimization problem is solved in the appendix. The optimal behavior of the households is described by:

- A set of optimality conditions for labor supply:

$$\psi \frac{1}{(1-h_1)} = \frac{1}{\hat{c}_1}(1-\tau)\hat{w}\varepsilon_1 + \beta^2 p_1 p_2 \frac{1}{\hat{c}_3}\frac{\theta}{2}(1-\tau)\hat{w}\varepsilon_1 \tag{26}$$

$$\psi \frac{1}{(1-h_2)} = \frac{1}{\hat{c}_2}(1-\tau)\hat{w}\varepsilon_2 + \beta p_2 \frac{1}{\hat{c}_3}\frac{\theta}{2}(1-\tau)\hat{w}\varepsilon_2 \qquad (27)$$

The optimality conditions for labor supply are often called the intratemporal optimality conditions in dynamic macroeconomic models. However, in this model, changes in labor supply have a direct intertemporal effect by influencing welfare in the last period of life through pension benefits; this is seen in the second term on the right hand side of equations (26) and (27). The optimality conditions for labor supply equate the marginal disutility of increasing labor supply with the sum of expected discounted marginal utility in the present period and when retired as a result of increased labor supply, i.e. the marginal benefit of increasing labor supply.

- A set of intertemporal optimality conditions for consumption/saving choice:

$$\frac{1}{\hat{c}_1} \geq \beta p_1 \frac{R}{(1+g)}\frac{1}{\hat{c}_2}, \quad \left(\frac{1}{\hat{c}_1} = \beta p_1 \frac{R}{(1+g)}\frac{1}{\hat{c}_2} \text{ if } a_2 > 0\right) \qquad (28)$$

$$\frac{1}{\hat{c}_2} \geq \beta p_2 \frac{R}{(1+g)}\frac{1}{\hat{c}_3}, \quad \left(\frac{1}{\hat{c}_2} = \beta p_2 \frac{R}{(1+g)}\frac{1}{\hat{c}_3} \text{ if } a_3 > 0\right) \qquad (29)$$

When (28)-(29) bind; these intertemporal optimality conditions equate the marginal disutility of giving up one unit of consumption in this period with the excepted discounted increase in next period marginal utility by saving one more unit today, adjusted for technology growth.

In addition, the following restrictions on household allocations must hold:

$$\hat{a}_2 \geq 0 \qquad (30)$$

$$\hat{a}_3 \geq 0 \qquad (31)$$

$$h_1 \in [0,1] \qquad (32)$$

$$h_2 \in [0,1] \qquad (33)$$

$$\hat{c}_1 \geq 0 \qquad (34)$$

$$\hat{c}_2 \geq 0 \qquad (35)$$

$$\hat{c}_3 \geq 0 \qquad (36)$$

Budget constraints

The budget constraints must bind at optimum because the agent has strictly increasing utility in both leisure and consumption. The set of budget constraints is:

$$\hat{c}_1 + (1+g)\hat{a}_2 = R\hat{b} + (1-\tau)\hat{w}\varepsilon_1 h_1 \tag{37}$$

$$\hat{c}_2 + (1+g)\hat{a}_3 = R(\hat{b} + \hat{a}_2) + (1-\tau)\hat{w}\varepsilon_2 h_2 \tag{38}$$

$$\hat{c}_3 = R(\hat{b} + \hat{a}_3) + \hat{S} \tag{39}$$

Social security

Pension benefits along the detrended balanced growth path are given by:

$$\hat{S} = \theta \frac{1}{2}\left((1-\tau)\hat{w}\varepsilon_1 h_1 + (1-\tau)\hat{w}\varepsilon_2 h_2\right) \tag{40}$$

The pension system budget balance along the detrended balanced growth path is given by:

$$\tau = \frac{\mu_3 \hat{S}}{\hat{w}(\mu_1 \varepsilon_1 h_1 + \mu_2 \varepsilon_2 h_2)} \tag{41}$$

Profit maximization

The representative firm's optimization problem is solved in the appendix. The optimal behavior of the representative firm implies that the following relations between factor prices and factor demands hold:

$$R = \alpha \frac{\hat{Y}}{\hat{K}} + 1 - \delta \tag{42}$$

$$\hat{w} = (1-\alpha)\frac{\hat{Y}}{\tilde{H}} \tag{43}$$

Market clearing conditions

The market clearing conditions along the detrended balanced growth path are described by:

- Capital market equilibrium implies:

$$\hat{K} = \mu_1 \hat{b} + \mu_2(\hat{a}_2 + \hat{b}) + \mu_3(\hat{a}_3 + \hat{b}) \tag{44}$$

- Labor market equilibrium implies:

$$\tilde{H} = (\mu_1 \varepsilon_1 h_1 + \mu_2 \varepsilon_2 h_2) \tag{45}$$

- Commodity market equilibrium implies:

$$\hat{C} + \hat{X} = \hat{Y} \tag{46}$$

- The following relations must hold:

$$\hat{C} = \mu_1 \hat{c}_1 + \mu_2 \hat{c}_2 + \mu_3 \hat{c}_3 \tag{47}$$

$$\hat{X} = (g + \eta + g\eta + \delta)\hat{K} \tag{48}$$

$$\hat{Y} = \hat{K}^{\alpha}\tilde{H}^{1-\alpha} \tag{49}$$

$$\hat{b} = \frac{1}{1+\eta}(\mu_1(1-p_1)\hat{a}_2 + \mu_2(1-p_2)\hat{a}_3) \tag{50}$$

4.4.2 Numerical Algorithm

The model's detrended balanced growth path equilibrium is characterized by the equations (26)-(50). To solve for the model's detrended balanced growth path equilibrium I apply the following procedure:

1. Assume that (30)-(36) are slack in equilibrium,
2. The equilibrium is then characterized by the 18 equations (26)-(29) and (37)-(50). The model contains a set of 17 endogenous variables, $\{\hat{c}_1, \hat{c}_2, \hat{c}_3, h_1, h_2, \hat{a}_2, \hat{a}_3, \hat{K}, \hat{X}, \hat{Y}, \hat{C}, H, \tau, \hat{S}, \hat{b}, R, \hat{w}\}$.
3. In order to have a system of 17 equations in the 17 endogenous variables we assume that the aggregate resource constraint, i.e. equation (46), holds for the solution to the system of equations defined by (26)-(29), (37)-(45) and (47)-(50). This system of 17 equations in the 17 endogenous variables is solved by using MATLAB's non-linear equations solver "fsolve" in MATLAB's optimization toolbox for some vector of starting values.
4. Check ex-post that (30)-(36) are in fact slack and that (46) holds for the solution obtained.
5. Check that the solution obtained is robust to various starting vectors.

4.5 Benchmark Results

Table 4: Calibration targets and benchmark results

Variable	Target	Benchmark Model
$\hat{K}/\hat{Y}$	0.1269	0.1269
Average Hours	0.3286	0.3286
$\hat{X}/\hat{K}$	1.6974	1.6975

Table 4 shows that we succeed in hitting the targets for the capital to output ratio, the investment to capital ratio and the average hours for employees that we observe in the data. The average hours for employees is calculated as $\frac{\mu_1 h_1 + \mu_2 h_2}{\mu_1 + \mu_2}$.

Demographics

The cohort shares produce a ratio of workers to pensioners of

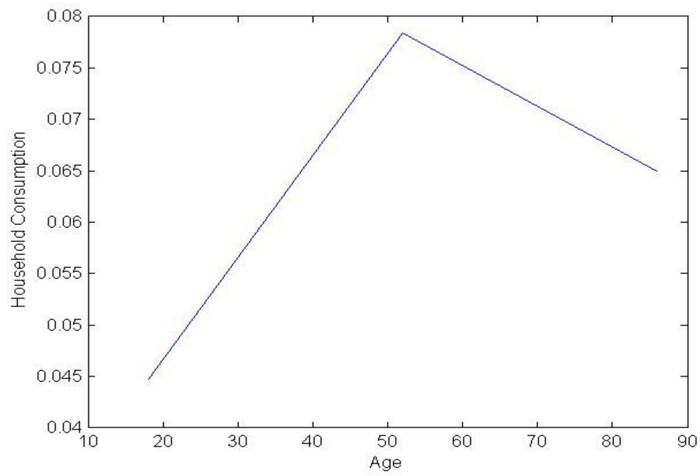
$$\frac{\mu_1 + \mu_2}{\mu_3} \approx 5.64,$$

which is in the neighborhood of 4.6, the actual ratio in the Norwegian data in 2005 (Stensnes et al., 2007), but substantially higher than the expected ratio of 2.7 in 2050 in the baseline scenario of demographic projections for Norway (Stensnes et al., 2007). We see that the model, calibrated to the historical data, cannot capture the expected future demographic situation. However, it seems reasonable to speculate that the model might produce predictions broadly in line with the projections for the future demographic situation by calibrating the model to the expected population growth rate and expected conditional survival probabilities.

4.5.1 Households' lifecycle profiles

Consumption

Figure 3: Households' lifecycle consumption profile



I have not located data on consumption sorted by age groups in the Norwegian data so it is not possible to asses how good the model predictions are under a PAYGo system relative to the data. However, as we can see in figure 3, the model does produce a hump shaped lifecycle consumption profile[4]. This is at least qualitatively consistent with what we observe in e.g. the US data (Fernandez-Villaverde and Krueger, 2007) and might expect to observe also in the Norwegian data.

Hours

Figure 4: Households' lifecycle profile of hours

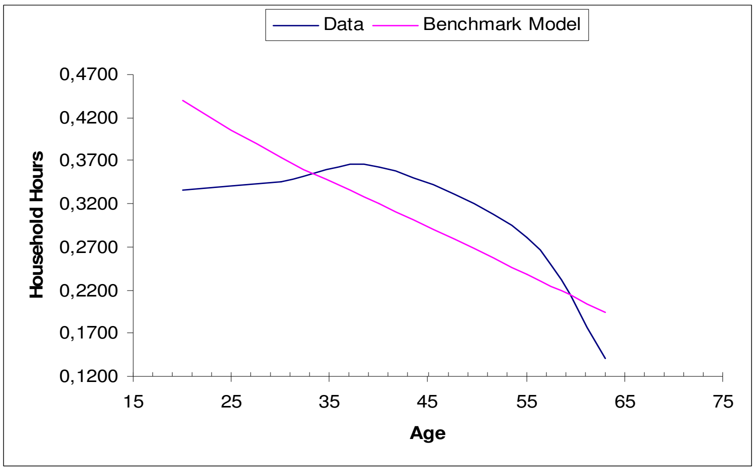


Figure 4 displays the household allocation of hours during the lifecycle as predicted by the model and the hours profile found in the data. As a proxy for the lifecycle hours profile in the data, I use figures published in Vaage (2003) concerning time allocation for men in the age range 20 to 66 in Norway during the year 2000. We see that the benchmark model predicts a downward sloping lifecycle profile of hours, i.e. that agents work less when middle aged than when young. Clearly, we cannot capture the hump shaped hours profile observed in the data when we interpolate linearly between only two observations over the lifecycle in the model, but the model predictions of the hours profile does seem quite good given the model's simplicity.

Sensitivity analysis of the efficiency units

To investigate how sensitive the hours profile is to the calibration of efficiency units, I produce the lifecycle profile of hours under two different calibrations below. Alternative calibration number 1 uses $\varepsilon_1 = 1, \varepsilon_2 = 1.1$. Alternative calibration number 2 uses $\varepsilon_1 = 1, \varepsilon_2 = 1.18$.

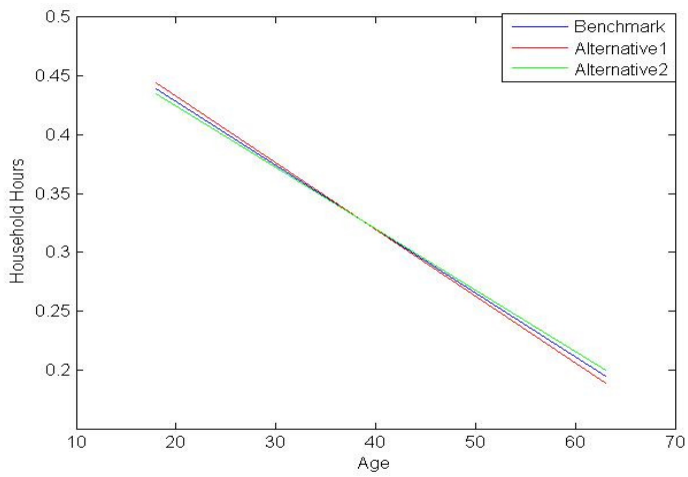
[4] We can observe a hump shaped life cycle consumption profile in OLG models with stochastic lifetimes when the effective discount factor, i.e. the factor that combines the unconditional survival probability with the subjective discount factor, is lower than the market rate of interest.

The disutility factor of labor and the subjective discount factor must be adjusted to hit the average working time and capital to output ratio that we observe in the data when we vary the efficiency units. All other parameters remain as in table 3. Table 5 displays the values of the subjective discount factor and the disutility factor under the two alternative calibrations.

Table 5: Alternative calibrations		
Alternative #	β	ψ
1	0.945	1.973
2	0.984	2.0235

In figure 5, we see that the lifecycle hours profile does not change a lot when we vary the efficiency units moderately. Since the lifecycle hours profile is relatively insensitive to moderate changes in efficiency units and since very large differences in efficiency units are considered unlikely, the rest of the discussion in this thesis refers to the original calibration.

Figure 5: Sensitivity analysis of the lifecycle hours profile



5 The Main Counterfactual Pension Reform: A Fully Funded Pension System

Chapter 5 describes and analyzes the model economy under a counterfactual pension system and compares the numerical results to the results obtained under the unfunded pension system in chapter 4. The counterfactual pension system in this chapter is of a fully funded, defined contribution and actuarial type. The focus of the analysis will be to study the differences in household allocations and the quantitative differences in the capital stock, output, aggregate hours and welfare along the detrended balanced growth path.

To quantity the differences in welfare of an agent born into the steady state of the two pension systems I follow the path often taken in the quantitative macro literature of computing the Consumption Equivalent Variation (CEV). The CEV is defined as the factor, κ, that consumption in all possible states of the model under the PAYGo system must be multiplied with in order to make the expected lifecycle utility of a newborn agent under the PAYGo system equal to the expected lifecycle utility of a newborn agent under the fully funded system, when we hold labor supply constant. I define expected lifecycle utility under the PAYGo system as EU^{PG} and expected lifecycle utility under the fully funded system as EU_{GE}^{FF}. In the appendix, I show that

$$\kappa = \exp\left\{ \frac{EU_{GE}^{FF} - EU^{PG}}{\left(1 + \beta p_1 + \beta^2 p_1 p_2\right)} \right\}.$$

The remainder of chapter 5 is structured in the following way:

- Section 5.1 modifies the previous model by replacing the unfunded pension system with a fully funded pension system.
- Section 5.2 defines the competitive equilibrium along the balanced growth path in this model.
- Section 5.3 describes how we simulate the model numerically.
- Section 5.4 discusses the long run results of the counterfactual pension reform.

5.1 Environment

The demographic environment, technology and the optimization program for the representative firm are the same as under a PAYGo system and are therefore not repeated here.

Households' problem

The households' optimization problem is identical to the households' optimization problem under the PAYGo system, except with regard to the definition of social security benefits, which I discuss below.

Social security

The pension system is a pure fully funded system and is run at zero cost. An individual, who is retired in period s, receives pension benefits S_s calculated as the return on payroll taxes paid in to the social security system over the life cycle:

$$S_s = \begin{pmatrix} R_{s-1} R_s \tau_{s-2} w_{s-2} \varepsilon_1 h_{1,s-2} \\ + R_s \tau_{s-1} w_{s-1} \varepsilon_2 h_{2,s-1} \end{pmatrix}. \tag{51}$$

Lump sum transfers

Since agents face stochastic lifetime, agents may leave unintended bequests when they die before the third life period. Furthermore, agents who die before the third period of life leave behind accumulated pension benefits. All accidental bequests and accumulated pension benefits of agents who die before retirement are immediately distributed as lump sum transfers to agents alive at no cost. In every period s, the following relation must then hold:

$$b_s = \frac{1}{1+\eta} \begin{pmatrix} \mu_1 (1 - p_1)(a_{2,s} + \tau_{s-1} w_{s-1} \varepsilon_1 h_{1,s-1}) \\ + \mu_2 (1 - p_2) \left(a_{3,s} + \begin{pmatrix} R_{s-1} \tau_{s-2} w_{s-2} \varepsilon_1 h_{1,s-2} \\ + \tau_{s-1} w_{s-1} \varepsilon_2 h_{2,s-1} \end{pmatrix} \right) \end{pmatrix}. \tag{52}$$

5.2 Competitive equilibrium

The sequential competitive equilibrium for the economy under the fully funded pension system is defined analogously to under the PAYGo system. The equilibrium consists of sequences of the social security tax rates and benefits $\{\tau_t, S_t\}_{t=0}^{\infty}$, lump sum transfers $\{b_t\}_{t=0}^{\infty}$, household allocations $\{c_{1,t}, c_{2,t+1}, c_{3,t+2}, a_{2,t+1}, a_{3,t+2}, h_{1,t}, h_{2,t+1}\}_{t=0}^{\infty}$, factor demands for the representative firm $\{K_t, H_t\}_{t=0}^{\infty}$ and factor prices $\{w_t, R_t\}_{t=0}^{\infty}$ such that:

1. Given factor prices, payroll tax rates and lump sum transfers, the household allocations solve the individuals' optimization problem.
2. Given factor prices, the factor demands solve the representative firm's optimization problem.
3. Factor prices are such that all markets clear, i.e. such that aggregate and individual behavior is consistent. Point 3 implies that equations (21)-(23), (2) and (4) must hold also under this model. The market clearing condition for capital market equilibrium must be modified to incorporate the funding of pension benefits. Equation (20) is then replaced by:

$$K_t = N_t \begin{pmatrix} \mu_1 b_t + \mu_2 \left(a_{2,t} + b_t + \tau_{t-1} w_{t-1} \varepsilon_1 h_{1,t-1} \right) \\ + \mu_3 \left(a_{3,t} + b_t + \begin{pmatrix} R_{t-1} \tau_{t-2} w_{t-2} \varepsilon_1 h_{1,t-2} \\ + \tau_{t-1} w_{t-1} \varepsilon_2 h_{2,t-1} \end{pmatrix} \right) \end{pmatrix} \tag{53}$$

4. The pension benefits paid out in each period to the retired agents equal their accumulated life cycle contribution to the pension system with interest. I.e. pension benefits are defined by equation (51).
5. Lump sum transfers equal accidental bequests and accumulated pension benefits of agents who die before retirement. I.e. lump sum transfers are defined by equation (52).

5.3 Numerical Simulation

5.3.1 The Detrended Balanced Growth Path Equilibrium

Given the model specification above, variables along the balanced growth path grow as under the benchmark model. Therefore, per capita and aggregate variables are detrended in exactly the same way as under the benchmark model.

The detrended competitive equilibrium is defined as the competitive equilibrium in section 5.2; we just replace the growing variables in the definition with the detrended variables. Along the detrended balanced growth path all detrended aggregate and per capita variables, in addition to the interest factor and per capita hours, are constant so time subscript can be dropped. As under the PAYGo system, the period equilibrium is characterized by a set of equations describing:

- The optimal behavior of households.
- The budget constraints of the households.
- The pension benefits.
- The optimal behavior of the representative firm.
- The market clearing conditions.

Since all quantities and prices along the detrended growth path are time invariant, solving for the one period equilibrium implies that we have recovered the whole equilibrium sequence.

Households' optimality conditions

The optimization problem for the consumer is solved in the appendix. The optimal behavior of households is described by:

- A set of optimality conditions for labor supply:

$$\psi \frac{1}{(1-h_1)} = \frac{1}{\hat{c}_1}(1-\tau)\hat{w}\varepsilon_1 + \beta^2 p_1 p_2 \frac{1}{\hat{c}_3} R^2 \tau \frac{\hat{w}}{(1+g)^2}\varepsilon_1 \qquad (54)$$

$$\psi \frac{1}{(1-h_2)} = \frac{1}{\hat{c}_2}(1-\tau)\hat{w}\varepsilon_2 + \beta p_2 \frac{1}{\hat{c}_3} R \tau \frac{\hat{w}}{(1+g)}\varepsilon_2 \qquad (55)$$

  The interpretations of the optimality conditions for labor supply are analogous to the interpretations of these conditions in the model under the unfunded pension system. The differences in the second term on the right hand side reflect the differences in calculation of pension benefits under the two pension systems.

- A set of intertemporal optimality conditions for consumption and saving choice:

$$\frac{1}{\hat{c}_1} \geq \beta p_1 \frac{R}{(1+g)}\frac{1}{\hat{c}_2}, \quad \left(\frac{1}{\hat{c}_1} = \beta p_1 \frac{R}{(1+g)}\frac{1}{\hat{c}_2} \text{ if } a_2 > 0\right) \qquad (56)$$

$$\frac{1}{\hat{c}_2} \geq \beta p_2 \frac{R}{(1+g)}\frac{1}{\hat{c}_3}, \quad \left(\frac{1}{\hat{c}_2} = \beta p_2 \frac{R}{(1+g)}\frac{1}{\hat{c}_3} \text{ if } a_3 > 0\right) \qquad (57)$$

  The interpretation of these optimality conditions is identical to under the unfunded system.

Budget constraints

The budget constraints must bind at optimum because the agents have strictly increasing utility in both leisure and consumption. The set of budget constraints is identical to under the benchmark model, i.e. we require equations (37)-(39) to hold also in this equilibrium.

Social Security

The pension benefits along the detrended balanced growth path are defined by:

$$\hat{S} = \left(R^2 \tau \frac{\hat{w}}{(1+g)^2} \varepsilon_1 h_1 + R\tau \frac{\hat{w}}{(1+g)} \varepsilon_2 h_2 \right) \tag{58}$$

Profit maximization

As under the benchmark model; the optimal behavior of the representative firm implies that equations (42)-(43) hold in equilibrium.

Market clearing conditions

The market clearing conditions are described by equations (45)-(49). In addition we require that the capital market clearing condition hold and that the lump sum transfers are adjusted to contain also the accumulated pension benefits of agents who die before retirement.

Capital market equilibrium along the detrended balanced growth path implies:

$$\hat{K} = \left(\begin{aligned} &\mu_1 \hat{b} + \mu_2 \left(\hat{a}_2 + \hat{b} + \tau \frac{\hat{w}}{(1+g)} \varepsilon_1 h_1 \right) \\ &+ \mu_3 \left(\hat{a}_3 + \hat{b} + R\tau \frac{\hat{w}}{(1+g)^2} \varepsilon_1 h_1 + \tau \frac{\hat{w}}{(1+g)} \varepsilon_2 h_2 \right) \end{aligned} \right) \tag{59}$$

Lump sum transfers along the detrended balanced growth path are described by:

$$\hat{b} = \frac{1}{1+\eta} \left(\begin{aligned} &\mu_1 (1-p_1) \left(a_2 + \tau \frac{\hat{w}}{(1+g)} \varepsilon_1 h_1 \right) \\ &+ \mu_2 (1-p_2) \left(a_3 + \left(R\tau \frac{\hat{w}}{(1+g)^2} \varepsilon_1 h_1 + \tau \frac{\hat{w}}{(1+g)} \varepsilon_2 h_2 \right) \right) \end{aligned} \right) \tag{60}$$

5.3.2 Numerical Algorithm

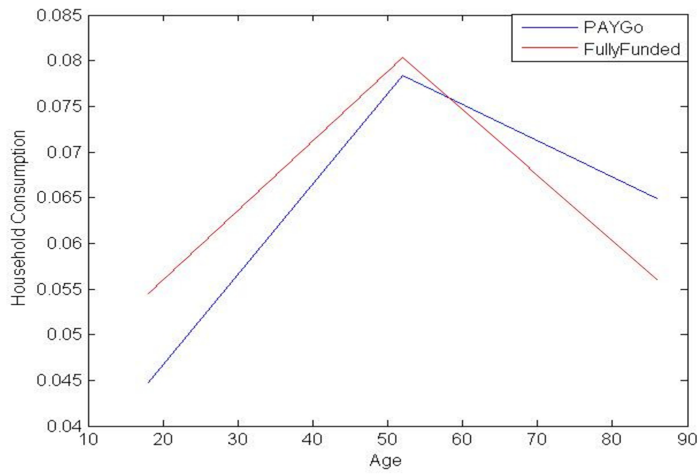
I simulate numerically for the detrended balanced growth path under the fully funded pension system using the same general algorithm as under the PAYGo system, i.e. by solving the system of equations defined by equations (37)-(39), (42)-(43), (45), (47)-(49) and (54)-(60) for the set of endogenous variables, $\{\hat{c}_1, \hat{c}_2, \hat{c}_3, h_1, h_2, \hat{a}_2, \hat{a}_3, \hat{K}, \hat{X}, \hat{Y}, \hat{C}, H, \hat{S}, \hat{b}, R, \hat{w}\}$. This system of nonlinear equations gives 16 equations in 16 endogenous variables. We check ex-post that (30)-(36) are in fact slack for the solution obtained and that the aggregate resource constraint holds, i.e. that (46) is satisfied, for the solution obtained.

5.4 Long Run Effects of the Pension Reform

5.4.1 Households' lifecycle profiles

Consumption

Figure 6: Households' lifecycle consumption profile



As we can see in figure 6, the model produces a hump shaped life cycle consumption profile also under the fully funded pensions system. However, the difference between the lifecycle consumption profiles under the two pension systems is significant. The households prioritize consumption earlier in life relative to later in life under the fully funded system. To see why the households frontload consumption over the life-cycle under the fully funded model, I rearrange the optimality conditions for consumption/savings decisions in the following way:

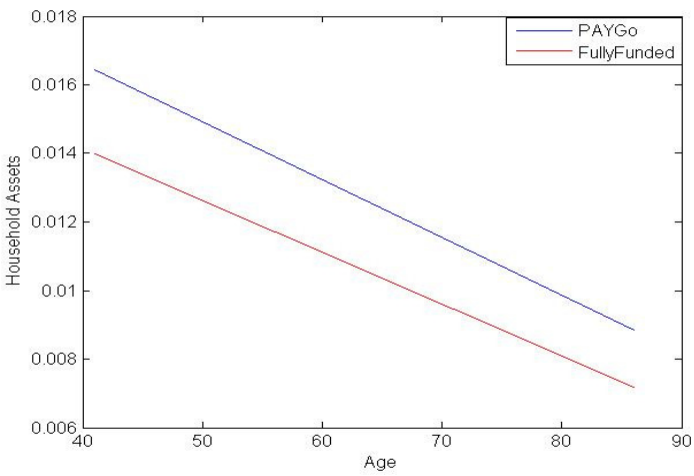
$$\frac{\hat{c}_{i+1}}{\hat{c}_i} = \beta p_i \frac{R}{(1+g)}, i \in \{1,2\}.$$

As we see in table 6, the interest factor is smaller under the fully funded system than under the PAYGo system. This means that the ratio $\frac{\hat{c}_{i+1}}{\hat{c}_i}$ must be smaller under the fully funded system for $i = 1,2$. The economic motivation is clear. Given the fact that the agent faces a smaller interest factor under the fully funded system, the gains from saving today in terms of future consumption will be smaller. Hence, the agent will choose to save less and consume more, in relative terms, today under the fully funded pension system.

Intended asset holdings

Figure 7 displays the households' intended asset holdings of an agent under the two pension system. We see that a household chooses to save less for all ages under the fully funded system compared to under the PAYGo system. This is consistent with our discussion concerning the lifecycle profile of consumption, i.e. that the consumer would like to save relatively less and consume relatively more when facing a lower return on savings.

Figure 7: Households' lifecycle intended asset holdings profile



Hours

In figure 8 we see that, compared to under the PAYGo system, the hours profile under the fully funded system is flatter. To understand why the households choose a flatter hours profile over the lifecycle under the fully funded system, we can use the households' optimality conditions to derive relations between the lifecycle hours profile and the interest rate. This is straightforward for the fully funded system, where the expression is:

$$\beta p_1 \frac{\varepsilon_1}{\varepsilon_2} \frac{R}{(1+g)} = \frac{(1-h_2)}{(1-h_1)}. \tag{NR1}$$

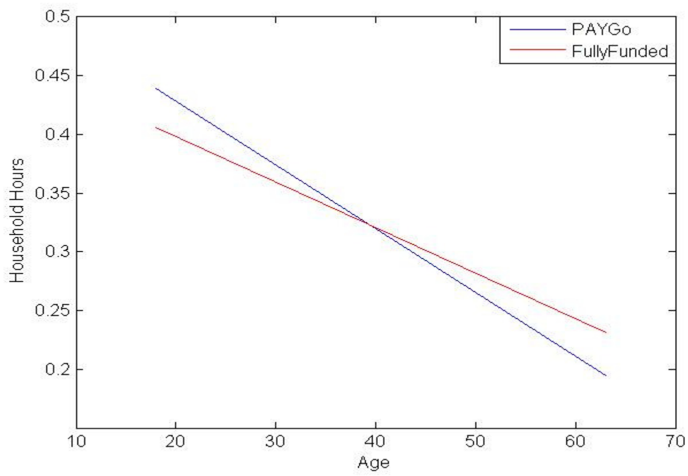
(NR1) is derived in the appendix. The economic intuition behind (NR1) is clear. A larger interest factor makes it more attractive to work relatively more early in life because the return on saving is better. The agent's relative supply of labor also depends on the ratio of the efficiency units. The agent's supply of labor when middle aged relative to when he is young will increase if the agent's earnings power when he is middle aged relative to when he is young increases, ceteris paribus. A similar expression under the PAYGo system is also derived in the appendix. The expression is:

$$\frac{(1-h_2)}{(1-h_1)} = \beta p_1 \frac{\varepsilon_1}{\varepsilon_2} \left(\frac{2R^2 + \theta(1+g)^2}{(1+g)(2R + \theta(1+g))} \right). \tag{NR2}$$

In (NR2) we see that households take into consideration the implicit return on the pension system in addition to the market rate of interest.

Given the fact that the agent faces a smaller interest factor under the fully funded system, he will choose to supply less labor to the market when he is young relative to when he is middle aged because the return on saving is lower.

Figure 8: Households' lifecycle profile of hours



5.4.2 Aggregate variables

Table 6 displays the numerical results for obtained under this counterfactual pension reform, together with the numerical results obtained under the PAYGo system and the relative difference between the steady state variables under these models.

Table 6: Comparing aggregate variables under the two pension systems

Variable	PAYGo	Fully funded	Relative difference[5]
τ	0.0935	0.0935	0 %
$\hat{b}$	0.0022	0.0037	68.2 %
$\hat{S}$	0.0293	0.0263	-10.2 %
$\hat{Y}$	0.0774	0.0872	12.7 %
$\hat{K}$	0.0098	0.0133	35.7 %
$\hat{C}$	0.0607	0.0646	6.4 %
$\hat{X}$	0.0167	0.0225	34.7 %
$\hat{K}/\hat{Y}$	0.1269	0.1523	20.0 %
$\tilde{H}$	0.2896	0.2904	0.3 %
R	3.2380	2.7260	-15.8 %
$\hat{w}$	0.1630	0.1831	12.3 %
Welfare	-7.9738	-7.7794	8.9 %

Capital stock

The per capita aggregate capital stock is 35.7 % higher under the fully funded system than under the PAYGo system. As we have seen above, intended asset holdings of the households are lower for all ages under the fully funded system so the large increase the in capital stock is driven by the funding of pension benefits.

Aggregate Hours

We have seen above that the household allocations of hours are lower when the agents are young and higher when the agents are middle aged under the fully funded system than under the PAYGo system. When these household allocations are aggregated by cohort shares and adjusted for efficiency weights, we see that the net effect on per capita aggregate hours is a 0.3 % increase under the fully funded system relative to under the PAYGo system.

[5] The relative difference between aggregate variables under the two systems is calculated as $\left(\frac{Z^{FF}}{Z^{PG}}-1\right)*100$, where Z^{FF} and Z^{PG} denote the steady state values of a variable under respectively the Fully Funded and the PAYGo pension system. The welfare under the two pension systems is calculated as the expected discounted lifecycle utility of a newborn agent under the different pension systems. The relative difference in welfare is calculated as the CEV, which is explained in the appendix.

Output

With aggregate hours and the aggregate capital stock both higher under the fully funded system than under the PAYGo system, it is no surprise that output is higher under the fully funded system. Since output is 12.7 % higher under the fully funded system, while aggregate hours are only marginally higher under the fully funded system, it is also clear that the main driver of the increased output is the big increase in the per capita aggregate capital stock under the fully funded system.

Capital to output ratio

We have seen that the increase in the capital stock is significantly larger than the increase in output under the fully funded model compared to under the PAYGo model. Therefore it is natural that the capital to output ratio is higher under the fully funded model than under the PAYGo model. The capital to output ratio is approximately 20 % higher along the detrended balanced growth path under the fully funded pension system than under the PAYGo pension system.

Welfare

We see in table 6 that consumption in all three potential periods of life under the PAYGo system must be increased by approximately 8.9 % in order to make a newborn agent indifferent between the expected lifecycle utility under the two pension systems along their respective balanced growth path. This means that the introduction of a fully funded pension system will have a large positive effect on the welfare of agents born into the new steady state of the economy.

Pension benefits

The pension benefits are 10.2 % lower under the fully funded system than under the PAYGo system due to the general equilibrium effect of a lower interest rate under the fully funded system. This can be seen by using the hours profile obtained under the fully funded model and the factor prices obtained under the benchmark model in equation (58), which would have given pension benefits of a value of 0.0315.

Tax rate

The payroll tax rate is determined endogenously as 9.35 % under the PAYGo system in order to balance the pension budget when the replacement rate of pension benefits to average lifecycle wage earnings net of taxes is calibrated to the Norwegian data. The same payroll tax rate is taken as exogenous under the fully funded system.

Lump sum transfers

We see that the lump sum transfers are 68.2 % higher under the fully funded system than under the PAYGo system, even though the intended asset holdings are lower for all ages under the fully funded system. We see that the inclusion of accumulated pension funds of agents who die before retirement in the lump sum transfers dominates the effect of lower unintended bequests under the fully funded system relative to under the PAYGo system.

Gross investments

Given the large increase in aggregate capital stock, it is no surprise that the gross investments increase by approximately the same percentage (34.7 %), when we look at equation (48).

Aggregate consumption

Per capita aggregate consumption is 6.4 % higher under the fully funded system than under the PAYGo system. This means that the effects of higher consumption for the young and middle aged agents dominates the effect of lower consumption for the retired agents when household allocations are aggregated by cohort shares.

Factor prices

We are studying a closed economy, general equilibrium model where factor prices clear markets. Factor prices are given by:

$$R = \alpha \frac{\hat{Y}}{\hat{K}} - \delta + 1$$

$$\hat{w} = (1-\alpha)\frac{\hat{Y}}{\tilde{H}}$$

As we have seen above, the ratio $\frac{\hat{Y}}{\hat{K}}$ is smaller while the ratio $\frac{\hat{Y}}{\tilde{H}}$ must clearly be larger under the fully funded pension system than under the PAYGo system. In this way, the market prices for the production factors reflect the relative scarcity of the factors under different pension systems.

Given the fact that the Norwegian economy is a small open economy, it might be reasonable to assume that the interest rate is, at least largely, independent of aggregate domestic saving. Therefore, there are good reasons to question the relevance of these results when considering pension reform in Norway. This issue will be considered in chapter 7.

6 An Alternative Counterfactual Pension Reform: Eliminating the Pension System

Several studies (Imrohoroglu et al., 1995 and 1999B) have found that the replacement rate which maximizes the expected lifecycle utility of an agent born into the steady state of an economy with an unfunded pension system is equal to zero. This chapter researches how our models measure up to this benchmark in the literature.

I set $\theta = 0$ in the system of equations (26)-(50) and simulate numerically for the steady state in exactly the same way as in section 4.4. This obviously determines the social security payroll tax rate and pension benefits as zero. The capital to output ratio and the aggregate efficient hours obtained under this numerical experiment are shown in table 7 in comparison to the results obtained under the fully funded system. The lifecycle consumption profile obtained under this numerical experiment is compared to the two previous models in figure 9. We show the analogous results for the lifecycle hours profile and intended asset holdings in respectively figure 10 and 11. Because the lifecycle profile of hours and consumption is identical under the fully funded pension system and when the public pension system is removed we suppress the results for the fully funded system in figure 9 and 10.

Table 7: Aggregate variables

Variable	Fully Funded	Eliminate Pension System
$\hat{K}/\hat{Y}$	0.1523	0.1523
$\tilde{H}$	0.2904	0.2904

The household allocations for consumption and hours, in addition to capital to output ratio and aggregate hours, are in fact identical to under the scenario with a fully funded pension system. This result is due to the fact that the households are indifferent between doing their saving for retirement directly through the private market or through the government run pension system when facing the same rate of return on their contribution.

When facing a positive social security payroll tax rate in a fully funded actuarial pension system, the agent will reduce individual saving for retirement on a one to one basis against the forced saving done through the government run pension system. Hence, the taxation will not entail a welfare loss for the agent as long as the value of the payroll taxes paid is not higher than what the agent would maximally choose to save for retirement in an economy without a pension system. In this sense, it is irrelevant whether we compare the results on welfare obtained under the PAYGo pension system with a positive replacement rate to an economy without a government run pension system or to an economy with a government run fully funded pension system, as long as the payroll tax rate in the fully funded system is not "too large".

Figure 9: Households' lifecycle consumption profile

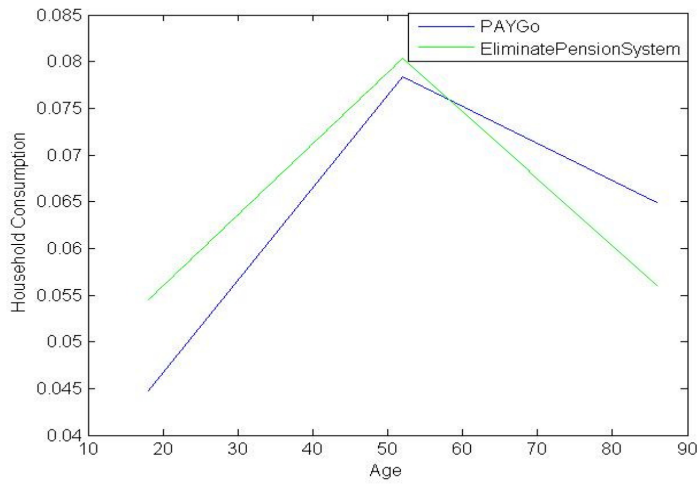


Figure 10: Households' lifecycle hours profile

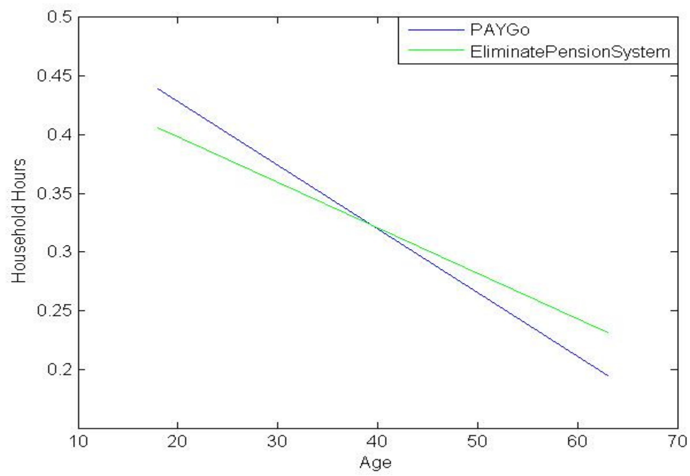
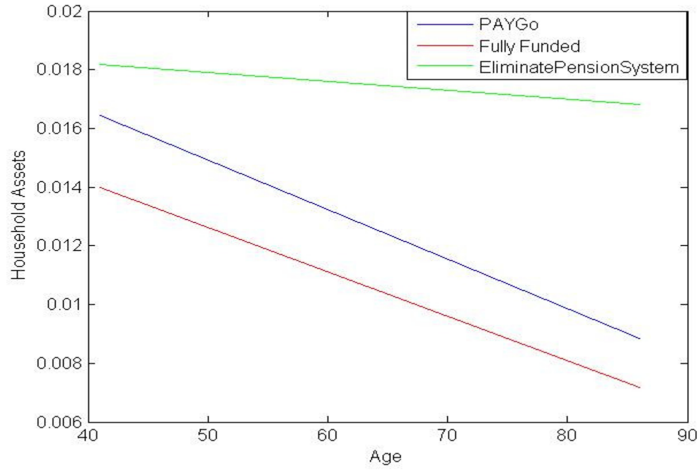


Figure 11: Households' lifecycle intended asset holdings profile



7 A Partial Equilibrium Experiment

Since Norway is a small, open economy, while the results so far have been obtained under the assumption of a closed economy, this chapter researches how sensitive the results are to this assumption. In order to perform such an inquiry, I construct a partial equilibrium model where the production sector of the economy is abstracted from. The pension system in the model is identical to the fully funded pension system outlined in chapter 5. I assume that, for the market clearing factor and commodity prices calculated under the benchmark model, the supply of the macro good and the factor demands are perfectly elastic also at the aggregate level. Domestic quantities of the macro good consumed, aggregate hours and aggregate saving is then de facto determined by the households' demand for the macro good and supply of factors respectively at these prices. This is represented graphically in figure 12, 13 and 14.

Figure 12: Illustrating the commodity market

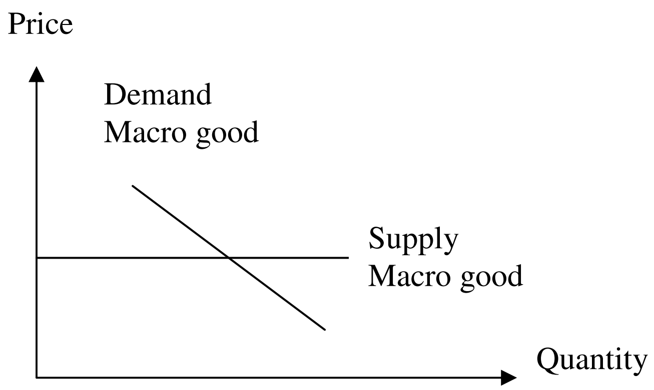


Figure 13: Illustrating the market for labor

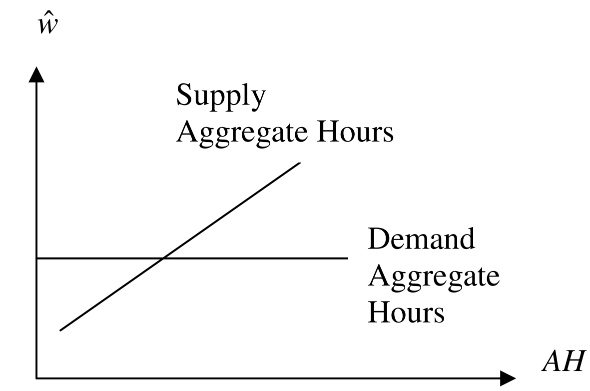
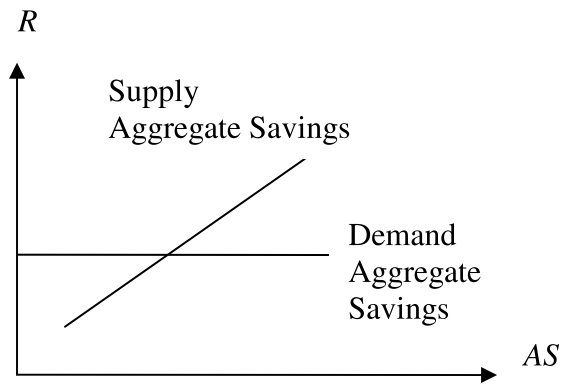


Figure 14: Illustrating the market for capital



The remainder of chapter 7 is structured in the following way:

- Section 7.1 outlines the partial equilibrium model with a fully funded pension system.
- Section 7.2 defines the competitive partial equilibrium in this model.
- Section 7.3 discusses the numerical algorithm used to simulate the model.
- Section 7.4 discusses the numerical results in relation to the results obtained in the benchmark model and the main counterfactual model, i.e. the model in chapter 5.

7.1 Environment

Demography

I assume the same demographic environment as in the previous model specifications.

Households' problem

The households' optimization problem is assumed to be identical to the households' optimization problem under the detrended fully funded general equilibrium, as it is found in the appendix. This can be justified by assuming that the wage rate is growing exogenously at rate g per period, such that household allocations of consumption and intended asset holdings grow at the same rate per period, and that we are studying a detrended version of this partial equilibrium. In this way, all relevant[6] variables are directly comparable across the general and partial equilibrium models.

[6] I.e. those variables which appear in all the models.

Social security

The pension benefits are defined as the pension benefits in the detrended fully funded general equilibrium model. I.e. pension benefits are defined by equation (58).

Lump sum transfers

As in the general equilibrium model; all accidental bequests and accumulated pension benefits of agents who die before retirement are immediately distributed as lump sum transfers to agents alive at no cost. The definition of the lump sum transfers under this partial equilibrium model is identical to the definition under the detrended fully funded general equilibrium model. I.e. they are defined by equation (60).

7.2 Competitive Partial Equilibrium

The competitive partial equilibrium is defined by a sequence of household allocations $\{\hat{c}_{1,t}, \hat{c}_{2,t+1}, \hat{a}_{2,t+1}, \hat{a}_{3,t+2}, h_{1,t}, h_{2,t+1}\}_{t=0}^{\infty}$, sequences of social security tax rates and benefits $\{\tau_t, \hat{S}_t\}_{t=0}^{\infty}$ and lump sum transfers $\{\hat{b}_t\}_{t=0}^{\infty}$, such that:

1. Given the constant factor and commodity prices, tax rates and lump sum transfers, the household allocations solve the households' optimization program.
2. The pension benefits are defined by equation (58).
3. Lump sum transfers are defined by equation (60).

All aggregate and per capita variables are constant so time subscript can be dropped. The competitive equilibrium is then characterized by a set of equations describing:

- The optimal behavior of households.
- The budget constraints of the households.
- The pension benefits.
- Lump sum transfers equal accidental bequests and accumulated pension benefits of agents who die before retirement.

Households' optimality conditions

The optimization program for the households is, under the model specification above, identical to the optimization program of the households under the general equilibrium model with the fully funded pension system. The optimal behavior of households is then described by:

- A set of optimality conditions for labor supply, i.e. equations (54)-(55).
- A set of intertemporal optimality conditions for consumption and saving choice, i.e. equations (56)-(57).

Budget constraints

The set of households' budget constraints is identical to under the previous models, therefore we require that (37)-(39) hold in equilibrium.

Social security

The pension system is identical to under the fully funded general equilibrium model and is described by equation (58).

Lump sum transfers

The lump sum transfers are defined as under the fully funded general equilibrium model and are described by equation (60).

Other relations

The restrictions on household allocations defined by equations (30)-(36) must hold also under this model.

In addition, I define the two following relations:

Aggregate saving:

$$AS = \begin{pmatrix} \mu_1 \hat{b} + \mu_2 \left(\hat{a}_2 + \hat{b} + \tau \frac{\hat{w}}{(1+g)} \varepsilon_1 h_1 \right) \\ + \mu_3 \left(\hat{a}_3 + \hat{b} + R\tau \frac{\hat{w}}{(1+g)^2} \varepsilon_1 h_1 + \tau \frac{\hat{w}}{(1+g)} \varepsilon_2 h_2 \right) \end{pmatrix} \tag{61}$$

Aggregate hours:

$$AH = (\mu_1 \varepsilon_1 h_1 + \mu_2 \varepsilon_2 h_2) \tag{62}$$

7.3 Numerical Algorithm

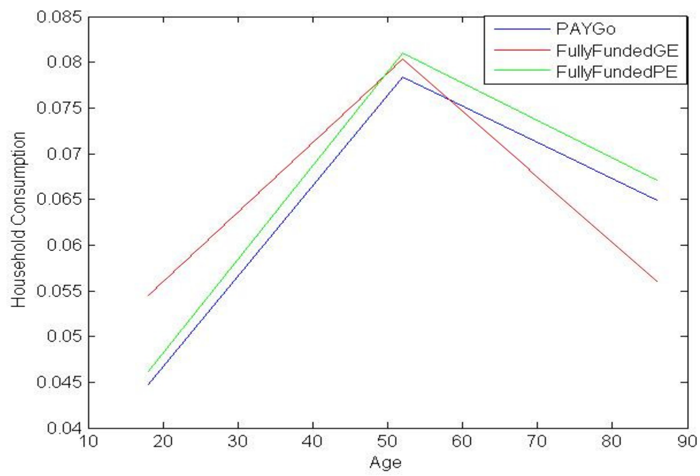
To solve the model, I use the same (relevant) parameter values as was calibrated earlier. I use the factor prices that were calculated numerically in the detrended balanced growth path under the benchmark model in chapter 4. The price of the macro good is equal to unity as in the previous numerical experiments and the social security tax rate is set equal to the rate obtained under the benchmark model, which was also used in the general equilibrium model of the closed economy under a fully funded pension system. The numerical solution to this model is found as the simultaneous solution to the system of non-linear equations defined by equations (37)-(39), (54)-(58) and (60)-(62), after which we check ex-post that (30)-(36) are satisfied. The system of equations contains 11 equations in a set of 11 endogenous variables, $\{\hat{c}_1, \hat{c}_2, \hat{c}_3, h_1, h_2, \hat{a}_2, \hat{a}_3, AS, AH, \hat{S}, \hat{b}\}$.

7.4 Numerical Results

7.4.1 Households' lifecycle profiles

Consumption and intended asset holdings

Figure 15[7]: Households' lifecycle consumption profile



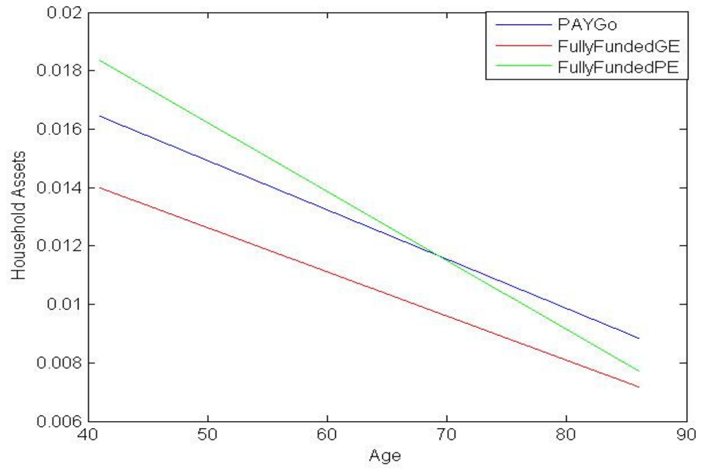
As I show in figure 15, the life cycle consumption profile has the same shape under the partial equilibrium fully funded model as under the benchmark model because the interest factor is identical under the two models. This is seen by once again rearranging the optimality conditions for consumption/savings choice as

[7] GE denotes the general equilibrium results; PE denotes the partial equilibrium results.

$$\frac{\hat{c}_{i+1}}{\hat{c}_i} = \beta p_i \frac{R}{(1+g)}, i \in \{1,2\}.$$

Since the interest factor is identical under the benchmark and partial equilibrium models, the ratio $\frac{\hat{c}_{i+1}}{\hat{c}_i}$ for $i \in \{1,2\}$ must also be identical under these two models. We see that households prioritize consumption earlier in life under the fully funded general equilibrium model because they face a lower rate of return on their savings than under the benchmark model and the partial equilibrium model. This reduced incentive to save under the fully funded general equilibrium model is seen also in figure 16, which displays the intended asset holdings over the lifecycle for the various models. The profiles for both the benchmark model and the fully funded partial equilibrium model lie above the profile for the fully funded general equilibrium model for all periods of life.

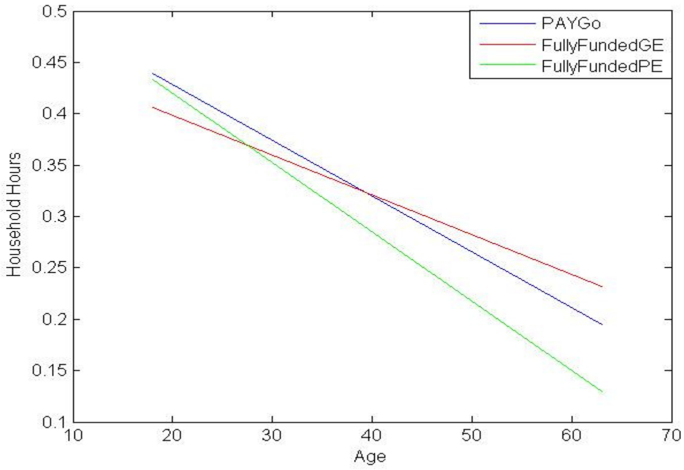
Figure 16: Households' lifecycle intended asset holdings profile



Hours

Figure 17 displays the lifecycle profile of hours for the three models. Under the partial equilibrium model, the households' lifecycle profile of labor supply is much steeper than under the previous models. Looking at (NR 1) and (NR 2), we see that since the households face a higher return on saving under the partial equilibrium fully funded model, they will choose to work more early in life relative to later in life.

Figure 17: Households' lifecycle hours profile



7.4.2 Aggregate variables

Table 8 displays the relative difference of certain macro variables under the two fully funded models compared to the results obtained under the benchmark model.

Table 8: Relative difference to the benchmark scenario

Variable	Fully Funded GE	Fully Funded PE
Welfare[8]	8.9 %	10.9 %
AH	0.3 %	-10.8 %
AS	35.7 %	53.1 %
$\hat{S}$	-10.2 %	1.0 %
$\hat{b}$	68.2 %	77.3 %
τ	0 %	0 %
$\hat{C}$	6.4 %	3.3 %

The differences between the results under the general and partial equilibrium models are due to the general equilibrium effects of changing factor prices. Four results are particularly interesting:

1. The quantitative increase in welfare is two percentage points larger under the partial equilibrium model than under the general equilibrium model.

[8] The quantitative increase in welfare is measured by the Consumption Equivalence Variation (CEV), as in chapter 5.

2. The relatively large (10.8 %) reduction in aggregate hours under the partial equilibrium model versus the minor (0.3 %) increase obtained under the general equilibrium fully funded model.
3. The quantitative increase in aggregate saving is approximately 17 percentage points larger under the partial equilibrium model than under the general equilibrium fully funded model.
4. The pension benefits are 1 % larger under the partial equilibrium model while they are 10.2 % smaller under the fully funded general equilibrium model.

Welfare

The quantitative increase in welfare for an agent born into the new steady state after the pension reform, as measured by the Consumption Equivalence Variation, is actually 2 percentage points larger under the partial equilibrium model than under the fully funded general equilibrium model. This is a somewhat surprising result, but means that holding factor prices constant, as in a small economy such as Norway, might actually serve to strengthen the case for pension reform when we consider the argument of welfare improvements for generations born after the new pension system has been phased in.

Aggregate hours

We have seen that the general equilibrium effect of a lower interest factor under the general equilibrium fully funded model makes the lifecycle profile of hours flatter, where the reduction in hours when the households are young is countered by an increase in hours when they are middle aged and most efficient. The net effect is a 0.3 % increase in per capita aggregate hours under the general equilibrium fully funded model.

Looking at the households' hours profiles under the different models in figure 17, we see that household hours are lower for all periods under the partial equilibrium model with a fully funded model than under the benchmark model with the PAYGo pension system. However, the difference between the hours that households supply to the labor market is larger when the agents are middle aged and therefore most efficient, than when the agents are young. We have seen above that the reason for agents choosing to work relatively less when they are middle aged under the partial equilibrium model is due to the fact that the return on saving is higher under that model. The net effect on per capita aggregate hours is a 10.8 % decrease under the partial equilibrium model.

Aggregate saving

The increase in pr capita aggregate saving is stronger under the partial equilibrium model than under the fully funded general equilibrium model. The increase under the partial equilibrium model is 53.1 %, while it is 35.7 % higher under the general equilibrium model.

As was discussed in chapter 5, the increase in per capita aggregate capital stock under the fully funded general equilibrium model relative to under the benchmark model is driven by pension funding, while the lifecycle profile of intended asset holdings under the fully funded general equilibrium model lies below the profile in the benchmark model for all ages.

We see in figure 16 that under the fully funded partial equilibrium model, the households' lifecycle profile of intended asset holdings lies above the profile under the PAYGo model when the agents are young and lies below the profile under the PAYGo model when the agents are middle aged. Define $\hat{a}_i^{FFPE}$ as the intended asset holdings of an age i individual in the fully funded partial equilibrium model. Define $\hat{a}_i^{PG}$ analogously along the detrended balanced growth path under the PAYGo general equilibrium model. Because[9] $\left|\hat{a}_2^{FFPE} - \hat{a}_2^{PG}\right| > \left|\hat{a}_3^{FFPE} - \hat{a}_3^{PG}\right|$, it is clear that the increase in aggregate saving is driven by both individual saving and pension funding under the fully funded partial equilibrium model.

Pension benefits

Figure 17 shows that the household hours profile under the partial equilibrium model lies below the profile under the benchmark model for all ages. Furthermore, the wage rate and interest factor under these two models is identical. That the pension benefits are 1 % larger under the partial equilibrium model than under the benchmark model means that the market rate of interest is higher than the implicit return on the PAYGo pension system when we hold factor prices constant.

7.4.3 Summary of Results

The results we have obtained in this chapter are consistent with the premise of holding factor prices constant at the levels obtained under the benchmark model. Relative to under the fully funded general equilibrium model, the households prioritize consuming less and saving more

[9] The numerical values of these household allocations are found in the appendix.

early in life and working more early in life under the benchmark and fully funded partial equilibrium model because they face a higher interest rate under these two models than under the fully funded general equilibrium model.

At the aggregate level we have seen that the pension benefits under the partial equilibrium fully funded model are higher than the benefits in the other models due to the higher return on the pension system in this model.

We have also seen that the long run increase in aggregate saving is stronger under the partial equilibrium model than under the fully funded general equilibrium model. The difference between the two models is due to the higher return on savings in the partial equilibrium model.

Although, given the higher rate of return on saving, we would expect to see a steeper lifecycle profile of hours under the partial equilibrium model than under the other models, the decrease in aggregate hours under the partial equilibrium model is certainly very strong and should be interpreted with caution. Institutional arrangements in the labor market might serve to weaken the possibility for such a large reduction in labor supply.

We have also seen that the welfare of an agent born into the different steady states increases considerable under both the partial equilibrium and general equilibrium fully funded models relative to the benchmark model. That welfare in their respective steady states is higher under the fully funded pension systems than under unfunded pension system is consistent with other research in the literature, though it is interesting to see that the quantitative increase in welfare might actually be stronger in an environment where factor prices are kept constant.

8 Conclusion

In this thesis we have concentrated on researching the long run macroeconomic and welfare effects of introducing a fully funded pension system in a model economy reproducing certain features of the Norwegian economy. We have seen that the introduction of such a pension system would have large long run positive effects on the nation's capital stock and the welfare of agents born into the new long run equilibrium in comparison to a scenario with a continued unfunded pension system. We have also seen that there are variations in how a fully funded pension system would affect aggregate variables and households' allocations, depending on whether we assume a general equilibrium closed economy or an economic environment with fixed factor and good prices.

The increase in per capita aggregate savings in the long run equilibrium has been shown to be robust to whether prices are fixed or determined in general equilibrium because it is to a large extent driven by the process of funding pension benefits.

Likewise, the increase in households' welfare in long run equilibrium is influenced by whether factor prices are fixed or determined in domestic factor markets, but the quantitative increase in expected lifecycle welfare is considerable under both alternatives. We also saw that, in the alternative with fixed prices, the quantitative increase in welfare was actually somewhat larger than in the general equilibrium model.

Relative to under the unfunded system, there were only marginal changes in aggregate labor supply in the fully funded general equilibrium model, but a relatively large reduction in aggregate labor supply in the partial equilibrium model. That aggregate labor supply does not unequivocally increase in the long run under a fully funded system compared to under an unfunded system is not particularly surprising. The unfunded system we studied was of a quasi-actuarially fair type, which implies that households would respond to relatively moderate changes in the rate of return on the pension system and the interest rate when determining the lifecycle profile of labor supply. There is no reason to assume, a priori, that these changes would necessarily have a net positive impact on households' labor supply. However, the decrease in aggregate hours under the fully funded partial equilibrium model is certainly very strong and should be interpreted with caution.

It is important to note that the models studied in this thesis only captures changes in households' labor supply during their existing work life. Changes in aggregate labor supply as a result of how the pension system influences households' retirement decisions cannot be captured in a model framework with an exogenous retirement decision.

Furthermore, the model framework used in this thesis does not capture the effects of a pension reform on intra-cohort heterogeneity. This concept figures prominently in the public debate and any consideration of an actual pension reform would have to take into account the effects on intra-cohort heterogeneity. However, this issue can be incorporated in OLG-models and is discussed in papers such as Conesa & Krueger (1999). It would require relatively minor modifications of the model used in this thesis to incorporate the problem of intra-cohort heterogeneity, although the computational cost of solving the model numerically would be higher.

We have not solved for the transition path between the long rung equilibriums in this model. Study of the transition path between long run steady states is important because transitional agents might be worse off as a result of pension reforms that are highly beneficial for the population in the long run. In light of earlier research in the literature we cannot expect the introduction of a fully funded pension system to be Pareto improving. However, from a political economy perspective, it would be interesting to see if it is possible to make a majority of the transitional agents better off by switching to a fully funded system in a model economy calibrated to Norwegian data.

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Appendix

Detrending

To show that aggregate variables, except hours, increase by $(1+g)(1+\eta)$ along the balanced growth path, begin by noting that along the balanced growth path household hours are constant. This implies that

$$\frac{H_{t+1}}{H_t} = \frac{N_{t+1}(\mu_1\varepsilon_1 h_{1,t+1} + \mu_2\varepsilon_2 h_{2,t+1})}{N_t(\mu_1\varepsilon_1 h_{1,t} + \mu_2\varepsilon_2 h_{2,t})}$$

$$\frac{H_{t+1}}{H_t} = \frac{N_{t+1}}{N_t}$$

$$\frac{H_{t+1}}{H_t} = (1+\eta)$$

I.e. aggregate efficient hours increase by the rate of population growth.

Given the fact that the labor augmenting technology grows at a rate of g, this implies that:

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha}}{K_t^{\alpha}}\left(\frac{A_{t+1}}{A_t}\frac{H_{t+1}}{H_t}\right)^{1-\alpha}$$

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha}}{K_t^{\alpha}}((1+g)(1+\eta))^{1-\alpha}. \tag{A1}$$

Define the growth factor of production and the aggregate capital stock as respectively:

$$\frac{Y_{t+1}}{Y_t} = G_Y$$

$$\frac{K_{t+1}}{K_t} = G_K.$$

We know that $\frac{K}{Y}$ is constant along the balanced growth path, which implies that

$$G_Y = G_K = G. \tag{A2}$$

Substituting (A2) into (A1) yields

$$G = G^{\alpha}((1+g)(1+\eta))^{1-\alpha}$$

$$G = (1+g)(1+\eta) \tag{A3}$$

I.e. aggregate capital stock and production grow at a rate of $(\eta + g + \eta g)$.

From (4) we know that

$$X_t = K_{t+1} - (1-\delta)K_t .$$

Along the balanced growth path, this can be written as

$$X_t = (\eta + g + \eta g)K_t ,$$

which implies that

$$\frac{X_{t+1}}{X_t} = \frac{(\eta + g + \eta g)K_{t+1}}{(\eta + g + \eta g)K_t}$$

$$\frac{X_{t+1}}{X_t} = G . \tag{A4}$$

Substituting (A3) into (A4), yields

$$\frac{X_{t+1}}{X_t} = (1+g)(1+\eta)$$

I.e. aggregate gross investment increases at a rate of $(\eta + g + \eta g)$.

To show that the wage rate increases at a rate of g along the balanced growth path, define:

$$w_{t+1} = \frac{Y_{t+1}}{H_{t+1}} \tag{A5}$$

$$w_t = \frac{Y_t}{H_t} \tag{A6}$$

Dividing (A5) by (A6) yields

$$\frac{w_{t+1}}{w_t} = \frac{Y_{t+1}}{H_{t+1}} \frac{H_t}{Y_t}$$

$$\frac{w_{t+1}}{w_t} = (1+g) . \tag{A7}$$

I.e. the wage rate grows at a rate of g .

Given a wage rate growing at a constant rate of g and that household hours remain constant along the balanced growth path, we would expect to see all other household allocations growing at a constant rate of g . This again implies that the lump sum transfers, b_t , and pension benefits, S_t , grow at a rate of g per period.

Finally, define GDP per capita as

$$\frac{Y_t}{N_t} = \frac{K_t^{\alpha}\left(A_t H_t\right)^{1-\alpha}}{N_t}.$$

The growth in GDP per capita is then

$$\frac{Y_{t+1}}{N_{t+1}}\frac{N_t}{Y_t} = \frac{K_{t+1}^{\alpha}\left(A_{t+1} H_{t+1}\right)^{1-\alpha}}{N_{t+1}} \frac{N_t}{K_t^{\alpha}\left(A_t H_t\right)^{1-\alpha}}$$

$$\frac{Y_{t+1}}{N_{t+1}}\frac{N_t}{Y_t} = \frac{(1+\eta)(1+g)}{(1+\eta)}$$

$$\frac{Y_{t+1}}{N_{t+1}}\frac{N_t}{Y_t} = (1+g)$$

I.e. the growth rate in GDP per capita along the balanced growth path is equal to g .

Firm's Optimization Problem

The representative firm's optimization problem, with detrended variables, is given by

$$\max_{\hat{K}_t, \hat{H}_t} \left\{ \hat{K}_t^{\alpha} \hat{H}_t^{1-\alpha} - \hat{w}_t \hat{H}_t - (R_t - 1 + \delta)\hat{K}_t \right\}$$

First order conditions for maximum:

$$\alpha\left(\frac{\hat{H}_t}{\hat{K}_t}\right)^{1-\alpha} - (R_t - 1 + \delta) = 0 \qquad \text{(A8)}$$

$$(1-\alpha)\left(\frac{\hat{K}_t}{\hat{H}_t}\right)^{\alpha} - \hat{w}_t = 0 \qquad \text{(A9)}$$

(A8) implies

$$R_t = \alpha\left(\frac{\hat{H}_t}{\hat{K}_t}\right)^{1-\alpha} - \delta + 1$$

(A9) implies

$$\hat{w}_t = (1-\alpha)\left(\frac{\hat{K}_t}{\hat{H}_t}\right)^{\alpha}$$

Households' Detrended Optimization Problem under the Benchmark Model

The consumer maximizes expected lifecycle utility

$$\max_{\left\{\substack{c_{1,t},c_{2,t+1},c_{3,t+2},h_{1,t},\\ h_{2,t+1},a_{2,t+1},a_{3,t+2}}\right\}} \left\{ \begin{array}{l} \log \hat{c}_{1,t} + \psi \log(1-h_{1,t}) \\ + \beta p_1 \left(\log \hat{c}_{2,t+1} + \psi \log(1-h_{2,t+1})\right) + \beta^2 p_1 p_2 \log \hat{c}_{3,t+2} \end{array} \right\} \quad \text{(A10)}$$

Subject to a set of budget constraints

$$\hat{c}_{1,t} + (1+g)\hat{a}_{2,t+1} = R_t \hat{b}_t + (1-\tau_t)\hat{w}_t \varepsilon_1 h_{1,t} \quad \text{(A11)}$$

$$\hat{c}_{2,t+1} + (1+g)\hat{a}_{3,t+2} = R_{t+1}\left(\hat{b}_{t+1} + \hat{a}_{2,t+1}\right) + (1-\tau_{t+1})\hat{w}_{t+1}\varepsilon_2 h_{2,t+1} \quad \text{(A12)}$$

$$\hat{c}_{3,t+2} = R_{t+2}\left(\hat{b}_{t+2} + \hat{a}_{3,t+2}\right) + \hat{S}_{t+2}. \quad \text{(A14)}$$

In addition, households take into account the future pension benefits when deciding allocations

$$\hat{S}_{t+2} = \frac{\theta}{2}\left((1-\tau_t)\hat{w}_t \varepsilon_1 h_{1,t} + (1-\tau_{t+1})\hat{w}_{t+1}\varepsilon_2 h_{2,t+1}\right). \quad \text{(A15)}$$

Furthermore, the restrictions on household allocations described by (30)-(36) must hold.

Assuming (30)-(36) are slack at optimum, the maximization problem described by (A10)-(A15) is solved by substituting (A15) for $\hat{S}_{t+2}$ in (A14) and forming a Lagrangian for the reduced problem:

$$L = \left(\begin{array}{l} \log \hat{c}_{1,t} + \psi \log(1-h_{1,t}) + \beta p_1 \left(\log \hat{c}_{2,t+1} + \psi \log(1-h_{2,t+1})\right) \\ + \beta^2 p_1 p_2 \log \hat{c}_{3,t+2} \\ - \lambda_t \left(\hat{c}_{1,t} + (1+g)\hat{a}_{2,t+1} - R_t \hat{b}_t - (1-\tau_t)\hat{w}_t \varepsilon_1 h_{1,t}\right) \\ - \lambda_{t+1}\left(\hat{c}_{2,t+1} + (1+g)\hat{a}_{3,t+2} - R_{t+1}\left(\hat{b}_{t+1} + \hat{a}_{2,t+1}\right) - (1-\tau_{t+1})\hat{w}_{t+1}\varepsilon_2 h_{2,t+1}\right) \\ - \lambda_{t+2}\left(\hat{c}_{3,t+2} - R_{t+2}\left(\hat{b}_{t+2} + \hat{a}_{3,t+2}\right) - \frac{\theta}{2}\left((1-\tau_t)\hat{w}_t \varepsilon_1 h_{1,t} + (1-\tau_{t+1})\hat{w}_{t+1}\varepsilon_2 h_{2,t+1}\right)\right) \end{array} \right)$$

The first order conditions for maximum are

$$\frac{\partial L}{\partial \hat{c}_{1,t}} = \frac{1}{\hat{c}_{1,t}} - \lambda_t = 0 \quad \text{(A16)}$$

$$\frac{\partial L}{\partial \hat{c}_{2,t+1}} = \beta p_1 \frac{1}{\hat{c}_{2,t+1}} - \lambda_{t+1} = 0 \quad \text{(A17)}$$

$$\frac{\partial L}{\partial \hat{c}_{3,t+2}} = \beta^2 p_1 p_2 \frac{1}{\hat{c}_{3,t+2}} - \lambda_{t+2} = 0 \quad \text{(A18)}$$

$$\frac{\partial L}{\partial h_{1,t}} = -\psi \frac{1}{\left(1-h_{1,t}\right)} + \lambda_t \left(1-\tau_t\right)\hat{w}_t \varepsilon_1 + \lambda_{t+2} \frac{\theta}{2}\left(1-\tau_t\right)\hat{w}_t \varepsilon_1 = 0 \tag{A19}$$

$$\frac{\partial L}{\partial h_{2,t+1}} = -\beta p_1 \psi \frac{1}{\left(1-h_{2,t+1}\right)} + \lambda_{t+1} \left(1-\tau_{t+1}\right)\hat{w}_{t+1} \varepsilon_2 + \lambda_{t+2} \frac{\theta}{2}\left(1-\tau_{t+1}\right)\hat{w}_{t+1} \varepsilon_2 = 0 \tag{A20}$$

$$\frac{\partial L}{\partial \hat{a}_{2,t+1}} = -\lambda_t \left(1+g\right) + \lambda_{t+1} R_{t+1} = 0 \tag{A21}$$

$$\frac{\partial L}{\partial \hat{a}_{3,t+2}} = -\lambda_{t+1} \left(1+g\right) + \lambda_{t+2} R_{t+2} = 0. \tag{A22}$$

By combining (A16), (A17) and (A21), and by combining (A17), (A18) and (A22) the following intertemporal optimality conditions are obtained:

$$\frac{1}{\hat{c}_{1,t}} = \beta p_1 \frac{R_{t+1}}{\left(1+g\right)} \frac{1}{\hat{c}_{2,t+1}} \tag{A23}$$

$$\frac{1}{\hat{c}_{2,t+1}} = \beta p_2 \frac{R_{t+2}}{\left(1+g\right)} \frac{1}{\hat{c}_{3,t+2}} \tag{A24}$$

By combining (A16), (A18) and (A19), and by combining (A17), (A18) and (A20) we obtain the following optimality conditions for labor supply:

$$\psi \frac{1}{\left(1-h_{1,t}\right)} = \frac{1}{\hat{c}_{1,t}} \left(1-\tau_t\right)\hat{w}_t \varepsilon_1 + \beta^2 p_1 p_2 \frac{1}{\hat{c}_{3,t+2}} \frac{\theta}{2}\left(1-\tau_t\right)\hat{w}_t \varepsilon_1 \tag{A25}$$

$$\psi \frac{1}{\left(1-h_{2,t+1}\right)} = \frac{1}{\hat{c}_{2,t+1}} \left(1-\tau_{t+1}\right)\hat{w}_{t+1} \varepsilon_2 + \beta p_2 \frac{1}{\hat{c}_{3,t+2}} \frac{\theta}{2}\left(1-\tau_{t+1}\right)\hat{w}_{t+1} \varepsilon_2 . \tag{A26}$$

Households' Detrended Optimization Problem under the Main Counterfactual Model

The consumer maximizes expected lifecycle utility

$$\max_{\left\{\substack{c_{1,t}, c_{2,t+1}, c_{3,t+2}, h_{1,t}, \\ h_{2,t+1}, a_{2,t+1}, a_{3,t+2}}\right\}} \left\{ \begin{array}{l} \log \hat{c}_{1,t} + \psi \log\left(1-h_{1,t}\right) \\ + \beta p_1 \left(\log \hat{c}_{2,t+1} + \psi \log\left(1-h_{2,t+1}\right)\right) + \beta^2 p_1 p_2 \log \hat{c}_{3,t+2} \end{array} \right\} \tag{A27}$$

Subject to a set of budget constraints

$$\hat{c}_{1,t} + \left(1+g\right)\hat{a}_{2,t+1} = R_t \hat{b}_t + \left(1-\tau_t\right)\hat{w}_t \varepsilon_1 h_{1,t} \tag{A28}$$

$$\hat{c}_{2,t+1} + \left(1+g\right)\hat{a}_{3,t+2} = R_{t+1}\left(\hat{b}_{t+1} + \hat{a}_{2,t+1}\right) + \left(1-\tau_{t+1}\right)\hat{w}_{t+1} \varepsilon_2 h_{2,t+1} \tag{A29}$$

$$\hat{c}_{3,t+2} = R_{t+2}\left(\hat{b}_{t+2} + \hat{a}_{3,t+2}\right) + \hat{S}_{t+2} \tag{A30}$$

In addition, households take into account the future pension benefits when deciding allocations

$$\hat{S}_{t+2} = \left(R_{t+1} R_{t+2} \tau_t \frac{\hat{w}_t}{(1+g)^2} \varepsilon_1 h_{1,t} + R_{t+2} \tau_{t+1} \frac{\hat{w}_{t+1}}{(1+g)} \varepsilon_2 h_{2,t+1} \right) \tag{A31}$$

Furthermore, the restrictions on household allocations described by (30)-(36) must hold.

Assuming (30)-(36) are slack at optimum, the maximization problem described by (A27)-(A31) is solved by substituting (A31) for $\hat{S}_{t+2}$ in (A30) and forming a Lagrangian for the reduced problem:

$$L = \left(\begin{array}{l} \log \hat{c}_{1,t} + \psi \log(1 - h_{1,t}) + \beta p_1 (\log \hat{c}_{2,t+1} + \psi \log(1 - h_{2,t+1})) \\ + \beta^2 p_1 p_2 \log \hat{c}_{3,t+2} \\ - \lambda_t \left(\hat{c}_{1,t} + (1+g) \hat{a}_{2,t+1} - R_t \hat{b}_t - (1-\tau_t) \hat{w}_t \varepsilon_1 h_{1,t} \right) \\ - \lambda_{t+1} \left(\hat{c}_{2,t+1} + (1+g) \hat{a}_{3,t+2} - R_{t+1} \left(\hat{b}_{t+1} + \hat{a}_{2,t+1} \right) - (1-\tau_{t+1}) \hat{w}_{t+1} \varepsilon_2 h_{2,t+1} \right) \\ - \lambda_{t+2} \left(\begin{array}{l} \hat{c}_{3,t+2} - R_{t+2} \left(\hat{b}_{t+2} + \hat{a}_{3,t+2} \right) \\ - \left(R_{t+1} R_{t+2} \tau_t \frac{\hat{w}_t}{(1+g)^2} \varepsilon_1 h_{1,t} + R_{t+2} \tau_{t+1} \frac{\hat{w}_{t+1}}{(1+g)} \varepsilon_2 h_{2,t+1} \right) \end{array} \right) \end{array} \right)$$

The first order conditions for maximum are:

$$\frac{\partial L}{\partial \hat{c}_{1,t}} = \frac{1}{\hat{c}_{1,t}} - \lambda_t = 0 \tag{A32}$$

$$\frac{\partial L}{\partial \hat{c}_{2,t+1}} = \beta p_1 \frac{1}{\hat{c}_{2,t+1}} - \lambda_{t+1} = 0 \tag{A33}$$

$$\frac{\partial L}{\partial \hat{c}_{3,t+2}} = \beta^2 p_1 p_2 \frac{1}{\hat{c}_{3,t+2}} - \lambda_{t+2} = 0 \tag{A34}$$

$$\frac{\partial L}{\partial h_{1,t}} = \left(\begin{array}{l} -\psi \frac{1}{(1-h_{1,t})} + \lambda_t (1-\tau_t) \hat{w}_t \varepsilon_1 \\ + \lambda_{t+2} R_{t+1} R_{t+2} \tau_t \frac{\hat{w}_t}{(1+g)^2} \varepsilon_1 \end{array} \right) = 0 \tag{A35}$$

$$\frac{\partial L}{\partial h_{2,t+1}} = \left(\begin{array}{l} -\beta p_1 \psi \frac{1}{(1-h_{2,t+1})} + \lambda_{t+1} (1-\tau_{t+1}) \hat{w}_{t+1} \varepsilon_2 \\ + \lambda_{t+2} R_{t+2} \tau_{t+1} \frac{\hat{w}_{t+1}}{(1+g)} \varepsilon_2 \end{array} \right) = 0 \tag{A36}$$

$$\frac{\partial L}{\partial \hat{a}_{2,t+1}} = -\lambda_t (1+g) + \lambda_{t+1} R_{t+1} = 0 \tag{A37}$$

$$\frac{\partial L}{\partial \hat{a}_{3,t+2}} = -\lambda_{t+1} (1+g) + \lambda_{t+2} R_{t+2} = 0 \tag{A38}$$

By combining (A32), (A33) and (A37), and by combining (A33), (A34) and (A38) the following intertemporal optimality conditions are obtained:

$$\frac{1}{\hat{c}_{1,t}} = \beta p_1 \frac{R_{t+1}}{(1+g)} \frac{1}{\hat{c}_{2,t+1}} \tag{A39}$$

$$\frac{1}{\hat{c}_{2,t+1}} = \beta p_2 \frac{R_{t+2}}{(1+g)} \frac{1}{\hat{c}_{3,t+2}} \tag{A40}$$

By combining (A32), (A34) and (A35), and by combining (A33), (A34) and (A36) we obtain the following optimality conditions for labor supply:

$$\psi \frac{1}{(1-h_{1,t})} = \frac{1}{\hat{c}_{1,t}} (1-\tau_t) \hat{w}_t \varepsilon_1 + \beta^2 p_1 p_2 \frac{1}{\hat{c}_{3,t+2}} R_{t+1} R_{t+2} \tau_t \frac{\hat{w}_t}{(1+g)^2} \varepsilon_1 \tag{A41}$$

$$\psi \frac{1}{(1-h_{2,t+1})} = \frac{1}{\hat{c}_{2,t+1}} (1-\tau_{t+1}) \hat{w}_{t+1} \varepsilon_2 + \beta p_2 \frac{1}{\hat{c}_{3,t+2}} R_{t+2} \tau_{t+1} \frac{\hat{w}_{t+1}}{(1+g)} \varepsilon_2 \tag{A42}$$

Deriving (NR1)

Substituting (A40) for $\frac{1}{\hat{c}_{2,t+1}}$ in (A42) and solving for $\psi \hat{c}_{3,t+2}$ yields

$$\psi \hat{c}_{3,t+2} = \beta p_2 \frac{R_{t+2}}{(1+g)} \hat{w}_{t+1} \varepsilon_2 (1-h_{2,t+1}) \tag{A47}$$

Substituting (A40) for $\frac{1}{\hat{c}_{2,t+1}}$ in (A39) yields

$$\frac{1}{\hat{c}_{1,t}} = \beta^2 p_1 p_2 \frac{R_{t+1} R_{t+2}}{(1+g)^2} \frac{1}{\hat{c}_{3,t+2}} \tag{A48}$$

Substituting (A48) for $\frac{1}{\hat{c}_{1,t}}$ in (A41) and solving for $\psi \hat{c}_{3,t+2}$ yields:

$$\psi \hat{c}_{3,t+2} = \beta^2 p_1 p_2 \frac{R_{t+1} R_{t+2}}{(1+g)^2} \varepsilon_1 (1-h_{1,t}) \tag{A49}$$

Setting (A47) equal to (A49), dropping time-subscripts along the balanced growth path and simplifying yields:

$$\frac{(1-h_2)}{(1-h_1)} = \beta p_1 \frac{\varepsilon_1}{\varepsilon_2} \frac{R}{(1+g)} \tag{NR1}$$

Deriving (NR2)

Substituting (A24) for $\frac{1}{\hat{c}_{2,t+1}}$ in (A26) and solving for $\psi \hat{c}_{3,t+2}$ yields

$$\psi \hat{c}_{3,t+2} = \beta p_2 (1-\tau_{t+1}) \hat{w}_{t+1} \varepsilon_2 \left(\frac{R_{t+2}}{(1+g)} + \frac{\theta}{2} \right) (1-h_{2,t+1}) \tag{A43}$$

Substituting (A24) for $\frac{1}{\hat{c}_{2,t+1}}$ in (A23) yields

$$\frac{1}{\hat{c}_{1,t}} = \beta^2 p_1 p_2 \frac{R_{t+1} R_{t+2}}{(1+g)^2} \frac{1}{\hat{c}_{3,t+2}} \tag{A44}$$

Substituting (A44) for $\frac{1}{\hat{c}_{1,t}}$ in (A25) and solving for $\psi \hat{c}_{3,t+2}$ yields:

$$\psi \hat{c}_{3,t+2} = \beta^2 p_1 p_2 (1-\tau_t) \hat{w}_t \varepsilon_1 \left(\frac{R_{t+1} R_{t+2}}{(1+g)^2} + \frac{\theta}{2} \right) (1-h_{1,t}) \tag{A45}$$

Setting (A43) equal to (A45), dropping time-subscripts along the balanced growth path and simplifying yields:

$$\frac{(1-h_2)}{(1-h_1)} = \beta p_1 \frac{\varepsilon_1}{\varepsilon_2} \frac{\left(\frac{R^2}{(1+g)^2} + \frac{\theta}{2} \right)}{\left(\frac{R}{(1+g)} + \frac{\theta}{2} \right)}$$

$$\Rightarrow \frac{(1-h_2)}{(1-h_1)} = \beta p_1 \frac{\varepsilon_1}{\varepsilon_2} \left(\frac{2R^2 + \theta(1+g)^2}{(1+g)(2R + \theta(1+g))} \right) \tag{NR2}$$

Deriving the consumption equivalent variation

Define $\hat{c}_i^{FF}$ as the consumption which maximized expected utility in the detrended balanced growth path under the fully funded pension system for an age $i \in \{1,2,3\}$ individual. h_j^{FF} is defined as the supply of labor which maximizes expected utility in the detrended balanced growth path under the fully funded system for an age $j \in \{1,2\}$ individual. $\hat{c}_i^{PG}$ and h_j^{PG} are defined analogously under the PAYGo pension system.

Expected utility for a newborn agent in the steady state under the PAYGo system is:

$$EU^{PG} = \begin{pmatrix} \log \hat{c}_1^{PG} + \psi \log\left(1 - h_1^{PG}\right) \\ + \beta p_1 \left(\log \hat{c}_2^{PG} + \psi \log\left(1 - h_2^{PG}\right)\right) + \beta^2 p_1 p_2 \log \hat{c}_3^{PG} \end{pmatrix}$$

Expected utility for a newborn agent in the steady state under the fully funded system is:

$$EU_{GE}^{FF} = \begin{pmatrix} \log \hat{c}_1^{FF} + \psi \log\left(1 - h_1^{FF}\right) \\ + \beta p_1 \left(\log \hat{c}_2^{FF} + \psi \log\left(1 - h_2^{FF}\right)\right) + \beta^2 p_1 p_2 \log \hat{c}_3^{FF} \end{pmatrix}$$

We wish to find the factor κ such that:

$$\log \hat{c}_1^{FF} + \psi \log\left(1 - h_1^{FF}\right) + \beta p_1 \left(\log \hat{c}_2^{FF} + \psi \log\left(1 - h_2^{FF}\right)\right) + \beta^2 p_1 p_2 \log \hat{c}_3^{FF}$$

$$=$$

$$\log \kappa\hat{c}_1^{PG} + \psi \log\left(1 - h_1^{PG}\right) + \beta p_1 \left(\log \kappa\hat{c}_2^{PG} + \psi \log\left(1 - h_2^{PG}\right)\right) + \beta^2 p_1 p_2 \log \kappa\hat{c}$$

Using elementary rules for natural logarithms we see that this implies

$$EU_{GE}^{FF} = \left(1 + \beta p_1 + \beta^2 p_1 p_2\right) \log \kappa + EU^{PG}$$

Solving for κ yields

$$\kappa = \exp\left\{ \frac{EU_{GE}^{FF} - EU^{PG}}{\left(1 + \beta p_1 + \beta^2 p_1 p_2\right)} \right\}.$$

Calibrating Efficiency Units

To calculate numerical estimates of the ratio of the efficiency units, I use data from the AKU-survey over the years 2001-2006. The data show the number of persons in the labor force and the average monthly full time equivalent earnings sorted by the age intervals shown in table 9.

Table 9: Age Intervals in the AKU-survey
-24 years
25-29 years
30-34 years
35-39 years
40-44 years
45-49 years
50-54 years
55-59 years
60 years or older

To fit the intervals shown in table 9 to the model periods in table 2, I assume, as a simplification, that the number of workers under 18 and over 63 is negligible. Furthermore, I assume that the numbers of workers within the interval $[40,44]$ years in the data are evenly distributed on each year in the interval. I.e. that the number of workers of e.g. age 40 equals the total number of workers in the interval $[40,44]$ years divided by five.

For each year we have data (i.e. 2001-2006), I calculate the average monthly full time equivalent earnings for the model periods in table 2, weighted by the number of workers in each age group. This gives the average earnings of an age 1 and age 2 individual. I then divide the average earnings of an age 2 individual with the average earnings of an age 1 individual. Finally, I calculate the average ratio of the average earnings of an age 2 individual to the average earnings of an age 1 individual over the years 2001-2006. This average ratio is the numerical estimate of $\frac{\varepsilon_2}{\varepsilon_1}$. In the data $\frac{\varepsilon_2}{\varepsilon_1} \approx 1.14$.

Household allocations

Table 10 displays the household allocations obtained under the benchmark and fully funded partial and general equilibrium models.

Table 10: Household allocations under various models

Variable	Benchmark	Fully Funded GE	Fully Funded PE
$\hat{c}_1$	0.0447	0.0545	0.0462
$\hat{c}_2$	0.0784	0.0803	0.0810
$\hat{c}_3$	0.0649	0.0560	0.0671
h_1	0.4395	0.4062	0.4336
h_2	0.1946	0.2317	0.1295
$\hat{a}_2$	0.0164	0.0140	0.0183
$\hat{a}_3$	0.0088	0.0072	0.0077

Data

Table 11: Survival Probabilities[10]

Age	Mortality Probability	Survival Probability
0	0.00307	0.99693
1	0.00027	0.99973
2	0.00022	0.99978
3	0.00007	0.99993
4	0.00007	0.99993
5	0.00009	0.99991
6	0.00002	0.99998
7	0.0001	0.9999
8	0.0001	0.9999
9	0.00007	0.99993
10	0.00011	0.99989
11	0.00014	0.99986
12	0.00016	0.99984
13	0.00011	0.99989
14	0.0001	0.9999
15	0.00003	0.99997
16	0.00031	0.99969
17	0.00032	0.99968
18	0.00036	0.99964
19	0.00049	0.99951
20	0.00056	0.99944
21	0.0005	0.9995
22	0.00061	0.99939
23	0.00061	0.99939
24	0.00053	0.99947
25	0.00061	0.99939

[10] The survival probabilities are found at http://www.ssb.no/dode/tab-2008-04-10-05.html. I consider the survival probabilities of both genders.

26	0.00059	0.99941
27	0.0007	0.9993
28	0.00076	0.99924
29	0.00061	0.99939
30	0.00047	0.99953
31	0.00054	0.99946
32	0.00066	0.99934
33	0.00062	0.99938
34	0.00056	0.99944
35	0.00073	0.99927
36	0.00081	0.99919
37	0.00083	0.99917
38	0.00068	0.99932
39	0.0008	0.9992
40	0.00081	0.99919
41	0.00099	0.99901
42	0.00108	0.99892
43	0.00124	0.99876
44	0.00149	0.99851
45	0.00187	0.99813
46	0.00153	0.99847
47	0.00194	0.99806
48	0.00228	0.99772
49	0.00229	0.99771
50	0.00258	0.99742
51	0.00262	0.99738
52	0.00334	0.99666
53	0.00359	0.99641
54	0.0034	0.9966
55	0.00397	0.99603
56	0.00439	0.99561
57	0.00477	0.99523
58	0.00488	0.99512

59	0.00631	0.99369
60	0.00687	0.99313
61	0.00705	0.99295
62	0.00811	0.99189
63	0.00855	0.99145
64	0.01025	0.98975
65	0.0108	0.9892
66	0.01191	0.98809
67	0.01332	0.98668
68	0.01396	0.98604
69	0.01527	0.98473
70	0.0163	0.9837
71	0.01884	0.98116
72	0.02072	0.97928
73	0.02254	0.97746
74	0.02648	0.97352
75	0.0305	0.9695
76	0.0314	0.9686
77	0.03909	0.96091
78	0.03991	0.96009
79	0.04579	0.95421
80	0.05464	0.94536
81	0.05722	0.94278
82	0.06761	0.93239
83	0.07509	0.92491
84	0.08018	0.91982
85	0.09856	0.90144

Table 12: Population growth rate		
Year	Population[11]	Growth rate
1970	3863221	0,0065
1971	3888305	0,0076
1972	3917773	0,0078
1973	3948235	0,0063
1974	3972990	0,0062
1975	3997525	0,0049
1976	4017101	0,0045
1977	4035202	0,0040
1978	4051208	0,0037
1979	4066134	0,0031
1980	4078900	0,0033
1981	4092340	0,0036
1982	4107063	0,0038
1983	4122511	0,0029
1984	4134353	0,0028
1985	4145845	0,0032
1986	4159187	0,0039
1987	4175521	0,0055
1988	4198289	0,0053
1989	4220686	0,0029
1990	4233116	0,0039
1991	4249830	0,0056
1992	4273634	0,0060
1993	4299167	0,0060
1994	4324815	0,0055
1995	4348410	0,0050
1996	4369957	0,0052
1997	4392714	0,0057
1998	4417599	0,0063

[11] The figures are found at http://statbank.ssb.no//statistikkbanken/default_fr.asp?PLanguage=1, choose paragraph no. 02, subparagraph 02.01, subparagraph 02.01.10 and table 05803: “Population, births, deaths, marriages, migration and population increase”. Choose “Population – Unit: Persons” for the years 1970-2006.

1999	4445329	0,0075
2000	4478497	0,0056
2001	4503436	0,0046
2002	4524066	0,0062
2003	4552252	0,0055
2004	4577457	0,0063
2005	4606363	0,0073
2006	4640219	
Average		0,0051

Table 13: Mainland GDP per capita growth rate

Year	Population[12]	Mainland GDP[13]	GDP per capita	GDP per capita growth rate
1970	3863221	4.11274E+11	106458.8332	0.0435
1971	3888305	4.31963E+11	111092.8798	0.0320
1972	3917773	4.49166E+11	114648.2964	0.0208
1973	3948235	4.62057E+11	117028.7483	0.0449
1974	3972990	4.85824E+11	122281.7072	0.0295
1975	3997525	5.03253E+11	125891.1451	0.0366
1976	4017101	5.2424E+11	130502.071	0.0230
1977	4035202	5.38738E+11	133509.5492	0.0107
1978	4051208	5.46655E+11	134936.2955	0.0270
1979	4066134	5.63482E+11	138579.2992	0.0238
1980	4078900	5.78715E+11	141880.1638	0.0088
1981	4092340	5.85744E+11	143131.8023	0.0025
1982	4107063	5.89308E+11	143486.4768	0.0109
1983	4122511	5.97966E+11	145048.9762	0.0399
1984	4134353	6.23608E+11	150835.693	0.0454
1985	4145845	6.53738E+11	157685.104	0.0234
1986	4159187	6.71157E+11	161367.3538	0.0206
1987	4175521	6.87675E+11	164692.0229	-0.0033
1988	4198289	6.89121E+11	164143.2974	-0.0129
1989	4220686	6.8389E+11	162032.9018	0.0040
1990	4233116	6.88676E+11	162687.7222	0.0116
1991	4249830	6.99407E+11	164572.9359	0.0196
1992	4273634	7.1711E+11	167798.6463	0.0193
1993	4299167	7.35338E+11	171041.9716	0.0299
1994	4324815	7.61823E+11	176151.5811	0.0232

[12] The figures for the population are found as in table 11.
[13] The figures are found at http://statbank.ssb.no//statistikkbanken/default_fr.asp?PLanguage=1, choose paragraph no. 09, subparagraph 09.01, "National accounts" and table 05112: "Production and uses, by kind of activity". Choose "Constant prices – Unit: Mill. NOK" Choose "Value added", "Mainland Norway" for the years 1970-2005 and multiply the figures by 1000000.

1995	4348410	7.83772E+11	180243.3533	0.0290
1996	4369957	8.10508E+11	185472.7632	0.0455
1997	4392714	8.51834E+11	193919.7498	0.0340
1998	4417599	8.85813E+11	200519.1055	0.0197
1999	4445329	9.08903E+11	204462.4819	0.0216
2000	4478497	9.3542E+11	208869.1809	0.0114
2001	4503436	9.51383E+11	211257.1379	0.0048
2002	4524066	9.60317E+11	212268.5655	0.0062
2003	4552252	9.72338E+11	213594.9416	0.0304
2004	4577457	1.00742E+12	220083.7714	0.0383
2005	4606363	1.05262E+12	228515.2082	
Average				0.0222

Table 14[14]: Calculating the Capital/GDP and Investment/Capital ratios

Year	GDP[15]	Fixed assets[16]	Gross fixed capital formation[17]	Capital/GDP	Investment/Capital
1970	84086	251081	21384	2.9860	0.0852
1971	95058	274273	24770	2.8853	0.0903
1972	105956	303935	27275	2.8685	0.0897
1973	119601	337113	30003	2.8186	0.0890
1974	137808	403991	36295	2.9315	0.0898
1975	158198	463775	43081	2.9316	0.0929
1976	179661	531470	49358	2.9582	0.0929
1977	202688	611587	58255	3.0174	0.0953
1978	218514	677353	64075	3.0998	0.0946
1979	233690	738978	64561	3.1622	0.0874
1980	259614	844732	71312	3.2538	0.0844
1981	294256	954018	78646	3.2421	0.0824
1982	327739	1070498	82173	3.2663	0.0768
1983	360682	1165857	85847	3.2324	0.0736
1984	398012	1263371	92051	3.1742	0.0729
1985	445487	1375161	103391	3.0869	0.0752
1986	502106	1532914	126252	3.0530	0.0824
1987	557571	1742131	138832	3.1245	0.0797
1988	592895	1938133	139655	3.2689	0.0721
1989	605148	1971705	120815	3.2582	0.0613
1990	624889	1966950	109202	3.1477	0.0555
1991	653840	1983530	104810	3.0337	0.0528
1992	679521	1989709	100937	2.9281	0.0507
1993	712302	2016934	99986	2.8316	0.0496
1994	749613	2092149	115506	2.7910	0.0552
1995	806858	2229122	135071	2.7627	0.0606

[14] GDP, Fixed assets and Grossed fixed capital formation are given in units of million Norwegian kroner at current prices.
[15] These figures are found at http://www.ssb.no/english/subjects/09/01/nr_en/, in table no. 9, view as CSV file, under Mainland Norway.
[16] These figures are found at http://www.ssb.no/english/subjects/09/01/nr_en/, in table no. 35, view as CSV file, under Mainland Norway.
[17] These figures are found at http://www.ssb.no/english/subjects/09/01/nr_en/, in table no. 26, view as CSV file, under Mainland Norway.

1996	851647	2343046	151422	2.7512	0.0646
1997	919034	2466937	170074	2.6843	0.0689
1998	992596	2607970	188910	2.6274	0.0724
1999	1045340	2749597	190745	2.6303	0.0694
2000	1113893	2931972	194107	2.6322	0.0662
2001	1179586	3112306	206641	2.6385	0.0664
2002	1224643	3221009	209887	2.6302	0.0652
2003	1274830	3334617	202723	2.6157	0.0608
2004	1355314	3562311	230041	2.6284	0.0646
2005	1451132	3810764	265234	2.6261	0.0696
2006	1575825	4190270	296671	2.6591	0.0708
2007	1708746	4623162	341957	2.7056	0.0740
Average				2.9196	0.0738

Table 15: Data[18] from the AKU survey used to calculate efficiency units

Year 2001		
	Employees covered by the survey	Monthly earnings (NOK)
-24 years	105321	17797
25-29 years	127649	22238
30-34 years	159093	24426
35-39 years	162103	25699
40-44 years	163046	26206
45-49 years	164976	26294
50-54 years	153828	26153
55-59 years	128479	26136
60 years or older	70746	25104
Year 2002		
	Employees covered by the survey	Monthly earnings (NOK)
-24 years	115754	18792
25-29 years	129385	23392
30-34 years	165713	25960
35-39 years	169158	27355
40-44 years	166428	28152
45-49 years	168753	28186
50-54 years	155648	28019
55-59 years	138213	27858
60 years or older	77113	26990

[18] The figures are found at http://statbank.ssb.no//statistikkbanken/default_fr.asp?PLanguage=1, choose paragraph no. 06, subparagraph 06.05, "Wage statistics. All employees" and table 05218: "Average monthly earnings for employees, full-time equivalents, by working hours, age-group and sex". Choose "Employees covers by the survey – Unit: Persons" and "Employees Monthly Earnings – Unit: NOK". Choose "All employed". Choose all age groups from "-24 years" to "60 years and older". Choose "Total" under gender. Choose years 2001 to 2006.

Year 2003		
	Employees covered by the survey	Monthly earnings (NOK)
-24 years	112724	19375
25-29 years	126405	24045
30-34 years	165268	26898
35-39 years	171745	28293
40-44 years	167796	29114
45-49 years	168955	29097
50-54 years	156370	28897
55-59 years	142633	28709
60 years or older	79134	28232
Year 2004		
	Employees covered by the survey	Monthly earnings (NOK)
-24 years	113212	19936
25-29 years	124270	24603
30-34 years	165560	27588
35-39 years	177508	29283
40-44 years	171211	30141
45-49 years	171726	30262
50-54 years	159316	30072
55-59 years	145677	29849
60 years or older	86970	29308
Year 2005		
	Employees covered by the survey	Monthly earnings (NOK)
-24 years	119970	20344
25-29 years	126870	25192

30-34 years	166594	28429
35-39 years	181123	30189
40-44 years	175642	31216
45-49 years	174235	31330
50-54 years	163427	31134
55-59 years	148633	30796
60 years or older	96096	30509
Year 2006		
	Employees covered by the survey	Monthly earnings (NOK)
-24 years	130212	21161
25-29 years	131391	26316
30-34 years	166937	29760
35-39 years	185356	31684
40-44 years	181805	32712
45-49 years	175983	33021
50-54 years	168015	32671
55-59 years	148818	32275
60 years or older	107348	32066