Monopoly Pricing under a Medicaid-Style Most-Favored-Customer Clause and Its Welfare Implication

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Abstract

To control Medicaid's increasing expenditure on reimbursement of outpatient prescription drugs, the Omnibus Budget Reconciliation Act of 1990 included a rebate program that featured a most favored customer (MFC) clause. This clause guarantees that Medicaid pays the minimum price offered in the market (minimum price provisioning or MPP) or a proportion of the average manufacturer price (average price provisioning or APP). We characterize the optimal pricing strategy of a third-degree price discriminating monopolist in response to the imposition of MPP or APP rules. Among our findings are conditions under which these rules result in higher prices charged in all markets. We also examine the effects of these rules on aggregate demand for the drug and on social welfare. In general, these rules may change aggregate demand and social welfare in either direction. We are able to provide some useful sufficient conditions. For example, imposing MPP increases social welfare if it results in higher aggregate demand.

1 Introduction

Medicaid is a U.S. government program to pay for health-care services for some low-income families and individuals. It is funded jointly by the federal and state governments. Growing concern over the rapid increase in Medicaid's spending for outpatient prescription drugs led to the enactment of the Medicaid rebate program in 1990. This rebate program, established by the Omnibus Budget Reconciliation Act (OBRA) of 1990, requires drug manufacturers to offer rebates to Medicaid based on the discounts offered to other large purchasers. This is a form of "most favored customer" (MFC) clause. In particular, Medicaid is only required to pay the manufacturer the minimum of the prices that the manufacturer charges any purchaser in the market (minimum price provisioning or MPP), or a percentage below the quantity-weighted average price it charges (average price provisioning or APP), whichever is lower.

As Medicaid consumers constitute a significant fraction of the whole market, the Medicaid rebate program provides drug manufacturers with a strategic incentive to alter their price distribution in the market. This article studies the optimal response of a monopolist to the imposition of these types of minimum price and average price MFC clauses. More specifically, we are interested in examining their effect on pricing when the monopolist practices third degree price discrimination across markets. We also examine how these rebates affect the total demand for the product as well as social welfare.

Drug manufacturers often practice price discrimination to increase profits. For single-source products (branded drugs with patent protection), suppliers enjoy a high degree of market power. Manufacturers can categorize various purchasers according to their price sensitivity, and charge each group a distinct price. This leads to a high level of price dispersion in the market. In 1991, before the rebate rule went into effect, nearly one-third of the single source drugs had a best price discount of at least 50 percent (i.e., the lowest price charged for the drug was less than half of the highest price charged) (Congressional Budget Office [3] , pp xi).

As of 2002, Medicaid constituted approximately 18.5% of the prescription drug market (Duggan and Scott Morton [7])[1]. As such a large purchaser, Medicaid might be expected to use its bargaining power to obtain relatively good prices. However, Medicaid was unable to do this as well as other large purchasers, in part because it reimbursed individual pharmacies and hospitals rather than purchasing in bulk from the manufacturer. To secure better prices for Medicaid patients, OBRA 90 included a voluntary program in which pharmaceutical manufacturers could enroll their product to have access to all state Medicaid formularies[2]. In return, drug manufacturers are required to pay rebates to state and federal Medicaid programs. The rebate rule has a fairly complex structure (and has been modified somewhat over time). As of 2006, manufacturers of branded products are required to sell to Medicaid at 84.9% of the Average Manufacturer Price (AMP), or the "best" price, whichever is lower (see Hearne [8]). AMP is a quantity-weighted average of the wholesale prices available to a member of the 'retail pharmaceutical trade'. The price at which the drug is sold to hospitals or HMOs are not counted in the calculation of AMP. The best price is simply the minimum price at which the product is sold to any purchaser, including hospitals and HMOs[3]. Generic

[1] As of 2006, those individuals over 65 who would have previously been covered by the Medicaid prescription drug program are now covered under Medicare's prescription drug program. Thus Medicaid's market share has likely shrunk somewhat recently. More recent numbers suggest a market share closer to 15% (Jacobson, Panangala and Hearne [9]).

[2] When this program was introduced, nearly all branded and generic drug manufacturers did enroll (Scott Morton [18]).

[3] The Veterans Administration (VA) and Department of Defense (DoD), being large purchasers, enjoy substantial discounts off the wholesale price. When the rebate program was originally enacted, these prices were included in the calculation of the best price. However, in 1992, Congress amended OBRA to exclude prices paid by VA, DoD and some other public purchasers from the calculation of best price.

products are not subject to the best price provision. The price of a generic product to Medicaid is required to be 89% of the AMP (see Hearne [8])[4].

The Medicaid rebate program was introduced to reduce Medicaid's expenditures on outpatient prescription drugs. Although this program seems to have succeeded in lowering Medicaid's inflation-adjusted drug expenditures (Congressional Budget Office [3]), its overall effects are not obvious. Pharmaceutical manufacturers should react to the rebate rule, thus potentially changing their price distribution across markets. What is the nature of this optimal price response? The savings to Medicaid, if any, would not generally be the same as those calculated without taking into account the change in optimal pricing strategy. Non-Medicaid purchasers are also affected by the rebate rule. For example, Duggan and Scott Morton [7] estimate that for the top 200 drug treatments, the average price of a non-Medicaid prescription would have been 13.3 percent lower in 2002 if the Medicaid MFC clause had not been in effect. The rebate rule also affects the manufacturer's profit adversely. It is important to examine the aggregate welfare effects of this cost-saving mechanism. Given the changes that take place as a consequence of the rebate rule, what happens to social welfare?

We analyze a model where a monopolist optimally determines her pricing strategy, subject to MPP or APP clauses. We examine these two types of MFC clauses separately. We also consider two assumptions about the demand of Medicaid participants for drugs: (i) Medicaid participants' demand curve is same as that of non-Medicaid consumers, and (ii) Medicaid participants' demand is completely inelastic and non-Medicaid consumers' demand is elastic. We consider only these two polar cases for simplicity. Medicaid consumers do not pay for their drugs directly (though in some states they do have small co-payments (Hearne [8])). Thus, their price sensitivity may be different from non-Medicaid purchasers' price sensitivity. On the other hand, Medicaid customers' purchases are influenced by physicians and others (including those running state drug formularies) as well as possible co-payments and so we think the case of elastic demand is worth investigating in addition to the inelastic case[5]. In examining the impact of the rebate rule on social welfare, we use Marshallian welfare, the sum of consumers' and producers' surplus, as our measure of social welfare.

What do we find? A quick preview of some of our results follows. Under MPP, the

[4] Average price provisioning is also subject to another restriction in terms of the inflation rate. If AMP rises faster than the inflation rate, an additional rebate, which is equal to the difference between the current AMP and the base year AMP increased by the consumer price index (CPI), is imposed. For a detailed discussion of the Medicaid rebate program, see Congressional Budget Office report [3]. Duggan and Scott Morton [7] point out that since price increases for any treatment are limited by CPI inflation, if the optimal price for a drug increases faster, there is an incentive to instead introduce and sell a new version of the same drug with a different dosage amount or type (e.g., liquid, capsule, tablet) that would have an unrestricted base price. They find evidence consistent with this behavior by drug manufacturers.

[5] Note that our work also applies to contracts in natural gas or international trade where the use of a most favored customer clause is common. In those applications, it is more natural to assume that the most favored customer's demand is also price sensitive.

minimum price charged always rises compared to the no regulation case. In fact, prices in *all* markets (weakly) rise if Medicaid and non-Medicaid consumers have the same demand characteristics (the "elastic" scenario). In contrast, if Medicaid demand is inelastic (the "inelastic" scenario), prices in all markets where the minimum is not charged will fall. In either scenario, the welfare effect of MPP may be good or bad. A useful sufficient condition for MPP to be welfare improving is that MPP raise aggregate quantity.

Under APP and elastic demand, prices in all markets move in the same direction. Prices increase if and only if the discount percentage off of average price is above a threshold. When MFCs' demand is inelastic, we provide conditions sufficient for prices to move together in each direction. Prices under APP are decreasing in the discount, and if the discount is high enough, APP will lower prices in every market. Thus large discounts have opposite effects on price movements under APP in the elastic versus inelastic cases. As with MPP, the welfare effect of imposing APP is ambiguous in general. If MFCs' demand is inelastic, then if prices in all markets increase, both welfare and aggregate quantity fall, while if all prices decrease this is welfare and quantity improving.

Understanding the effects of these regulations is not simply of interest for evaluation of Medicaid policy, but is also important as a guide to future regulation. For example, recently there has been debate about the appropriate regulatory regime to govern drug purchases and reimbursement under Medicare, the US government program of health insurance for the elderly (Jacobson, Panangala and Hearne [9]).

1.1 Related Literature

The literature related to the Medicaid rebate program and its rebate rules has been primarily empirical. The only theoretical models of monopoly behavior under these rules that we know of are in the brief theory sections of Scott Morton [18] and Congressional Budget Office [3]. The seminal Scott Morton [18] is the most closely related to our analysis, as we borrow the third degree price discrimination structure, the focus on the polar cases of elastic and inelastic Medicaid demand, and the definitions of the APP and MPP rules from her model. We build on and extend her theoretical analysis in a number of ways. First, we do not limit our analysis to the case of linear market demand curves – we allow general downward sloping, continuously differentiable demands. Second, we provide a full characterization of the solution for the minimum price provision problem. Even in the special case of linear demand, this solution only coincides with that in Scott Morton [18] under additional and restrictive assumptions. Moreover, we are able to describe conditions that determine whether prices increase or decrease when an average price provision rule is imposed. Finally, we analyze how these clauses could affect aggregate demand and social welfare, an aspect not studied in Scott Morton ([18], [19]) or Congressional Budget Office [3]. This last aspect of our work has close connections with the literature on the welfare effects of third degree price discrimination by a monopolist. The effect of price discrimination on social welfare was first studied by Robinson [14]. Schmalensee [16] reexamined the problem, and provided a sufficient condition for welfare to decrease under uniform pricing as compared to third degree price discrimination[6]. He shows that uniform pricing can lead to a decrease in welfare only if it leads to an decrease in aggregate demand. As stated above we show a similar result for MPP – imposition of MPP can lead to a decrease in welfare only if it leads to a decrease in aggregate demand. Schmalensee's techniques prove useful in our analysis.

The empirical work on the Medicaid rebate program includes two United States General Accounting Office (GAO) studies ([23], [24]), a Congressional Budget Office report [3], Scott Morton ([18], [19]) and Duggan and Scott Morton [7]. All of these papers find some evidence of post-rebate rule increases in drug prices for non-Medicaid buyers. GAO [23] studied how Veterans Affairs (VA) and Department of Defense (DoD) prescription drug prices had changed, while GAO [24] examined drug prices to health maintenance organizations (HMOs) and hospitals. In both cases price increases were observed, but the GAO could not determine whether the price growth was attributable to the rebate rules. The Congressional Budget Office [3] report concluded that although the rebate rule lowered Medicaid expenditure, it increased the prices paid by some purchasers in the private sector. Scott Morton [18] finds that the price of branded products facing generic competition rose. For generic drugs, the increase in price is higher as markets become more concentrated. Scott Morton [19] finds that products with higher ex-ante price dispersion show a greater increase in price when the rebate rule is in effect, consistent with the theory. Duggan and Scott Morton [7], as mentioned above, estimate that the average price of a non-Medicaid prescription would have been 13.3 percent lower in 2002 if the Medicaid MFC clause had not been in effect. They also find an increase in new drug introductions for the purpose of raising prices in reaction to a provision in the OBRA 90 legislation that ties increases in existing drug prices to inflation.

Rules like MPP have been studied in a number of other contexts. The impact of similar most favored customer clauses in oligopoly settings has been studied extensively in the theoretical literature. Most of the research explores the situation where the sellers strategically exploit the clause to soften price competition. See for example Besanko and Lyon [1], Cooper [4], Cooper and Fries [5], Png [12], Png and Hershleifer [13], and Salop [15]. Spier [20] studies uses of MFC-type clauses in settlement of litigation. The use of MPP with long term contracts has been studied by Butz [2] in the context of durable goods monopoly. Butz analyzes how MPP can be used to facilitate commitment not to reduce price in the future, and thereby sustain the monopoly price for the product. In his analysis, MPP is used as a strategic device by the monopolist in its intertemporal game with consumers to change consumer demand by changing beliefs about future prices. Thus even in the monopoly context, the emphasis has been on strategic effects. Our analysis differs substantially from those

[6] Varian [25] and Schwartz [17] are able to generalize Schmalensee's result.

mentioned in this paragraph because our focus is on the unilateral/own-price effects of such clauses rather than the strategic effects operating through competitor or consumer reaction. In particular, none of our pricing or welfare results may be derived from this literature.

This paper is organized as follows. Section 2 provides a simple example (with three markets and linear demands with a common slope) of price changes and welfare effects under the two different rebate rules in the elastic case of identical Medicaid and non-Medicaid demand. In section 3, we describe the general model and specify the monopolist's objective function under the two rules. In section 4, we solve the optimization problem under MPP and examine its welfare implications. We do this for both the elastic and inelastic MFC demand cases. Section 5 carries out a similar investigation for the APP rule. Section 6 concludes.

2 Example

We start with a simple numerical example to demonstrate the effect of these rebate rules on prices, demand and welfare when MFC and non-MFC consumers have the same demand characteristics.

Example 1 *Let us assume there are three submarkets with demand curves* $D_1(p) = 90 - 5p, D_2(p) = 100 - 5p$ *and* $D_3(p) = 200 - 5p$ *respectively. Assume the marginal cost of producing each unit of the product is* \$2 *and, in each market, let us suppose* 20% *of the consumers are protected by an MFC clause. Further, assume that the demand characteristics of consumers covered by the MFC clause are the same as those who are not. In the absence of an MFC clause, the monopolist charges the profit maximizing prices in each market, which are* $p_1 = 10, p_2 = 11$ *and* $p_3 = 21$*. The monopolist makes a profit of \$2530 (\$320, \$405 and \$1805 from submarkets 1, 2 and 3 respectively). With minimum price provisioning for the MFCs, optimal prices will be* $p_1 = 11.45, p_2 = 11.45$ *and* $p_3 = 21$*. The monopolist's total profit is reduced to \$2427.27. This increase in prices (strict increase in* p_1 *and* p_2 *with* p_3 *remaining same) as a consequence of the MPP is a general result. As the MPP clause means that some higher valuation consumers are now paying the minimum price, there is incentive to raise the minimum price. In the following section, we will show that in submarkets where this minimum price is not charged, the monopolist's optimal policy would be to maintain the profit maximizing price. Furthermore, the markets where the minimum price is charged will be shown to be all markets where the profit maximizing price is below an endogenously determined threshold.*

We now turn to an average price provision MFC clause (with a 15% discount off the quantity-weighted average price) in this example. With this APP clause, the monopolist's optimal pricing strategy will be $p_1 = 10.07, p_2 = 11.07$ *and* $p_3 = 21.07$ *and it will earn a total profit of* \$2455.7*. MFCs pay* 85% *of the quantity-weighted average price (*\$16.14*), which is* \$13.72*. In* *this example, prices in all markets increase under APP. The same will be true in any example with linear demand curves with a common slope. In general, we will show that under APP all prices must move in a common direction, but may either increase or decrease. Moreover, we will describe exactly when they will do one or the other.*

The monopolist's total profit is reduced under either clause. The intuition is straightforward. The optimal solution for the case without an MFC clause coincides with the solution of a less constrained optimization problem, namely, the situation where the monopolist is allowed to charge different prices to MFCs and non-MFCs in each market without any restriction. Both MPP and APP put additional restrictions on the prices offered and reduce the size of the feasible set of solutions.

Table 1

		No MFC	*MPP*	*APP*
	Mkt1	*\$10*	*\$11.45*	*\$10.07*
Price	*Mkt2*	*\$11*	*\$11.45*	*\$11.07*
	Mkt3	*\$21*	*\$21*	*\$21.07*
Profit		*\$2530*	*\$2427.27*	*\$2455.7*

What happens to quantities purchased and to overall welfare? In the absence of an MFC clause, the aggregate quantity purchased is 180*. Overall welfare, as measured by the sum of producer's and consumers' surplus, is* \$3795*. Under MPP, aggregate quantity is unchanged, but overall welfare rises by* \$51.36 *due to the reallocation of units from the lower valuation markets* 1 *and* 2 *to the higher valuation MFC consumers from market* 3*. For linear demand, this example generalizes, in that quantity purchased will stay the same and overall welfare will always increase. With more general demand structures aggregate quantity may change and welfare may increase or decrease. Under APP, in this example aggregate quantity increases slightly (to* 180.002*) and overall welfare increases by* \$36.88*. As with MPP, for more general demand structures, quantity may change in either direction as may overall welfare.*

3 The Model

Consider a monopolist selling a single good in n different markets, indexed by i. Each market i has a downward sloping non-negative continuously differentiable demand curve $q_i(p_i)$ for the product, where p_i is the price charged in market i. Assume demand is zero if p_i becomes large enough. We assume that the monopolist cannot discriminate between consumers within a market, but it can prevent arbitrage by consumers between markets. For simplicity, we consider a linear cost function $C(q) = cq$. We also assume there are gains from trade in all markets, i.e., $q_i(c) > 0$. This framework depicts a simple model of third degree price discrimination. We analyze the consequences of MFC clauses in this environment. In particular, we will discuss two types of MFC provisions: (i) Minimum price provision (MPP), and (ii) Average price provision (APP). Recall that under MPP, MFCs pay the minimum of the prices charged in the n markets. Under APP, MFCs pay a fraction $(1-\alpha)$ of the (non-MFC) quantity-weighted average of the prices charged in the n markets. The discount from the average, α, is a parameter in our model. The presence of a MFC provision divides consumers in each market into two categories: MFCs and non-MFCs. If all consumers in market i were non-MFCs, the demand function in market i would be q_i, as above. If all consumers in market i were MFCs, the demand function in market i would be q_i^M. More generally, some fraction of consumers in each market will fall into each category. For simplicity, we assume that the fraction of MFCs in each market is the same and we denote this fraction by $\gamma \in [0,1]$.

The monopolist's total profit can be written as

$$\Pi = (1-\gamma)\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \gamma \sum_{i=1}^{n}(p_i^M - c)q_i^M(p_i^M), \tag{3.1}$$

where p_i and p_i^M denote the prices paid by non-MFCs and MFCs in market i, respectively. Under MPP, p_i^M is $p_{\min} \equiv \min\{p_1, \ldots, p_n\}$. Under APP,

$$p_i^M = (1-\alpha)p_q \equiv p_{q,\alpha} \text{ where } p_q = \frac{(1-\gamma)\sum_{i=1}^{n} p_i q_i(p_i)}{(1-\gamma)\sum_{i=1}^{n} q_i(p_i)} = \frac{\sum_{i=1}^{n} p_i q_i(p_i)}{\sum_{i=1}^{n} q_i(p_i)}.$$

As mentioned in the Introduction, we study two scenarios: (i) when the MFCs' demand curve is the same as that of non-MFCs, i.e., $q_i^M = q_i$ for all i, and (ii) when the MFCs' demand is completely inelastic, i.e., $q_i^M = z_i > 0$ for constants z_i.

We will assume throughout our analysis that all n markets are served even after the MFC provisions are imposed. In an elastic demand framework, a sufficient condition for this is that every market is served with positive output under the optimal uniform monopoly price. We also assume that demand in each market is such that profit in that market (assuming no MFC) is a strictly concave function of price in that market whenever demand is positive. This assumption ensures that a solution of the profit maximization problem may be found by solving the first-order conditions. Under MPP, this assumption makes the first-order condition both necessary and sufficient for a solution of the profit maximization problem. For APP, stronger conditions are required for an analogous result. We defer further discussion of that case to Section 5. Formally, the following are imposed for the remainder of the paper unless explicitly stated otherwise:

Assumption 1 *Demand of non-MFCs is positive in every market at the optimal prices in the unconstrained, MPP and APP problems.*

Assumption 2 $(p-c)q_i(p)$ *is strictly concave in* p *whenever* $q_i(p) > 0$.

4 Minimum Price Provision

4.1 Elastic MFC demand

As a point of comparison, it is useful to begin our analysis by looking at the profit maximization problem for the monopolist when there is no provision for MFCs. Without any regulation, the monopolist cannot (and would not want to) discriminate between MFCs and non-MFCs within each market. The monopolist chooses prices to maximize

$$\Pi = \sum_{i=1}^{n} (p_i - c) q_i(p_i). \tag{4.1}$$

Let p_i^m denote the first best monopoly price in market i. Under Assumption 2, p_i^m is the unique solution to the equation

$$(p_i^m - c) q_i'(p_i^m) + q_i(p_i^m) = 0.$$

Without loss of generality, assume $p_1^m < p_2^m < \ldots < p_n^m$[7]. Denote the uniform monopoly price (profit maximizing price under no discrimination) by p^u, which is the unique solution to

$$\sum_{i=1}^{n} \left[(p - c) q_i'(p) + q_i(p) \right] = 0.$$

Following Robinson's (Robinson [14]) terminology, we call those markets with $p_i^m > p^u$ *strong* markets. Let S be the set of the corresponding indices. Similarly, *weak* markets are markets with $p_i^m < p^u$ and W denotes the set of the corresponding indices. Let I be set of indices of those *intermediary* markets where $p_i^m = p^u$. Observe that $p_1^m < p_n^m$ and Assumption 2 together imply that there is always at least one strong and one weak market.

We now examine the problem under MPP. Since MFCs and non-MFCs in market i have identical demand $q_i(\cdot)$, equation (3.1) implies the monopolist's profit maximization problem becomes:

$$\max_{\{p_i\}_{i=1}^n} (1 - \gamma) \sum_{i=1}^{n} (p_i - c) q_i(p_i) + \gamma \sum_{i=1}^{n} (p_{\min} - c) q_i(p_{\min}). \tag{4.2}$$

The following result describes the form of the optimal solution of this problem.

Proposition 1 *Suppose Assumptions* 1 *and* 2 *hold. The unique solution of the maximization problem under MPP will be of the form* $(\hat{p}, \ldots, \hat{p}, p_{k+1}^m, \ldots, p_n^m)$ *where* $k \in \{1, 2, \ldots, n-1\}$ *is the smallest value such that* $\hat{p} \in [p_k^m, p_{k+1}^m)$ *and* $k \in \{1, 2, \ldots, n-1\}$.

[7] If this is strictly violated, simply reindex the markets so that their numbering agrees with the monopoly price induced order. In cases where there is equality in monopoly prices across markets, a similar analysis can be carried out by first combining these markets into one. To see this, let us consider a situation where $p_1^m < \ldots < p_k^m = p_{k+1}^m < \ldots < p_n^m$. If we define a market indexed by k' by combining market k and market $k+1$ such that $q_{k'} = q_k + q_{k+1}$, then $p_{k'}^m$ remains the same as p_k^m or p_{k+1}^m. This returns us to a situation where strict inequality is maintainted among the optimal individual monopoly prices in each of these markets.

Proof. Let a solution vector be $(\tilde{p}_1, \ldots, \tilde{p}_n)$ and $J = \{i \in \{1, 2, \ldots, n\} | \, \tilde{p}_i = \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}\}$.

Claim 1: If $j \notin J$, then $\tilde{p}_j = p_j^m$.

If $j \notin J$, then $\tilde{p}_j > \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$. $\tilde{p}_j$ is also the solution of the optimization problem: $\max_p (p-c)\, q_j(p)$ such that $p \geq \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$.

If $p_j^m > \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$, and as p_j^m maximizes $(p-c)\, q_j(p)$ globally, $\tilde{p}_j = p_j^m$.

If $p_j^m \leq \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$, then $(p-c)\, q_j(p)$ being concave in p, is maximized at $p = \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$ over the range $\{p : p \geq \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}\}$. This implies that $\tilde{p}_j = \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$, or, $j \in J$. which is ruled out.

Claim 2: If $j \in J, l \notin J$, then $j < l$.

If not, let us suppose $\exists\, l \notin J$ and $j \in J$ such that $j > l$.

Then, Claim 1 suggests $\tilde{p}_l = p_l^m$. Moreover, $p_l^m > \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$ since $l \notin J$. As $j > l$, we have $p_j^m > p_l^m > \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$. Therefore, $j \notin J$. Contradiction.

Claim 3: $\min\{\tilde{p}_1, \ldots, \tilde{p}_n\} \in [p_k^m, p_{k+1}^m)$ for $k = \max J$.

By Claim 1, $\tilde{p}_{k+1} = p_{k+1}^m > \min\{\tilde{p}_1, \ldots, \tilde{p}_n\}$. Suppose $\tilde{p}_k < p_k^m$. Then, the monopolist could strictly increase profits by setting $\tilde{p}_k = p_k^m$. This increases profits from the non-MFC customers in market k, and leaves all other terms in the profit expression unchanged.

Claim 4: $k < n$.

Suppose $k = n$. Then $\min\{\tilde{p}_1, \ldots, \tilde{p}_n\} = p^u$, the uniform monopoly price. Since $p_n^m > p^u$, this contradicts Claim 3.

Claims 1, 2, 3 and 4 together yield that the solution is of the desired form.

It remains to show that the solution is unique. Suppose $(\check{p}_1, \ldots, \check{p}_n)$ is a different solution. It can differ from $(\tilde{p}_1, \ldots, \tilde{p}_n)$ only in the choice of k and $\hat{p}$. We now show that there is a unique profit maximizing choice of k and $\hat{p}$ so that the existence of such different solutions is not possible. For any fixed k, it follows from Assumption 2 that there is a unique profit maximizing price which satisfies $\max_p \sum_{i=1}^{k}(p-c)q_i(p) + \gamma \sum_{i=k+1}^{n}(p-c)q_i(p)$. Call this $\hat{p}(k)$. Suppose that there exist $k_1 < k_2$ such that $\hat{p}(k_1) \in [p_{k_1}^m, p_{k_1+1}^m)$ and $\hat{p}(k_2) \in [p_{k_2}^m, p_{k_2+1}^m)$ as was shown to be required for profit maximization by the first part of this proof. By revealed preference, profits from the first k_1 markets and the MFCs from the remaining markets are strictly higher when charging $\hat{p}(k_1)$ rather than $\hat{p}(k_2)$. Since $\hat{p}(k_2) \in [p_{k_2}^m, p_{k_2+1}^m)$, profits from the non-MFCs in markets $k_1+1, \ldots, k_2$ would be higher my charging the monopoly prices in those markets. Combining these facts implies that profits are higher with $k = k_1$ and $\hat{p} = \hat{p}(k_1)$ than with $k = k_2$ and $\hat{p} = \hat{p}(k_2)$. This shows that a profit maximizing solution of the required form must be unique. ■

From Proposition 1, it is apparent that prices in all markets weakly increase under MPP as compared to no regulation, and that they strictly increase only in an initial segment of markets. In the markets with strict increase, prices rise exactly to the minimum price under MPP. They rise because, under MPP, the price in these markets now also serves MFCs from the higher valuation markets. In contrast, for markets with prices above the minimum under MPP there is no reason to deviate from the original monopoly price, as the consumers facing these prices have the same demand characteristics as without MPP. Notice that the number of markets where the minimum price is charged should be determined endogenously along with the level of that minimum price. To fully understand pricing under MPP, therefore, we must explore how both k and $\hat{p}$ are determined. To do this, we construct an alternative optimization problem and show that its optimal solution coincides with the optimal solution of the original problem (4.2). We then derive properties of the optimal minimum price and cutoff k by studying the first order condition of this modified problem.

In Claim 3 of the proof of Proposition 1, the lower bound on $p_{\min}$ was derived by constructing a feasible and profitable deviation in k. Given this argument and Proposition 1, the maximization problem (4.2) may be rewritten as the following problem of maximizing with respect to k and $p_{\min}$, where only the upper bound on $p_{\min}$ is imposed:

$$\begin{aligned} &\max_{p<p_{k+1}^m, k\in\{1,2,\dots,n-1\}} (1-\gamma)\left(\sum_{i=1}^{k}(p-c)q_i(p)+\sum_{i=k+1}^{n}(p_i^m-c)q_i(p_i^m)\right) \\ &\qquad +\gamma\sum_{i=1}^{n}(p-c)q_i(p) \\ &= \max_{p<p_{k+1}^m, k\in\{1,2,\dots,n-1\}} \sum_{i=1}^{k}(p-c)q_i(p)+\gamma\sum_{i=k+1}^{n}(p-c)q_i(p) \\ &\qquad +(1-\gamma)\sum_{i=k+1}^{n}(p_i^m-c)q_i(p_i^m). \end{aligned} \tag{4.3}$$

Let p^*, k^* solve (4.3). Consider the following optimization problem:

$$\max_p \sum_{i=1}^{k^*}(p-c)q_i(p)+\gamma\sum_{i=k^*+1}^{n}(p-c)q_i(p). \tag{4.4}$$

Lemma 1 *The unique solution of (4.4) is p^*.*

Proof. By strict concavity, this problem has a unique solution – call it p'. By inspection, if $p' < p_{k^*+1}^m$ then $p' = p^*$. Otherwise, the monopolist could strictly increase profits by setting $p = p'$ (instead of p^*) in (4.3). Suppose $p' \geq p_{k^*+1}^m$. Then $p^* < p'$. Since p' optimizes a strictly concave function, any increase in p^*, no matter how small, will increase $\sum_{i=1}^{k^*}(p-c)q_i(p)+\gamma\sum_{i=k^*+1}^{n}(p-c)q_i(p)$. But some increase in p^* is always feasible in problem (4.3) as $p^* < p_{k^*+1}^m$ and so could be increased at least some amount and still remain the minimum. Thus, optimality of p^* in (4.3) is contradicted and it cannot be that $p' \geq p_{k^*+1}^m$. Therefore, $p' < p_{k^*+1}^m$ and $p' = p^*$. ■

Observe that if γ were to change, this would give rise to different optimal p^* and k^*. Therefore let us denote the functions yielding the corresponding k^*, p^* for each possible γ

by $k(\gamma)$ and $\hat{p}(\gamma)$ respectively. $k(\gamma)$ and $\hat{p}(\gamma)$, therefore, satisfy the first order condition (in price) of (4.4):

$$\sum_{i=1}^{k(\gamma)}[(\hat{p}(\gamma)-c)q_i'(\hat{p}(\gamma))+q_i(\hat{p}(\gamma))]+\gamma\sum_{i=k(\gamma)+1}^{n}[(\hat{p}(\gamma)-c)q_i'(\hat{p}(\gamma))+q_i(\hat{p}(\gamma))]=0. \qquad (4.5)$$

This equation shows how the minimum price is determined if MPP is imposed. For sufficiently low values of γ, price increases only in market 1 and $k(\gamma)=1$. As γ increases, the demand corresponding to the minimum price increases, which has a first order increasing effect on the profit. Since profit is a concave function in prices (by Assumption 2) and p_k^m is strictly less than p_{k+1}^m for every k, the monopolist increases the minimum price with an increase in γ. For sufficiently high values of γ, the minimum price exceeds the monopoly price in market 2, and the monopolist increases prices in both the markets. Thus, $k(\gamma)$ is weakly increasing in γ and $\hat{p}(\gamma)$ is strictly increasing in γ.

Lemma 2 *Suppose Assumptions* 1 *and* 2 *hold.* $k(\gamma)$ *is weakly increasing and* $\hat{p}(\gamma)$ *is strictly increasing in* γ*, the fraction of MFCs in the population.*

Proof. In the Appendix. ■

The following proposition states that prices would increase, compared to the case of no MFC clause, only in weak markets. The key to this is showing that the monopolist will never set the minimum price above the uniform monopoly price.

Proposition 2 *Suppose Assumptions* 1 *and* 2 *hold. If MPP is imposed, prices weakly increase in all markets. However, prices strictly increase only in an initial segment of weak markets, i.e., in markets where the monopoly price is lower than a cutoff value below the uniform monopoly price. Moreover, the optimal minimum price,* $\hat{p}(\gamma)$*, is bounded above by the uniform monopoly price and is strictly below this price when the fraction of MFCs,* γ*, is strictly less than* 1*.*

Proof. When $\gamma=1$, the demand corresponding to the minimum price is $\sum_{i=1}^{n} q_i(p)$, i.e. total consumer demand. Hence, $\hat{p}(1)=p^u$. Lemma 2 shows that $\hat{p}(\gamma)$ is an strictly increasing function of γ. Hence, $\hat{p}(\gamma)<p^u$, for any $\gamma\in[0,1)$. The rest of the statements now follow directly from Proposition 1. ■

4.2 Welfare analysis (when MFCs' demand is elastic)

We now turn to the welfare effects of MPP. Let Q be the total quantity produced by the monopolist. We use the classical Marshallian welfare measure, consumers' surplus plus producers' surplus, as a measure of social welfare[8]:

$$\begin{aligned} W &= (1-\gamma)\sum_{i=1}^{n}\{\int_{p_i}^{\infty} q_i(v)dv + (p_i - c)q_i(p_i)\} \\ &+\gamma\sum_{i=1}^{n}\{\int_{p_{\min}}^{\infty} q_i(v)dv + (p_{\min} - c)q_i(p_{\min})\}. \end{aligned}$$

In order to analyze the welfare effect of this clause, we will do the following. We join the two price vectors $(p_1^m, \ldots, p_n^m)$ and $(\hat{p}, \ldots, \hat{p}, p_{k(\gamma)+1}^m, \ldots, p_n^m)$ on the plane $\mathbb{R}^n$ by a piecewise smooth curve such that every point on the curve is a solution of a different optimization problem, where the problems are parametrized by $\tau \in [0,1]$. We study how Q and W change as τ varies and prices move along the curve.

Formally, we define a set of optimization problem indexed by τ, and denoted by OP_τ, such that OP_0 is identical to the monopolist's profit maximization problem with no MPP (i.e., the problem in (4.1)), and OP_1 is the monopolist's profit maximization problem with MPP. Specifically, for any $\tau \in [0,1]$,

$$\begin{aligned} OP_\tau \quad &: \quad \max_{\{p_i\}_{i=1}^n} \ (1-\tau)\sum_{i=1}^{n}(p_i - c)q_i(p_i) \\ &\quad +\tau\left[(1-\gamma)\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \gamma\sum_{i=1}^{n}(p_{\min} - c)q_i(p_{\min})\right] \\ &= \max_{\{p_i\}_{i=1}^n} \ (1-\tau\gamma)\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \tau\gamma\sum_{i=1}^{n}(p_{\min} - c)q_i(p_{\min}) \end{aligned}$$

$Q(\tau)$ and $W(\tau)$ define the total production and the measure of welfare respectively, when the monopolist is solving OP_τ. As is evident from the above equation, OP_τ is identical to the profit maximization problem with a $\tau\gamma$ fraction of MFCs. The solution of OP_τ will be of the form as described in Proposition 1 with γ replaced by $\tau\gamma$. In the remainder of this subsection, we will, for simplicity, change parameters from $\tau\gamma$ to t, where t takes values in the interval $[0,\gamma]$. As t is increased from zero to γ, $\hat{p}(t)$ moves from the optimal pre-MPP minimum price p_1^m to the optimal post-MPP minimum price $\hat{p}(\gamma)$. We can, therefore, compare aggregate demand and welfare at these two extreme points $t = 0$ and $t = \gamma$ by studying dQ/dt and dW/dt.

Definition 1 *Given $\gamma \in [0,1]$, define γ_j, $j = 1,2,3,\ldots,k(\gamma)+1$ by $\gamma_1 \equiv 0$, $\gamma_{k(\gamma)+1} \equiv \gamma$ and, for $j = 2,3,\ldots,k(\gamma)$, $\gamma_j \equiv \min\{\gamma' : k(\gamma') \geq j\}$, the argument at which $k(\cdot)$ first changes its value from $j-1$ to j.*

Observe that $0 = \gamma_1 < \gamma_2 < \ldots < \gamma_{k(\gamma)} \leq \gamma_{k(\gamma)+1} = \gamma$, and for any $t \in (\gamma_{j-1}, \gamma_j)$, $j = 2,3,\ldots,k(\gamma)+1$, $\hat{p}(t)$ solves the equation (4.5) with $k(\gamma)$ and $\hat{p}(\gamma)$ replaced by $j-1$ and $\hat{p}(t)$ respectively. In the interval (γ_{j-1}, γ_j), $\hat{p}(t)$ is a differentiable function of t. Since $\hat{p}(t)$ is

[8]See Schmalensee [16] and Varian [25] for discussions on the legitimacy of this measure.

a differentiable function over the interval $[0, \gamma]$ except for possibly finitely many points, we can measure the change in Q and W by ΔQ and ΔW respectively, where

$$\begin{aligned} \Delta Q &= Q(\gamma) - Q(0) = \sum_{i=2}^{k(\gamma)+1} Q(\gamma_i) - Q(\gamma_{i-1}) = \sum_{i=2}^{k(\gamma)+1} \int_{\gamma_{i-1}}^{\gamma_i} (dQ/dt)dt \\ and,\ \Delta W &= \sum_{i=2}^{k(\gamma)+1} \int_{\gamma_{i-1}}^{\gamma_i} (dW/dt)dt \end{aligned}$$

The following equation (derived in the Appendix) expresses dQ/dt in terms of prices and demands. For any $t \in (\gamma_{j-1}, \gamma_j),\ j = 2, 3, \ldots k(\gamma) + 1$,

$$dQ/dt = \sum_{i=1}^{j-1} q_i'(\hat{p}(t))\hat{p}'(t) + t \sum_{i=j}^{n} q_i'(\hat{p}(t))\hat{p}'(t) + \sum_{i=j}^{n} [q_i(\hat{p}(t)) - q_i(p_i^m)] \tag{4.6}$$

$$\begin{aligned} &= (-\frac{1}{2})(\hat{p}(t) - c)\hat{p}'(t) \left[\sum_{i=1}^{j-1} q_i''(\hat{p}(t)) + t \sum_{i=j}^{n} q_i''(\hat{p}(t)) \right] \\ &- \frac{1}{2} \sum_{i=j}^{n} \left[(\hat{p}(t) - c)q_i'(\hat{p}(t)) + q_i(\hat{p}(t)) \right] + \sum_{i=j}^{n} [q_i(\hat{p}(t)) - q_i(p_i^m)] \end{aligned} \tag{4.7}$$

To better understand this expression, note that as the fraction of MFC consumers, t, changes from $t_0 \in (\gamma_{j-1}, \gamma_j)$ to $t_0 + \delta \in (\gamma_{j-1}, \gamma_j)$ for small $\delta > 0$, the only change in the optimal prices is that the minimum price increases. This affects three classes of consumers who will pay the new minimum price: all consumers in the minimum price markets, the fraction, t_0, of consumers in markets where the monopoly price is charged who were MFC consumers before the change and the marginal fraction of consumers, δ, in markets where the monopoly price is charged switched from non-MFC to MFC status. The three terms in the first line of equation (4.6) give the change in quantity for these three classes of consumers respectively.

After simplifying, (4.6) can be written as (4.7). The marginal change in quantity, dQ/dt, can be either positive or negative. The first term in the expression after the second equals sign can be positive or negative, depending on whether demand curves are concave or convex. The second term (without the negative sign) is positive, as $\hat{p}(t)$ is less than p_i^m for $i \geq j$. The third term is always positive as we have assumed negatively sloped demand. Note further that if dQ/dt is positive (negative) for some value of t in the interval $[0, \gamma]$, this does not imply dQ/dt is always positive (negative) over the whole interval. The following example illustrates a case where ΔQ is positive and shows how to modify it so that ΔQ would be negative.

Example 2 *Let us consider an example with* n *distinguishable markets where in every market other than the first one, we have a linear demand curve. In particular, we consider* $q_i(p) = a_i - bp,\ i = 2, 3 \ldots n$ *and* $a_2 < a_3 < \ldots < a_n$*. We do not specify any functional form* *for $q_1(p)$ except for the restriction that $q_1(p)$ is a strictly concave function of p. In market $i,\ i = 2, 3, \ldots, n,$ the optimal monopoly price is $(a_i + bc)/2b$. Thus, for large enough values $a_2, \ldots, a_n,$ we will have $p_1^m < p_2^m < \ldots < p_n^m$. Let us consider an interval (γ_{j-1}, γ_j), for some $j \in \{2, 3, \ldots k(\gamma) + 1\}$. For $t \in (\gamma_{j-1}, \gamma_j)$, the optimal minimum price, $\hat{p}(t)$, lies between $(a_{j-1}+bc)/2b$ and $(a_j+bc)/2b$. Hence, $\hat{p}(t)$ is equal to $(a+bc)/2b$ for some $a,\ a_{j-1} < a \le a_j$. The second term plus the third term in equation (4.6) then becomes*

$$\begin{aligned} & (-\frac{1}{2}) \sum_{i=j+1}^{n} \{\frac{bc-a}{2} + \frac{2a_i - a - bc}{2}\} + \sum_{i=j+1}^{n} \{\frac{2a_i - a - bc}{2} - \frac{a_i - bc}{2}\} \\ = \quad & (-\frac{1}{2}) \sum_{i=j+1}^{n} (a_i - a) + \sum_{i=j+1}^{n} (\frac{a_i - a}{2}) = 0. \end{aligned}$$

This holds for any $t \in (\gamma_{j-1}, \gamma_j)$, and for any $j \in \{2, 3, \ldots, k(\gamma) + 1\}$. However, the first term of (4.6) is always positive for any such t, as $q_1(p)$ is strictly concave in p and all other q_i are linear. We, therefore, have $dQ/dt > 0$ for every $t \in (\gamma_{j-1}, \gamma_j)$. Integrating over the intervals (γ_{j-1}, γ_j) and summing over j, we get $\Delta Q > 0$. A similar example with a strictly convex demand function $q_1(p)$ will give $\Delta Q < 0$.

We now analyze the change in welfare due to MPP. Differentiating W with respect to t for $t \in (\gamma_{j-1}, \gamma_j)$, we get (derived in the Appendix):

$$\begin{aligned} dW/dt \quad = \quad & (\hat{p}(t) - c) \left[\sum_{i=1}^{j-1} q_i'(\hat{p}(t)) \hat{p}'(t) + t \sum_{i=j}^{n} q_i'(\hat{p}(t)) \hat{p}'(t) \right] \qquad (4.8) \\ & + \sum_{i=j}^{n} (\hat{p}(t) - c) \{q_i(\hat{p}(t)) - q_i(p_i^m)\} + \sum_{i=j}^{n} \{ \int_{\hat{p}(t)}^{p_i^m} \{q_i(v) - q_i(p_i^m)\} dv \\ = \quad & (\hat{p}(t) - c) \frac{dQ}{dt} + \sum_{i=j}^{n} \{ \int_{\hat{p}(t)}^{p_i^m} \{q_i(v) - q_i(p_i^m)\} dv. \qquad (4.9) \end{aligned}$$

To better understand this expression, note that as the fraction of MFC consumers, t, changes from $t_0 \in (\gamma_{j-1}, \gamma_j)$ to $t_0 + \delta \in (\gamma_{j-1}, \gamma_j)$ for small $\delta > 0$, the only change in the optimal prices is that the minimum price increases. This affects three classes of consumers who will pay the new minimum price: all consumers in the minimum price markets, the fraction, t_0, of consumers in markets where the monopoly price is charged who were MFC consumers before the change and the marginal fraction of consumers, δ, switched from non-MFC to MFC status in markets where the monopoly price is charged. For the minimum price markets there will be a decrease in social welfare as price increases and moves further away from the competitive price c. The first term in (4.8) measures the marginal change in welfare in these minimum price markets. In the markets where the monopoly price is charged, there will also be a similar decrease in welfare related to those consumers who were already covered by the MFC provision as the minimum price increases. The second term in (4.8) measures the marginal change in welfare for this portion of the monopoly price markets. For the marginal fraction of consumers who switched from non-MFC to MFC status, there will be a gain in consumer surplus, as their price is reduced from the monopoly price p_i^m for their market and the minimum price $\hat{p}(t)$. Part of this gain, however, is simply a transfer from the drug manufacturer. The net gain in social welfare for this section of the market is given by the third term in (4.8). After simplifying, (4.8) can be written as (4.9). The second term in (4.9) is always positive. Therefore, dW/dt is positive if $dQ/dt \geq 0$. Note that $(\hat{p}(t) - c)$ is bounded below by $(p_1^m - c)$, which is positive. Integrating over the intervals (γ_{j-1}, γ_j) and summing over j, we get $\Delta W > (p_1^m - c)\Delta Q$. Hence, we have the following useful sufficient condition for MPP to increase welfare:

Proposition 3 *Suppose Assumptions* 1 *and* 2 *hold. Welfare increases under MPP if MPP leads to a weakly higher level of output.*

However, the effect on welfare is not unambiguous when $\Delta Q < 0$. Welfare may increase if the decrease in Q is not high enough to overcome the gains (as quantified by the second term in equation (4.9)) from the drop in price of the incremental units sold to MFC consumers in monopoly price markets under MPP. Otherwise it will decrease. The next example illustrates the possibility of a fall in welfare.

Example 3 *We consider an example in the framework of Example 2. Consider* $n = 2$. *Let* $q_1(p) = a_1 - bp + dp^2$ *and* $q_2(p) = a_2 - bp$ *be the demand functions in markets* 1 *and* 2 *respectively. Consider parameter values,* $\{a_1 = 40, a_2 = 60, b = 5, c = 2, d = 0.15\}$ *and let the fraction of MFCs,* γ, *be* 0.15. *Without MPP, the optimal monopoly prices are* $\{p_1^m = 6.52, p_2^m = 7\}$. *Aggregate demand and total welfare are* 38.7652 *and* 288.744 *respectively. With MPP in effect, the monopolist charges* $\{\hat{p} = 6.6394, p_2^m = 7\}$, *where* $\hat{p}$ *is the optimal minimum price. Aggregate demand and welfare decrease to* 38.6857 *units and* 288.444 *units respectively.*

4.3 Inelastic MFC demand

Here we assume that MFCs' demand for the product is completely inelastic. They demand a fixed quantity of the product while non-MFCs have a downward sloping demand curve. The monopolist, therefore, charges $(p_1, \ldots, p_n)$ to maximize

$$(1-\gamma)\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \gamma\sum_{i=1}^{n}(p_i - c)z_i. \tag{4.10}$$

Defining $m_i \equiv \frac{\gamma z_i}{1-\gamma}$, and substituting yields

$$(1-\gamma)\left[\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \sum_{i=1}^{n}(p_i - c)m_i\right].$$

Under MPP, this objective function becomes

$$(1-\gamma)\left[\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \sum_{i=1}^{n}(p_{\min} - c)m_i\right]. \tag{4.11}$$

Unlike before, we define p_i^m as the optimal monopoly price if facing only the non-MFCs in market i and, similarly, p^u as the uniform monopoly price that would be optimal if the monopolist ignored the inelastic MFCs in all markets. We order the markets so that $p_1^m < p_2^m < \ldots < p_n^m$. Note that this ordering of the markets is on the basis of the optimal monopoly prices when facing the non-MFC consumers only, and that this is different from the way we ordered markets in the previous section. Here, p^u solves the equation

$$\sum_{i=1}^{n}\{(p-c)q_i'(p)+q_i(p)\}=0,$$

and each p_i^m solves

$$(p-c)q_i'(p)+q_i(p)=0.$$

The following condition will be useful in characterizing the optimal solution under MPP:

$$\sum_{i=1}^{n} m_i + \sum_{i=1}^{n}\{(p_n^m-c)q_i'(p_n^m)+q_i(p_n^m)\} \geq 0. \qquad \text{(Condition A)}$$

If the same price is being charged in all markets, the left-hand side of Condition A is the derivative of the profit function with respect to price evaluated at a price of p_n^m. Therefore, given strict concavity, Condition A is necessary and sufficient for the optimal uniform price to be above p_n^m. Note that, by definition, $\{(p_n^m-c)q_n'(p_n^m)+q_n(p_n^m)\}$ is zero, whereas, by concavity, $\{(p_n^m-c)q_i'(p_n^m)+q_i(p_n^m)\}$ is negative for any other i. The next result describes the optimal solution under MPP.

Proposition 4 *Suppose Assumptions* 1 *and* 2 *hold. If Condition A is violated, the solution of the profit maximization problem under MPP will be of the form*

$$(\underbrace{\hat{p},\ldots,\hat{p}}_{k\ times}, p_{k+1}^m,\ldots,p_n^m)$$
$$\textit{where } \hat{p} \in [p_k^m, p_{k+1}^m) \textit{ for some } k \in \{1,2,\ldots,n-1\}$$

where $\hat{p} \in [p_k^m, p_{k+1}^m)$ for some $k \in \{1,2,\ldots,n-1\}$. If Condition A holds, the solution of the profit maximization problem under MPP will be of the form $(\underbrace{\hat{p},\ldots,\hat{p}}_{n\ times})$ where $\hat{p} \geq p_n^m$.

We do not provide an explicit proof for this result, as the arguments are essentially the same as in the proof of Proposition 1. Two comments are worth noting in this regard. First, Assumption 2 remains sufficient to guarantee strict concavity of the profit function in the minimum price. The reason for this is that the inelastic part of the profit is linear in price whereas Assumption 2 says that the elastic portion (corresponding to non-MFCs in those markets where the minimum price is charged) is strictly concave in price. Second, as before, in any market where the minimum price is not charged, the monopolist will try to extract the monopoly profit from the non-MFCs as long as that optimal price is above the minimum price. However, if and only if the inelastic demand is sufficiently large (as described in Condition A), the monopolist will find it profitable to keep raising the minimum price until it exceeds the optimal monopoly price for the non-MFCs in every market. In that case, the monopolist will end up charging the same price in every market and that price will be weakly higher than p_n^m.

An interesting fact is, unlike the elastic demand case, prices may decrease in some of the markets under MPP compared to no MPP. In those markets where the minimum price is not charged, the monopolist will optimally extract the monopoly profit by charging the monopoly price for the non-MFCs. In those markets, before MPP is imposed, the optimal price was higher than the optimal monopoly price based on only the non-MFC section (this is because of the fact that if demand in a market is composed of both elastic and inelastic demands, then the optimal monopoly price for the combined market is higher than the optimal monopoly price for the elastic demand section only). Therefore, these prices decrease under MPP. However, prices cannot fall in all markets under MPP, since, in particular, prices in market 1 (where the minimum is charged under MPP) must rise because of the fact that this price will be paid by inelastic consumers in all markets.

To study properties of the minimum price, it is useful to derive the equation that characterizes the minimum price as a function of the fraction of MFCs. To do this, we construct an alternative optimization problem and show that its optimal solution coincides with the optimal solution of the original problem (4.11). We then derive properties of the optimal minimum price by studying the first order condition of this modified problem.

Given Proposition 4, the maximization problem (4.11) may be rewritten as the following problem of maximizing with respect to k and $p_{\min}$, where only an upper bound on $p_{\min}$ is imposed:

$$\max_{p<p_{k+1}^m, k\in\{1,2,\ldots,n-1,n\}} \sum_{i=1}^{k}(p-c)q_i(p) + \sum_{i=k+1}^{n}(p_i^m - c)q_i(p_i^m) + \sum_{i=1}^{n}(p-c)m_i \tag{4.12}$$

where p_{n+1}^m defined as ∞.[9]

Let p^* and k^* solve (4.12). We now show that the unique solution to the following unconstrained optimization problem is $p = p^*$:

$$\max_{p} \sum_{i=1}^{k^*}(p-c)q_i(p) + \sum_{i=1}^{n}(p-c)m_i. \tag{4.13}$$

By strict concavity, this problem has a unique solution – call it p'. By inspection, if $p' < p_{k^*+1}^m$ then $p' = p^*$. Otherwise, the monopolist could strictly increase profits by setting $p = p'$

[9] Unlike the elastic demand scenario, prices in all markets may change. In order to accommodate this effect, we consider the possibility that k may equal n. Since in that case, the value of k could even be greater than p_n^m, we set the upper limit as infinity (by setting $p_{n+1}^m = \infty$).

(instead of p^*) in (4.12). Can it be that $p' \geq p^m_{k^*+1}$ for some $k^* \in \{1, 2, \dots, n-1\}$? Then $p^* < p'$. Since p' optimizes a strictly concave function, any increase in p above p^*, no matter how small, will increase $\sum_{i=1}^{k^*}(p-c)q_i(p) + \sum_{i=1}^{n}(p-c)m_i$. But some increase is always feasible in problem (4.12), as $p^* < p^m_{k^*+1}$ and so could be increased at least some amount and still remain the minimum. This would contradict the optimality of p^* in (4.12) and so it cannot be that $p' \geq p^m_{k^*+1}$. Therefore, $p' < p^m_{k^*+1}$ and $p' = p^*$.

Observe that if the inelastic demand levels $m_1, m_2, \dots, m_n$ were to change, this would give rise to different optimal p^* and k^*. Therefore let us denote the functions yielding the corresponding p^*, k^* for each possible $m = (m_1, m_2, \dots, m_n)$ by $\hat{p}(m)$ and $k(m)$ respectively. $k(m)$ and $\hat{p}(m)$, therefore, solve the first order condition (in price) of (4.13):

$$\sum_{i=1}^{k(m)} \{(\hat{p}(m) - c)q_i'(\hat{p}(m)) + q_i(\hat{p}(m))\} + \sum_{i=1}^{n} m_i = 0. \tag{4.14}$$

This condition allows us to show that if MPP is imposed the minimum price will increase and, in markets where the minimum price is not charged, prices will fall.

Proposition 5 *Suppose Assumptions* 1 *and* 2 *hold. The minimum price charged by the monopolist increases under MPP. In those markets where the minimum price is not charged, prices decrease under MPP.*

Proof. First, we show that the minimum price increases after MPP is imposed. If $k(m) = n$, then the result trivially follows as the monopolist will charge the optimal uniform monopoly price which is always greater than the monopoly price in one of the markets. Let us assume $k(m) = k^*$ for some $k^* \in \{1, 2, \dots, n-1\}$. By Proposition 4, the minimum price will always be charged in market 1. Let $\tilde{p}_1$ denote the price that was charged in market 1 before MPP was introduced. Then, $\tilde{p}_1$ must solve the equation

$$(\tilde{p}_1 - c)q_i'(\tilde{p}_1) + q_i(\tilde{p}_1) + m_1 = 0. \tag{4.15}$$

If we can show that $\tilde{p}_1 \leq \hat{p}(m)$, this will complete the first part of the proof. To see that $\tilde{p}_1 \leq \hat{p}(m)$, consider the function

$$S(p) = \sum_{i=1}^{k^*} \{(p-c)q_i'(p) + q_i(p)\} + \sum_{i=1}^{n} m_i = 0.$$

By Assumption 2, $S(p)$ is decreasing in p.

For every $i = 2, 3, \dots k^*$, $(\tilde{p}_1 - c)q_i'(\tilde{p}_1) + q_i(\tilde{p}_1) + m_i \geq 0$. Hence, $S(\tilde{p}_1) \geq 0$. From (4.14), we get that $\tilde{p}_1 \leq \hat{p}(m)$ since $S(p)$ is decreasing in p.

The second part of the proposition, stating that prices decrease under MPP in those markets where the minimum price is not charged, directly follows from Proposition 4. ∎

4.4 Welfare Analysis (when MFCs' demand is inelastic)

As before, we parametrize a set of optimization problems OP_τ, so that OP_0 is identical to the monopolist's profit maximization problem with no MPP, and OP_1 is the monopolist's profit maximization problem with MPP. Formally, for $\tau \in [0,1]$, we define OP_τ as follows:

$$\begin{aligned} OP_\tau \quad : \quad & \max_{\{p_i\}_{i=1}^n} (1-\gamma)\left[\begin{array}{c} \sum_{i=1}^n (p_i - c) q_i(p_i) + \tau \sum_{i=1}^n (p_{\min} - c) m_i \\ +(1-\tau)\sum_{i=1}^n (p_i - c) m_i \end{array}\right] \\ = \quad & \max_{\{p_i\}_{i=1}^n} (1-\gamma)\left[\sum_{i=1}^n (p_i - c)\{q_i(p_i) + (1-\tau)m_i\} + \sum_{i=1}^n (p_{\min} - c)\tau m_i\right]. \end{aligned} \tag{4.16}$$

Note that the above optimization problem is similar to the profit maximizing problem under MPP (4.11) if we replace the term $q_i(p_i)$ by $\{q_i(p_i) + (1-\tau)m_i\}$ and m_i by τm_i respectively in (4.11). Hence, the solution to OP_τ will be of the form given in Proposition 4. Let $\hat{p}(\tau)$ denote the optimal minimum price in problem OP_τ, and $p_i^*(\tau)$ denote the optimal monopoly price in a market with demand function $\{q_i(p_i)+(1-\tau)m_i\}$. As before, we partition the interval $[\tau_1 = 0 < \tau_2 < \ldots < \tau_n \leq 1]$ such that over the interval (τ_{j-1}, τ_j), the optimal solution is $(\hat{p}(\tau), \ldots, \hat{p}(\tau), p_j^*(\tau), \ldots, p_n^*(\tau))$. $\hat{p}(\tau)$ is an increasing function of τ, whereas $p_{j+l}^*(\tau)$, $l = 0, 1, 2, \ldots, n-j$, are decreasing in τ. For $\tau \in (\tau_{j-1}, \tau_j)$, $Q(\tau)$, total output corresponding to OP_τ, is given by

$$Q(\tau) = (1-\gamma)\left[\sum_{i=1}^{j-1} q_i(\hat{p}(\tau)) + \sum_{i=j}^{n} q_i(p_i^*(\tau)) + \sum_{i=1}^{n} m_i\right], \tag{4.17}$$

and, $W(\tau)$, the Marshallian welfare resulting from OP_τ, is given by

$$\begin{aligned} W(\tau) \quad = \quad & (1-\gamma)\left[\sum_{i=1}^n \{(\hat{p}(\tau) - c)\tau m_i + \int_{\hat{p}(\tau)}^{\bar{M}} \tau m_i \, dv\}\right. \\ & + \sum_{i=1}^{j-1} \{(\hat{p}(\tau) - c)(q_i(\hat{p}(\tau)) + (1-\tau)m_i) + \int_{\hat{p}(\tau)}^{\bar{M}} (q_i(v) + (1-\tau)m_i) \, dv\} \\ & \left. + \sum_{i=j}^{n} \{(p_i^*(\tau) - c)(q_i(p_i^*(\tau)) + (1-\tau)m_i) + \int_{p_i^*(\tau)}^{\bar{M}} (q_i(v) + (1-\tau)m_i) \, dv\}\right] \\ = \quad & (1-\gamma)\left[\begin{array}{c} \sum_{i=1}^n \left(\bar{M} - c\right) m_i + \sum_{i=1}^{j-1}\{(\hat{p}(\tau) - c) q_i(\hat{p}(\tau)) \\ + \int_{\hat{p}(\tau)}^{\infty} q_i(v) dv\} + \sum_{i=j}^{n}\{(p_i^*(\tau) - c) q_i(p_i^*(\tau)) + \int_{p_i^*(\tau)}^{\infty} q_i(v) dv\} \end{array}\right] \end{aligned}$$

In this calculation, we assume that the upper bound of the price while measuring the inelastic consumers' welfare is given by $\bar{M}$, some finite number large enough so that $q_i(\bar{M}) = 0$ for all i. This is equivalent to saying that demand isn't really inelastic, but rather is inelastic until price hits $\bar{M}$, and zero thereafter. Otherwise, consumer surplus for the inelastic consumers will always be infinity and our welfare measure would not be sensitive to changes in prices. Observe that the first term after the second equals sign is the consumer plus producer surplus associated with the inelastic consumers, and that this is a constant – changes in price simply change the split between these consumers and the producer. Below we will be interested in changes in welfare, rather than levels. For this purpose, one may then ignore the inelastic part of the market and the conclusions are insensitive to the choice of $\bar{M}$.

As we show in the Appendix, for any $\tau \in (\tau_{j-1}, \tau_j)$, $dQ/d\tau$ can be written in terms of the second derivative of the demand curves as follows:

$$dQ/d\tau = (1-\gamma)\left(\sum_{i=1}^{j-1} q_i'(\hat{p}(\tau))\hat{p}'(\tau) + \sum_{i=j}^{n} q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)\right) \tag{4.18}$$

$$= -\frac{1-\gamma}{2}\left(\begin{array}{c} \sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i''(\hat{p}(\tau))\hat{p}'(\tau) \\ +\sum_{i=j}^{n}(p_i^*(\tau)-c)q_i''(p_i^*(\tau))p_i^{*\prime}(\tau) \end{array}\right). \tag{4.19}$$

To better understand the terms in (4.18), note that the monopolist's objective function in the constructed optimization problem (4.16) is a convex combination of the objective function in the pre-MPP case and the objective function in the post-MPP case with weights $1-\tau$ and τ respectively. The demand of MFCs is fixed at $(1-\gamma)\sum_{i=1}^{n} m_i$ and is independent of τ. As τ changes, the optimal minimum price $\hat{p}(\tau)$ and the optimal prices $p_i^*(\tau)$ in the monopoly price markets change. The effect of these price changes on aggregate demand comes only through the change in non-MFC demand. The first term and the second term in (4.18) measure the change in aggregate demand for non-MFCs in markets where the minimum price is charged and in markets where the minimum price is not charged respectively.

We can rewrite (4.18) as (4.19) (derived in the Appendix). Note that $\hat{p}'(\tau)$ is positive by an argument similar to that in the proof of Lemma 2. Also, $p_i^{*\prime}(\tau)$ is negative since, as τ increases, the inelastic portion of the demand corresponding to consumers paying $p_i^*(\tau)$ falls whereas the elastic portion remains unchanged. Therefore $dQ/d\tau$ may be either positive or negative and may be non-monotone in τ. In the special case of linear demand, $dQ/d\tau = 0$.

For any $\tau \in (\tau_{j-1}, \tau_j)$, $dW/d\tau$ is given by

$$dW/d\tau = (1-\gamma)\,[\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i'(\hat{p}(\tau))\hat{p}'(\tau) + \sum_{i=j}^{n}(p_i^*(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)], \tag{4.20}$$

$$= (\hat{p}(\tau)-c)dQ/d\tau + (1-\gamma)\sum_{i=j}^{n}(p_i^*(\tau)-\hat{p}(\tau))q_i'(p_i^*(\tau))p_i^{*\prime}(\tau). \tag{4.21}$$

Social welfare for the inelastic section of the consumers is fixed at $(1-\gamma)\sum_{i=1}^{n}\left(\bar{M}-c\right)m_i$ and is independent of τ. As τ changes, the effect of price changes on social welfare comes only through the changes in welfare for the elastic demand section of markets. The first term and the second term in (4.20) measure the change in aggregate demand for non-MFCs in markets where the minimum price is charged and in markets where the minimum price is not charged respectively.

We can rewrite (4.20) as (4.21) (derived in the Appendix). As $(p_i^*(\tau)-\hat{p}(\tau))(q_i'(p_i^*(\tau))p_i^{*\prime}(\tau))$ is strictly positive for every $i \in \{j+1,\ldots,n\}$, it follows that $dW/d\tau > (\hat{p}(\tau)-c)dQ/d\tau$.

Integrating over the intervals (τ_{j-1}, τ_j) and summing over j, we get $\Delta W > (\hat{p}(\tau) - c)\Delta Q$. Hence, we obtain the same useful sufficient condition for MPP to increase welfare as in the elastic demand case:

Proposition 6 *Suppose Assumptions* 1 *and* 2 *hold. Welfare increases under MPP in the inelastic demand framework if MPP results in a weakly higher aggregate demand.*

5 Average Price Provision

5.1 Elastic MFC demand

We now analyze the situation when an average price provision is in effect and MFCs' demand for the product is elastic. We assume that both MFCs and non-MFCs in market i have identical demand $q(\cdot)$. By definition, an average price provision guarantees that MFCs pay a proportion of the quantity-weighted average price. Under APP, the monopolist's profit maximization problem becomes:

$$\max_{\{p_i\}_{i=1}^n} \Pi = (1-\gamma)\sum_{i=1}^n (p_i - c)q_i(p_i) + \gamma \sum_{i=1}^n (p_{q,\alpha} - c)q_i(p_{q,\alpha}), \tag{5.1}$$

$$\text{where, } p_{q,\alpha} = (1-\alpha)p_q \text{ and } p_q = \frac{\sum_{i=1}^n p_i q_i(p_i)}{\sum_{i=1}^n q_i(p_i)}.$$

As a point of comparison, it is useful to begin our analysis by looking at the solution of the unconstrained problem when there is no provision for MFCs. Without any regulation, the monopolist chooses prices to maximize

$$\sum_{i=1}^n (p_i - c)q_i(p_i).$$

As before, let p_i^m denote the unique (under Assumption 2) value of p_i that solves the equation $q_i(p_i) + (p_i - c)q_i'(p_i) = 0$. In particular, p_i^m is the monopoly price in market i. In the elastic demand framework, the monopolist charges the monopoly prices p_i^m in market i if no average price provision is in effect.

Let $\hat{p}_i$ denote the equilibrium price charged in market i after APP in imposed. Then $\hat{p} = (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_n)$ solves the first-order conditions:

$$(1-\gamma)[(p_i - c)q_i'(p_i) + q_i(p_i)] +$$
$$\gamma(\frac{d}{dp_i}p_{q,\alpha})[\sum_{j=1}^n \{q_j(p_{q,\alpha}) + (p_{q,\alpha} - c)q_j'(p_{q,\alpha})\}] = 0 \text{ for } i = 1, 2, \ldots, n. \tag{5.2}$$

Any prices satisfying (5.2) will maximize profits, (5.1), if the objective function is strictly concave in $(p_1, p_2, \ldots, p_n)$ whenever demand is positive in all markets. Strict concavity of this profit function under APP is more stringent than Assumption 2. The following condition (together with Assumption 2) is sufficient for strict concavity of the monopolist's profit function under APP:

Assumption 3 $\sum_{i=1}^{n}(p_{q,\alpha}-c)q_i(p_{q,\alpha})$ *is weakly concave in* $(p_1, p_2, \ldots, p_n)$.

Under Assumptions 2 and 3, there exists a unique $(p_1, p_2, \ldots, p_n)$ that solves the monopolist's problem under APP. For the analyses that follow, we assume that this condition holds, and we characterize the behavior of the unique solution. The following lemma shows that the monopolist always charges a price above the marginal cost in every market after APP is imposed.

Lemma 3 *Suppose Assumptions 2 and 3 hold. At the solution of (5.1), prices are above the marginal cost in every market, i.e.,* $c < \hat{p}_i$ *for all* $i = 1, 2, \ldots, n$.

Proof. See Appendix. ■

Knowledge of the sign of $\frac{d}{dp_i}p_{q,\alpha}$ is useful for determining how the solution of the APP problem (5.1) compares to the unconstrained monopolist's solution, $(p_1^m, p_2^m, \ldots, p_n^m)$. The next two results show that $\frac{d}{dp_i}p_{q,\alpha}$ is strictly positive at the unconstrained solution and at least weakly positive at the APP solution.

Lemma 4 *Suppose Assumption 2 holds.* $\frac{d}{dp_i}p_q\Big|_{p=(p_1^m, p_2^m, \ldots, p_n^m)} > 0$ *for* $i = 1, 2, \ldots, n$.

Proof. See Appendix. ■

Lemma 5 *Suppose Assumptions 1, 2 and 3 hold. At the solution of (5.1),* $\frac{d}{dp_i}p_{q,\alpha}$ *is positive for all* i.

Proof. See Appendix. ■

We are now in a position to compare the APP prices, $\hat{p}$, to the unconstrained prices. By Lemma 5, and the fact that $\sum_{j=1}^{n}\{q_j(p_{q,\alpha}) + (p_{q,\alpha}-c)q_j'(p_{q,\alpha})\}$ is independent of i, the first term in (5.2), $(p_i - c)q_i'(p_i) + q_i(p_i)$, has the same sign at $\hat{p}$ for all i. At the solution of the unconstrained problem, this term is equal to zero for all i. Hence, by concavity (Assumption 2), when APP is imposed, prices in all markets move in the same direction compared to the unconstrained optimal prices. By inspection of (5.2), if $\sum_{j=1}^{n}\{q_j(p_{q,\alpha}) + (p_{q,\alpha}-c)q_j'(p_{q,\alpha})\}$ at $\hat{p}$ is positive (negative) then prices increase (decrease) under APP. The dividing line between these directions is the uniform monopoly price, p^u, as $\sum_{j=1}^{n}\{q_j(p^u) + (p^u - c)q_j'(p^u)\} = 0$ by definition. Hence, we have the following proposition:

Proposition 7 *Suppose Assumptions* $1, 2$ *and* 3 *hold. Prices in all markets move in the same direction from unconstrained profit maximizing prices when APP is imposed. Under APP, the discounted quantity-weighted average price is lower than the uniform monopoly price if and only if prices increase compared to the unconstrained case.*

The second part of the above proposition provides necessary and sufficient conditions for prices to increase when APP is imposed. Since these conditions require observing prices under APP, however, they would not allow a policy maker to evaluate the effect of imposing APP before actually implementing it. In our next result, we provide alternative necessary and sufficient conditions that depend only on unconstrained prices and the uniform monopoly price – neither of which depends on observing behavior under APP. Let p_q^m denote the quantity-weighted average price for the unconstrained problem, i.e., $p_q^m = \sum_{i=1}^n p_i^m q_i(p_i^m)/\sum_{i=1}^n q_i(p_i^m)$. Similarly, $p_{q,\alpha}^m \equiv (1-\alpha)p_q^m$.

Proposition 8 *Suppose Assumptions* $1, 2$ *and* 3 *hold. Prices (strictly) increase under APP if and only if* $p_{q,\alpha}^m \, (<) \leq p^u$*, i.e., if and only if the discounted quantity-weighted average price when the monopolist is unconstrained, is below the uniform monopoly price.*

Proof. Suppose $p_{q,\alpha}^m < p^u$. Then, by Assumption 2, $\sum_{i=1}^n \{q_i(p_{q,\alpha}) + (p_{q,\alpha} - c)q_i'(p_{q,\alpha})\} > 0$. By Lemma 4, $\frac{d}{dp_i}p_{q,\alpha} > 0$ for $i = 1, 2, \ldots, n$ at $(p_1^m, p_2^m, \ldots, p_n^m)$. Furthermore, at $(p_1^m, p_2^m, \ldots, p_n^m)$, $(p_i - c)q_i'(p_i) + q_i(p_i) = 0$ for $i = 1, 2, \ldots, n$. Therefore, (5.2) evaluated at $(p_1^m, p_2^m, \ldots, p_n^m)$ is strictly positive. Under Assumptions 2 and 3, problem (5.1) is globally concave, thus, (5.2) strictly positive at $(p_1^m, p_2^m, \ldots, p_n^m)$ implies $\hat{p}_i > p_i^m$ for $i = 1, 2, \ldots, n$. Therefore, prices strictly increase under APP.

Suppose $p_{q,\alpha}^m = p^u$. Then, since $\sum_{i=1}^n \{q_i(p_{q,\alpha}) + (p_{q,\alpha} - c)q_i'(p_{q,\alpha})\} = 0$ and $(p_i - c)q_i'(p_i) + q_i(p_i) = 0$ for $i = 1, 2, \ldots, n$, (5.2) evaluated at $(p_1^m, p_2^m, \ldots, p_n^m)$ is zero, and by global concavity, this solves (5.1) and prices do not change under APP.

Suppose $p_{q,\alpha}^m > p^u$. Then, by Assumption 2, $\sum_{i=1}^n \{q_i(p_{q,\alpha}) + (p_{q,\alpha} - c)q_i'(p_{q,\alpha})\} < 0$. By Lemma 4, $\frac{d}{dp_i}p_{q,\alpha} > 0$ for $i = 1, 2, \ldots, n$ at $(p_1^m, p_2^m, \ldots, p_n^m)$. Furthermore, at $(p_1^m, p_2^m, \ldots, p_n^m)$, $(p_i - c)q_i'(p_i) + q_i(p_i) = 0$ for $i = 1, 2, \ldots, n$. Therefore, (5.2) evaluated at $(p_1^m, p_2^m, \ldots, p_n^m)$ is strictly negative. Under Assumptions 2 and 3, problem (5.1) is globally concave, thus, (5.2) strictly negative at $(p_1^m, p_2^m, \ldots, p_n^m)$ implies $\hat{p}_i < p_i^m$ for $i = 1, 2, \ldots, n$. Therefore, prices strictly decrease under APP. ■

The basic intuition for the result is that under APP, MFCs pay a uniform price, namely the discounted quantity-weighted average of prices charged in different markets. The monopolist, therefore, all else equal, prefers to set prices so that the discounted quantity-weighted average price is close to the uniform monopoly price. When the unconstrained prices leave this average below the uniform monopoly price, APP pushes prices up and when unconstrained prices put this average above the uniform monopoly price, APP pushes prices down.

We can use the above result to relate the effect of APP to the discount, α, given off of average price. For high discounts, it is more likely that the discounted quantity-weighted average price at the pre-APP prices would be lower than the uniform monopoly price. Let α^* denote the discount for which the discounted quantity-weighted average price for the unconstrained problem is the same as the uniform monopoly price, i.e., $p^m_{q,\alpha^*} = p^u$. Observe that $\alpha^* = 1 - \frac{p^u}{p^m_q}$. From Proposition 8, we see that the monopolist increases price in every market if and only if the discount is above α^*.

Corollary 1 *Prices increase in every market when APP is imposed if and only if the discount α is greater than α^*.*

Is there anything more we can say about the effect of the discount parameter on prices? Let $\hat{p}_i(\alpha)$ denote the equilibrium price charged in market i after APP with a discount parameter $\alpha \in [0,1]$ is imposed. If the discount α is less than α^*, we will see that optimal prices under APP are strictly increasing in the discount α. To see the intuition behind this result, suppose that the current discount is $\alpha_1 < \alpha^*$. By the corollary, the optimal price in market i under APP is therefore strictly less than the monopoly price p^m_i. Consider the effect of a marginal increase in discount starting from α_1. We argue that the monopolist can increase profit by increasing prices from $\hat{p}(\alpha_1)$. As the discount increases, the discounted quantity-weighted average price computed at $\hat{p}(\alpha_1)$ falls. On the other hand, by increasing prices slightly in every market, the monopolist can increase the discounted quantity-weighted average price (this is because $\frac{d}{dp_i}\hat{p}(\alpha_1)_{q,\alpha} \geq 0$). By increasing prices in this way, the monopolist can keep the discounted quantity-weighted average price close to $\hat{p}(\alpha_1)_{q,\alpha_1}$. This movement in prices will thus not affect the monopolist's profit from the MFC section of the market. But an increase in prices will always increase the monopolist's profit from the non-MFC section of the market, since the original price $\hat{p}_i(\alpha_1)$ in market i was less than the monopoly price p^m_i. This is why the monopolist would prefer to increase prices as the discount α increases, as long as α is less than α^*. This line of argument does not hold for $\alpha > \alpha^*$ since an increase in prices does not increase profit from the non-MFC section of the market anymore. All we can say for these higher discounts is that whatever movements in prices occur as α moves above α^* are eventually reversed, because when the discount reaches 100% ($\alpha = 1$), prices return to the optimal unconstrained prices, just as they were at $\alpha = \alpha^*$.

Proposition 9 *Suppose Assumptions 1, 2 and 3 hold. For $\alpha < \alpha^*$ (i.e., if APP decreases prices), optimal prices under APP are strictly increasing in the discount α.*

5.2 Welfare Analysis (When MFCs' demand is elastic)

Aggregate demand as well as social welfare may move in either direction with the imposition of APP. To see this, we first consider the special case where $p^m_{q,\alpha} = p^u$. Then, by Proposition 8, the monopolist is not going to change prices, as at the unconstrained monopoly prices it is extracting monopoly rent from both the MFC and the non-MFC consumers even after APP is imposed. In this case, there will not be any change in aggregate demand or in welfare in the non-MFC section of the markets. However, MFCs in both markets will now be paying the uniform monopoly price instead of paying the individual monopoly prices for their markets. Therefore, for the MFC consumers, moving to APP is effectively a move from perfect third degree price discrimination to uniform pricing. From Schmalensee ([16], pp 244-245), we know that under such a scenario aggregate demand can move in either direction, depending on the curvature of the demand curves, and an increase in aggregate demand is sufficient for an increase in welfare. Recall that for MPP, we showed that a similar result was true quite generally. Unfortunately, the same is not true for evaluating the effect of APP. Specifically, once we leave the special case where the move to APP does not affect prices, there does not seem to be a simple condition relating changes in aggregate demand to changes in welfare.

To understand this, we briefly analyze changes in demand and welfare under the move to APP.

In order to analyze the welfare effect of APP, we will do the following. We join the two price vectors, giving the optimal prices in the unconstrained case and under APP respectively, $(p_1^m, \ldots, p_n^m)$ and $(\hat{p}_1, \ldots, \hat{p}_n)$ on the plane $\mathbb{R}^n$ by a piecewise smooth curve such that every point on the curve is a solution of a different optimization problem, where the problems are parametrized by $\tau \in [0,1]$. We study how aggregate demand and welfare, Q and W, change as τ varies and prices move along the curve.

Formally, define a set of optimization problems indexed by τ, and denoted by OP_τ, such that OP_0 is identical to the monopolist's unconstrained profit maximization problem, and OP_1 is the monopolist's profit maximization problem with APP. Specifically, for any $\tau \in [0,1]$,

$$\begin{aligned} OP_\tau \quad : \quad & \max_{\{p_i\}_{i=1}^n} \ (1-\tau)\sum_{i=1}^n (p_i - c) q_i(p_i) \\ & +\tau \left[(1-\gamma)\sum_{i=1}^n (p_i - c) q_i(p_i) + \gamma \sum_{i=1}^n (p_{q,\alpha} - c) q_i(p_{q,\alpha}) \right] \\ = \quad & \max_{\{p_i\}_{i=1}^n} \ (1-\tau\gamma)\sum_{i=1}^n (p_i - c) q_i(p_i) + \tau\gamma \sum_{i=1}^n (p_{q,\alpha} - c) q_i(p_{q,\alpha}) \end{aligned}$$

$Q(\tau)$ and $W(\tau)$ define the total production and social welfare respectively, when the monopolist is solving OP_τ. As is evident from the above equation, OP_τ is identical to the profit maximization problem under APP with a $\tau\gamma$ fraction of MFCs.

As $\tau \in [0,1]$, $\tau\gamma \in [0,\gamma]$. For simplicity, we replace $\tau\gamma$ by t in the above objective function, and restrict its range to $[0,\gamma]$. For such t, let $\hat{p}_i(t)$ be the unique solution to $OP_{\frac{t}{\gamma}}$ and let $\hat{p}_{q,\alpha}(t)$ give the corresponding discounted quantity-weighted average prices. As t is increased from zero to γ, $\hat{p}_i(t)$ moves from the optimal pre-APP price in market i, p_i^m, to the optimal price in market i under APP, $\hat{p}_i(\gamma)$. Similarly, let $Q(t)$ and $W(t)$ denote the corresponding aggregate demand and social welfare respectively. Thus,

$$
\begin{aligned}
Q(t) &= (1-t)\sum_{i=1}^{n} q_i(\hat{p}_i(t)) + t\sum_{i=1}^{n} q_i(\hat{p}_{q,\alpha}(t)) \\
W(t) &= (1-t)\sum_{i=1}^{n}\{(\hat{p}_i(t)-c)q_i(\hat{p}_i(t)) + \int_{\hat{p}_i(t)}^{\infty} q_i(v)dv\} \\
&\quad + t\sum_{i=1}^{n}\{(\hat{p}_{q,\alpha}(t)-c)q_i(\hat{p}_{q,\alpha}(t)) + \int_{\hat{p}_{q,\alpha}(t)}^{\infty} q_i(v)dv\}.
\end{aligned}
$$

We can compare aggregate demand and welfare at the two extreme points $t=0$ and $t=\gamma$ by studying dQ/dt and dW/dt. Observe that

$$
\begin{aligned}
\frac{dQ}{dt} &= \left[\sum_{i=1}^{n}\{q_i(\hat{p}_{q,\alpha}(t)) - q_i(\hat{p}_i(t))\}\right] + \left[(1-t)\sum_{i=1}^{n}\{q_i'(\hat{p}_i(t))\,\hat{p}_i'(t)\right] \\
&\quad + \left[t\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right)\sum_{i=1}^{n} q_i'(\hat{p}_{q,\alpha}(t))\right],
\end{aligned}
\tag{5.3}
$$

and

$$
\begin{aligned}
\frac{dW}{dt} &= (\hat{p}_{q,\alpha}(t)-c)\frac{dQ}{dt} + \left[\sum_{i=1}^{n}\int_{\hat{p}_{q,\alpha}(t)}^{\hat{p}_i(t)}\{q_i(v)-q_i(\hat{p}_i(t))\}dv\right] \\
&\quad - \left[(1-t)\sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-\hat{p}_i(t))q_i'(\hat{p}_i(t))\,\hat{p}_i'(t)\right].
\end{aligned}
\tag{5.4}
$$

Depending on the curvature of the demand curve, it is possible for $\frac{dQ}{dt}$ to be of any sign. In equation (5.4), the first term is the same sign as $\frac{dQ}{dt}$ and the second term is positive for any negatively sloped demand curves. However, the third term in (5.4) can be of any sign and may be large enough to outweigh the first two terms. Thus, the relation between dQ/dt and dW/dt is not unambiguous. Next, we give an example where social welfare decreases while aggregate demand increases.

Example 4 *We reconsider the previous example, now with the imposition of APP. As before, let $q_1(p) = 40-5p+0.15p^2$, $q_2(p) = 60-5p$ and $c=2$. Without APP, the monopolist charges $p_1^m = 6.52$ and $p_2^m = 7$ in markets 1 and 2 respectively. In this example, for any positive values of α, $p_{q,\alpha}^m$ is less than the uniform monopoly price $p^u = 6.85$. Therefore, for any discount α, the introduction of APP results in price increases in both markets. Figures 1 and 2 plot aggregate demand and social welfare against the fraction of MFCs, t, over the range $[0, 0.2]$ with discount $\alpha = 0.1$. One can see that aggregate demand increases as there are more MFCs while social welfare decreases. Thus, for these parameters, introduction of APP lowers welfare but raises aggregate quantity. For higher values of α (for example, $\alpha = 0.3$), both aggregate demand as well as welfare increase with t in this example, and thus APP increases both.*

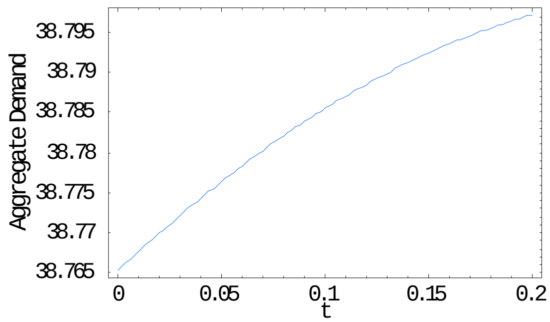

Figure 1: Aggregate demand vs. fraction of MFCs

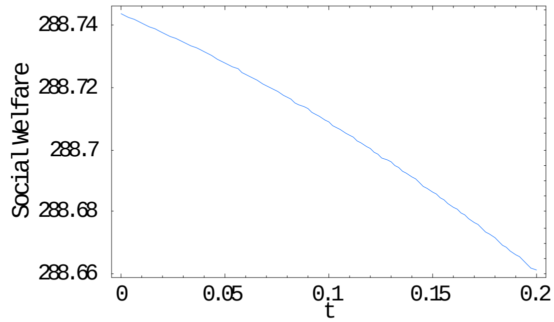

Figure 2: Social welfare vs. fraction of MFCs

5.3 Inelastic MFC Demand

Assume MFCs' demand for the product is completely inelastic. They demand a fixed quantity of the product while non-MFCs have a downward sloping demand curve. The monopolist, therefore, chooses $(p_1, \ldots, p_n)$ to maximize

$$(1-\gamma)\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \gamma\sum_{i=1}^{n}(p_i - c)z_i.$$

Defining $m_i \equiv \frac{\gamma z_i}{1-\gamma}$ and substituting yields

$$(1-\gamma)\left[\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \sum_{i=1}^{n}(p_i - c)m_i\right].$$

Under APP, this objective function becomes

$$(1-\gamma)\left[\sum_{i=1}^{n}(p_i - c)q_i(p_i) + \sum_{i=1}^{n}(p_{q,\alpha} - c)m_i\right]. \tag{5.5}$$

Without any restriction on the prices, the monopolist's optimal pricing strategy is to charge $p^* = (p_1^*, \ldots, p_n^*)$ where p_i^* solves

$$q_i(p_i) + (p_i - c)q_i'(p_i) + m_i = 0. \tag{5.6}$$

Denote the optimal price vector under APP by $\hat{p} = (\hat{p}_1, \ldots, \hat{p}_n)$. $\hat{p}$ solves the system of equations

$$q_i(p_i) + (p_i - c)q_i'(p_i) + (\sum_{j=1}^{n} m_j)(\frac{d}{dp_i}p_{q,\alpha}) = 0 \text{ for each } i. \tag{5.7}$$

Any prices satisfying (5.7) will maximize profits under APP if the objective function (5.5) is strictly concave in $(p_1, \ldots, p_n)$ whenever demand is positive in all markets. Strict concavity of this profit function under APP is more stringent than Assumption 2. The following condition (together with Assumption 2) is sufficient for strict concavity of the monopolist's profit function under APP:

Assumption 4 *$p_{q,\alpha}$ is weakly concave in $(p_1, \ldots, p_n)$.*

Under Assumptions 2 and 4, there exists a unique $(p_1, \ldots, p_n)$ that solves the monopolist's problem under APP. For the analyses that follow, we assume that this condition holds, and we characterize the behavior of the unique solution. Our next result provides some sufficient conditions for all prices to increase (and also for all prices to decrease) because of the introduction of an APP rule. These sufficient conditions are potentially useful in policy planning because they involve only pre-APP price and demand information (and thus information that is knowable before the policy decision about APP is undertaken).

Proposition 10 *Assume MFC demand is inelastic and assumptions 1, 2 and 4 hold. With the imposition of APP, prices in all markets increase if $\frac{d}{dp_i}p_{q,\alpha}$, computed at unconstrained optimal monopoly prices p^*, is greater than $m_i/\sum_{j=1}^n m_j$ for all i. Conversely, if $\frac{d}{dp_i}p_{q,\alpha}$, computed at p^* is smaller than $m_i/\sum_{j=1}^n m_j$ for all i, APP decreases prices in all markets.*

Proof. Suppose at p^*, $\frac{d}{dp_i}p_{q,\alpha} > m_i/\sum_{j=1}^n m_j$ for each i. Therefore, at $p = p^*$, the left-hand side of (5.7) is positive for each i. Since Assumptions 2 and 4 ensure that the optimization problem under APP is globally strictly concave, it must be that $\hat{p}_i > p_i^*$ for each i.
Similarly, suppose at p^*, $\frac{d}{dp_i}p_{q,\alpha} < m_i/\sum_{j=1}^n m_j$ for each i. Then, at $p = p^*$, the left-hand side of (5.7) is negative for each i. Global strict concavity then implies $\hat{p}_i < p_i^*$ for each i. ■

How the prices will change under APP if some of the average price derivatives are greater than $m_i/\sum_{j=1}^n m_j$, while others are not, is not obvious. However, one can show that $\frac{d}{dp_i}p_{q,\alpha}$ decreases with α for all prices. As a result, for high values of α, $\frac{d}{dp_i}p_{q,\alpha}$, computed at pre-APP optimal monopoly prices p^* will be smaller than $m_i/\sum_{j=1}^n m_j$ for each $i = 1, 2, ...n$. Therefore, APP will decrease prices in all markets if the discount from average price is high enough. The next result quantifies this sufficient condition.

Corollary 2 *For sufficiently high values of α, the imposition of APP decreases prices in every market. In particular, this occurs if α is greater than* $\max\{1 - \frac{m_i/\sum_{j=1}^n m_j}{\frac{d}{dp_i}p_{q|p=p^*}}, i = 1, 2, ...n\}$.

Since the pre-APP optimal price p_i^* in market i is strictly above the monopoly price for non-MFCs in market i, there is a tension between wanting to reduce prices for the non-MFCs and making the discounted quantity-weighted average price as high as possible to extract surplus from the MFCs. For high discounts (i.e., $(1 - \alpha)$ small), the change in the discounted quantity-weighted average price from any given change in prices is low (since $\frac{d}{dp_i}p_{q,\alpha} = (1 - \alpha)\frac{d}{dp_i}p_q$), and the MFC effect is correspondingly small. This is why reducing prices dominates for high values of α.

The above intuition also suggests that optimal prices under APP decrease with the discount parameter, α. Since MFCs' demand is inelastic, in every market the monopolist charges a price above the monopoly price for non-MFCs in that market. Therefore, at the optimal price in market i, when considering a decrease in price, the marginal gain in profit from the non-MFCs in market i exactly counterbalances the marginal loss in profit coming from the MFCs. As the discount increases, the marginal loss in profit from the MFCs when decreasing price in market i, measured by $(\sum_{j=1}^{n} m_j)(\frac{d}{dp_i} p_{q,\alpha})$, decreases. The marginal gain in profit from the non-MFCs in market i is unaffected. Therefore, as the discount increases, the monopolist finds it optimal to decrease prices. The next proposition (proved in the Appendix) formalizes this reasoning.

Proposition 11 *Assume MFC demand is inelastic and Assumptions 1, 2 and 4 hold. Optimal prices under APP are strictly decreasing in α.*

5.4 Welfare Analysis (When MFCs' demand is inelastic)

As before, we parametrize a set of optimization problems OP_τ, so that OP_0 is identical to the monopolist's profit maximization problem with no MPP, and OP_1 is the monopolist's profit maximization problem with MPP. Formally, for $\tau \in [0,1]$, we define OP_τ as follows:

$$\begin{aligned} OP_\tau \quad &: \quad (1-\gamma)\left[\begin{array}{c} \max_{\{p_i\}_{i=1}^n} \sum_{i=1}^n (p_i - c) q_i(p_i) + \tau \sum_{i=1}^n (p_{q,\alpha} - c) m_i \\ +(1-\tau)\sum_{i=1}^n (p_i - c) m_i \end{array}\right] \qquad (5.8) \\ &= \quad (1-\gamma)\left[\max_{\{p_i\}_{i=1}^n} \sum_{i=1}^n (p_i - c)\{q_i(p_i) + (1-\tau) m_i\} + \sum_{i=1}^n (p_{q,\alpha} - c)\tau m_i\right]. \end{aligned}$$

Note that the above optimization problem is similar to the profit maximizing problem under APP (5.5) if we replace the term $q_i(p_i)$ by $\{q_i(p_i)+(1-\tau)m_i\}$ and m_i by τm_i respectively in (5.5). Let $\hat{p}(\tau) = (\hat{p}_1(\tau), \ldots, \hat{p}_n(\tau))$ and $\hat{p}_{q,\alpha}(\tau)$ denote the optimal price in problem OP_τ and the discounted quantity-weighted average price computed at the optimal price vector $\hat{p}(\tau)$ respectively (Given our notation used to denote the pre-APP optimal monopoly price and post-APP optimal price, we have $\hat{p}(0) = p^*$ and $\hat{p}(1) = \hat{p}$).

For $\tau \in [0,1]$, $Q(\tau)$, aggregate demand corresponding to OP_τ, is given by

$$Q(\tau) = (1-\gamma)\left[\sum_{i=1}^n q_i(\hat{p}_i(\tau)) + \sum_{i=1}^n m_i\right], \qquad (5.9)$$

and, $W(\tau)$, the Marshallian welfare resulting from OP_τ, is given by

$$\begin{aligned} W(\tau) \quad &= \quad (1-\gamma)\left[\begin{array}{c} \sum_{i=1}^n \left\{\begin{array}{c} (\hat{p}_i(\tau) - c)(q_i(\hat{p}_i(\tau)) + (1-\tau) m_i) \\ + \int_{\hat{p}_i(\tau)}^{\bar{M}} (q_i(v) + (1-\tau) m_i) dv \end{array}\right\} \\ + \sum_{i=1}^n \left\{(\hat{p}_{q,\alpha}(\tau) - c)\tau m_i + \int_{\hat{p}_{q,\alpha}(\tau)}^{\bar{M}} \tau m_i dv\right\} \end{array}\right] \qquad (5.10) \\ &= \quad (1-\gamma)\left[\begin{array}{c} \sum_{i=1}^n \{(\hat{p}_i(\tau) - c)(q_i(\hat{p}_i(\tau)) + \int_{\hat{p}_i(\tau)}^{\infty} (q_i(v)) dv\} \\ + \sum_{i=1}^n \left(\bar{M} - c\right) m_i \end{array}\right] \qquad (5.11) \end{aligned}$$

In this calculation, we assume that the upper bound of the price while measuring the inelastic consumers' welfare is given by $\bar{M}$, some finite number large enough so that $q_i(\bar{M}) =$

0 for all i. This is equivalent to saying that demand isn't really inelastic, but rather is inelastic until price hits $\bar{M}$, and zero thereafter. Otherwise, consumer surplus for the inelastic consumers will always be infinity and our welfare measure would not be sensitive to changes in prices. Observe that the last term in (5.11) is the consumer plus producer surplus associated with the inelastic consumers, and that this is a constant – changes in price simply change the split between these consumers and the producer. Below we will be interested in changes in welfare, rather than levels. For this purpose, one may then ignore the inelastic part of the market and the conclusions are insensitive to the choice of $\bar{M}$.

As τ is increased from 0 to 1, $\hat{p}_i(\tau)$ moves from the optimal pre-APP price in market i, p_i^*, to the optimal price in market i under APP, $\hat{p}_i$. We can, therefore, compare aggregate demand and welfare at these two extreme points $\tau = 0$ and $\tau = 1$ by studying $dQ/d\tau$ and $dW/d\tau$. Observe that

$$\frac{dQ}{d\tau} = (1-\gamma)\sum_{i=1}^{n} q_i'(\hat{p}_i(\tau))\hat{p}_i'(\tau) \tag{5.12}$$

and

$$\begin{aligned}\frac{dW}{d\tau} &= (1-\gamma)\sum_{i=1}^{n}\left\{(\hat{p}_i(\tau)-c)q_i'(\hat{p}_i(\tau))\hat{p}_i'(\tau) + \hat{p}_i'(\tau)\left(q_i\left(\hat{p}_i(\tau)\right) - q_i\left(\hat{p}_i(\tau)\right)\right)\right\} && (5.13)\\ &= (1-\gamma)\sum_{i=1}^{n}(\hat{p}_i(\tau)-c)q_i'(\hat{p}_i(\tau))\hat{p}_i'(\tau) && (5.14)\end{aligned}$$

To better understand the terms in (5.12) and (5.14), note that the monopolist's objective function in the constructed optimization problem (5.8) is a convex combination of the objective function in pre-APP case and the objective function in post-APP case with weights $1-\tau$ and τ respectively. The demand of MFCs in the optimization problem (5.8) is fixed at $(1-\gamma)\sum_{i=1}^{n} m_i$ and is independent of τ. As τ changes, the optimal price $\hat{p}_i(\tau)$ in market i changes and the effect of these price changes on aggregate demand comes only through the change in non-MFC's demand. The change in non-MFC's demand is measured by (5.12). Similarly, social welfare for the inelastic section of the consumers in the optimization problem (5.8) is fixed at $(1-\gamma)\sum_{i=1}^{n}\left(\bar{M}-c\right)m_i$ and is independent of τ. As τ changes, the effect of price changes on social welfare comes only through the changes in social welfare for the elastic demand section of markets. This change is measured by (5.14).

Since prices may move in either direction, change in $\frac{dQ}{d\tau}$ is not unidirectional. However, the conditions for all prices to move in the same direction (either all to increase or all to decrease) stated in Proposition 10 provide sufficient conditions for APP to increase or decrease aggregate demand and welfare. From (5.12), we see that aggregate demand decreases (increases) if prices in every market increase (decrease) (since $q_i'(\hat{p}_i(\tau)) \leq 0$ for every market i). Since the monopolist always charges a price above the marginal cost c, welfare decreases (increases) if prices in every market increase (decrease). Hence, we have the following sufficient conditions for APP to decrease (increase) welfare:

Proposition 12 *Assume MFCs demand is inelastic and Assumptions* $1, 2$ *and* 4 *hold. With the imposition of APP, aggregate demand and welfare decrease (increase) if* $\frac{d}{dp_i} p_{q,\alpha}$*, computed at pre-APP optimal prices* p^**, is greater (smaller) than* $m_i / \sum_{j=1}^{n} m_j$ *for all* i*.*

6 Conclusion

6.1 Other Applications

Though the motivation for this paper mainly comes from the MFC clauses that are featured in the Medicaid reimbursement policy, our model is closely related to a broad class of contractual problems featuring similar clauses. Such clauses are used in contractual agreements in different industries (e.g. agreements between health care providers and health practitioners[10] (see Martin [10]), and most favored nation clauses in legal settlements (see Spier [20], [21]). In a selling context, they come in the form of minimum price protection that obligates a seller of a product or a service provider to treat the buyers (who are otherwise distinguishable) symmetrically in their pricing decision. Though the exact form of such agreements does not always match our formulation, we can often accommodate monopoly versions of these problems into our model with only a slight reformulation. Here, we provide a few examples.

a) Long term trading contracts with price protection: This type of contract is often present in markets where market power is on the side of the buyer. Applications include natural gas contracts (see e.g. Crocker and Lyon [6]) and other utility contracts. Sellers often sign contractual agreements with large buyers (or buyers with large sellers) to provide the buyers (or sellers) with price protection over an extended time period . We can accommodate this problem in our set up in the following way. Consider this as an n period problem, where demand may change from period to period. A section of buyers, treated as most favored customers, will be paying the minimum price that prevails over the n periods. However, the seller is allowed to charge different prices in different periods to other customers. As long as it is not possible to substitute demand in one period for demand in another, we can treat these n different periods as n different markets with distinct demand curves. If the section of most favored customers remains a fixed fraction of the total consumers in every market, this formulation will directly fit our model.

b) Exogenous shift of consumers between markets: Consider the example of an electronics

[10] Ocean State, a for-profit HMO, filed an antitrust case against Blue Cross and Blue Shield of Rhode Island, another health care provider, for introducing a Prudent Buyer policy that involves a most favored customer contract with physicians. The Prudent Buyer policy ensured that Blue Cross would not pay more for the services of its physician providers than what its providers are accepting from other health care companies, including Ocean State.

goods manufacturer who sells her product in different locations through retailers. Retailers differ in their bargaining power, depending on the size and elasticity of their individual markets. Assuming a high level of search cost, this would typically result in high dispersion in retail prices. Now consider an exogenous mechanism that can reduce the search cost for a section of consumers. For example, with the growth of web based transactions, almost every retailer now maintains a web site that allows online purchase of electronics goods. Not everybody can easily access or feels comfortable using that market, but for those who do, search cost is reduced to a large extent. Assuming that the fraction of consumers who may exercise the online purchasing option remains relatively constant across different markets, this implies that a section of consumers from every market now pay the minimum price (ignoring differences in retailer service provision and return policies).

What is important from a theoretical perspective is that these contractual agreements or exogenous shifts in location of consumers create a cross-market effect among the individual market prices in the monopolist's objective function. In each market, a fraction of the consumers is now paying a price that is connected to the prices charged in other markets. We precisely deal with the situation where this cross-market connection is induced through one of two different forms of price protection: MPP or APP.

6.2 Summary

Our analysis shows how the MPP and APP rebate rules affect a monopolist's optimal pricing strategy as well as social welfare under third-degree price discrimination. Under MPP, the minimum price charged always rises compared to the no regulation case. In fact, prices in *all* markets (weakly) rise if Medicaid and non-Medicaid consumers have the same demand characteristics. In contrast, if Medicaid demand is inelastic, prices in all markets where the minimum is not charged will fall. In either scenario, the welfare effect of MPP may be good or bad. A useful sufficient condition for MPP to be welfare improving is that MPP raise aggregate quantity.

Under APP and elastic demand, prices in all markets move in the same direction. Prices increase if and only if the discount percentage off of average price is above a threshold. When MFCs' demand is inelastic, we provide conditions sufficient for prices to move together in each direction. Prices under APP are decreasing in the discount, and if the discount is high enough, APP will lower prices in every market. Thus large discounts have opposite effects on price movements under APP in the elastic versus inelastic cases. As with MPP, the welfare effect of imposing APP is ambiguous in general. If MFCs' demand is inelastic, then if prices in all markets increase, both welfare and aggregate quantity fall, while if all prices decrease this is welfare and quantity improving.

The analysis of these policies is surprisingly intricate, even in a relatively simple setting such as ours. This suggests that great care is needed when implementing such MFC rules and that making provisions for data collection to support follow-up empirical work measuring the pricing and demand response has high potential value in avoiding mistakes or helping fine-tune the policy. Some theoretical issues that we have not addressed here, such as incorporating demand uncertainty, second-degree price discrimination and the effect on dynamic R&D incentives for the manufacturer are also interesting topics for future work to explore.

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7 Appendix

Proof of Lemma 2. :

We first show that $k(\gamma)$ is an (weakly) increasing function of γ. Let us take $0 \leq \gamma_1 < \gamma_2$. Note that $\hat{p}(\gamma)$ and $k(\gamma)$ solve equation (4.5) for $\gamma = \gamma_1,\ \gamma_2$. Thus,

$$\begin{aligned} & \sum_{i=1}^{k(\gamma_1)} \{(\hat{p}(\gamma_1) - c)q_i'(\hat{p}(\gamma_1)) + q_i(\hat{p}(\gamma_1))\} + \gamma_1 \sum_{i=k(\gamma_1)+1}^{n} \{(\hat{p}(\gamma_1) - c)q_i'(\hat{p}(\gamma_1)) + q_i(\hat{p}(\gamma_1))\} \\ = & \sum_{i=1}^{k(\gamma_2)} \{(\hat{p}(\gamma_2) - c)q_i'(\hat{p}(\gamma_2)) + q_i(\hat{p}(\gamma_2))\} + \gamma_2 \sum_{i=k(\gamma_2)+1}^{n} \{(\hat{p}(\gamma_2) - c)q_i'(\hat{p}(\gamma_2)) + q_i(\hat{p}(\gamma_2))\}. \end{aligned} \tag{7.1}$$

We will now show that $k(\gamma_2) < k(\gamma_1)$ contradicts (7.1). If $k(\gamma_2) < k(\gamma_1)$, from Proposition 1, it follows that $\hat{p}(\gamma_2) < \hat{p}(\gamma_1)$. By Assumption (2), we have

$$(\hat{p}(\gamma_1) - c)q_i'(\hat{p}(\gamma_1)) + q_i(\hat{p}(\gamma_1)) < (\hat{p}(\gamma_2) - c)q_i'(\hat{p}(\gamma_2)) + q_i(\hat{p}(\gamma_2)) \tag{7.2}$$

for every $i = 1, 2, ...n$. Furthermore, from Proposition 1, it follows that

$$\begin{aligned} \{(\hat{p}(\gamma) - c)q_i'(\hat{p}(\gamma)) + q_i(\hat{p}(\gamma))\} &< 0 \text{ for } i = 1, 2, \ldots k(\gamma) - 1 \\ &\leq 0 \text{ for } i = k(\gamma) \end{aligned}$$

and

$$\{(\hat{p}(\gamma) - c)q_i'(\hat{p}(\gamma)) + q_i(\hat{p}(\gamma))\} > 0 \text{ for } i = k(\gamma) + 1, \ldots n.$$

Hence,

$$\begin{aligned}
& \gamma_2 \sum_{i=k(\gamma_2)+1}^{n} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} \\
> \ & \gamma_1 \sum_{i=k(\gamma_2)+1}^{n} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} \\
(\text{as } \gamma_1 < \ & \gamma_2 \text{ and } \sum_{i=k(\gamma_2)+1}^{n} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} > 0) \\
> \ & \gamma_1 \sum_{i=k(\gamma_1)+1}^{n} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} \text{ (as } k(\gamma_2) < k(\gamma_1)) \\
> \ & \gamma_1 \sum_{i=k(\gamma_1)+1}^{n} \{(\hat{p}(\gamma_1)-c)q_i'(\hat{p}(\gamma_1))+q_i(\hat{p}(\gamma_1))\} \\
& (\text{by (7.2)}).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\sum_{i=1}^{k(\gamma_2)} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} &> \sum_{i=1}^{k(\gamma_2)} \{(\hat{p}(\gamma_1)-c)q_i'(\hat{p}(\gamma_1))+q_i(\hat{p}(\gamma_1))\} \\
&\geq \sum_{i=1}^{k(\gamma_1)} \{(\hat{p}(\gamma_1)-c)q_i'(\hat{p}(\gamma_1))+q_i(\hat{p}(\gamma_1))\}.
\end{aligned}$$

But then

$$\begin{aligned}
& \sum_{i=1}^{k(\gamma_2)} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} + \gamma_2 \sum_{i=k(\gamma_2)+1}^{n} \{(\hat{p}(\gamma_2)-c)q_i'(\hat{p}(\gamma_2))+q_i(\hat{p}(\gamma_2))\} \\
> & \sum_{i=1}^{k(\gamma_1)} \{(\hat{p}(\gamma_1)-c)q_i'(\hat{p}(\gamma_1))+q_i(\hat{p}(\gamma_1))\} + \gamma_1 \sum_{i=k(\gamma_1)+1}^{n} \{(\hat{p}(\gamma_1)-c)q_i'(\hat{p}(\gamma_1))+q_i(\hat{p}(\gamma_1))\}
\end{aligned}$$

contradicting the equality in (7.1). This proves $k(\gamma)$ is (weakly) increasing in γ.

To complete the proof, we must show that $\hat{p}(\gamma)$ is a strictly increasing function of γ. Fix $\gamma_1 < \gamma_2$. By what we proved above, $k(\gamma_1) \leq k(\gamma_2)$. If $k(\gamma_1) < k(\gamma_2)$, Proposition 1 directly implies that $\hat{p}(\gamma_1) < p^m_{k(\gamma_1)+1} \leq p^m_{k(\gamma_2)} \leq \hat{p}(\gamma_2)$. If $k(\gamma_1) = k(\gamma_2) = k$, then on the interval $[\gamma_1, \gamma_2]$, $\hat{p}(\gamma)$ solves the equation

$$\sum_{i=1}^{k} \{(\hat{p}(\gamma)-c)q_i'(\hat{p}(\gamma))+q_i(\hat{p}(\gamma))\} + \gamma \sum_{i=k+1}^{n} \{(\hat{p}(\gamma)-c)q_i'(\hat{p}(\gamma))+q_i(\hat{p}(\gamma))\} = 0. \tag{7.3}$$

Let us define the function $\pi_i(x) = (x-c)\,q_i(x)$ for $x > 0$. Then Equation (7.3) can be rewritten as

$$\sum_{i=1}^{k} \pi_i'(\hat{p}(\gamma)) + \gamma \sum_{i=k+1}^{n} \pi_i'(\hat{p}(\gamma)) = 0.$$

Differentiating with respect to γ over the interval $[\gamma_1, \gamma_2]$, we get $\sum_{i=1}^{k} \pi_i''(\hat{p}(\gamma))\hat{p}'(\gamma) + \gamma \sum_{i=k+1}^{n} \pi_i''(\hat{p}(\gamma))\hat{p}'(\gamma) + \sum_{i=k+1}^{n} \pi_i'(\hat{p}(\gamma)) = 0$. As $\pi_i''(\hat{p}'(\gamma)) < 0$ (by Assumption 2) and

$\sum_{i=k+1}^{n} \pi_i'(\hat{p}(\gamma))$ is strictly positive (follows from Proposition 1), we must have $\hat{p}'(\gamma) > 0$. Hence $\hat{p}$ is an increasing function of γ. ■

Derivation of equation (4.6):

Proof. For $t \in (\gamma_{j-1}, \gamma_j)$, $\hat{p}(t)$ solves

$$\frac{d}{dp}[\sum_{i=1}^{j-1}(p-c)q_i(p) + t\sum_{i=j}^{n}(p-c)q_i(p)] \mid_{p=\hat{p}(t)} = 0,$$

$$\text{or, } \sum_{i=1}^{j-1}\{(\hat{p}(t)-c)q_i'(\hat{p}(t)) + q_i(\hat{p}(t))\} + t\sum_{i=j}^{n}\{(\hat{p}(t)-c)q_i'(\hat{p}(t)) + q_i(\hat{p}(t))\} = 0$$

Differentiating with respect to t,

$$\begin{aligned}
&\sum_{i=1}^{j-1}\left\{(\hat{p}(t)-c)q_i''(\hat{p}(t))\hat{p}'(t) + 2q_i'(\hat{p}(t))(\hat{p}'(t)\right\} \\
&+t\sum_{i=j}^{n}\left\{(\hat{p}(t)-c)q_i''(\hat{p}(t))\hat{p}'(t) + 2q_i'(\hat{p}(t))\hat{p}'(t)\right\} \\
&+\sum_{i=j}^{n}\{(\hat{p}(t)-c)q_i'(\hat{p}(t)) + q_i(\hat{p}(t))\} = 0,
\end{aligned}$$

or,

$$\begin{aligned}
&2\hat{p}'(t)[\sum_{i=1}^{j-1} q_i'(\hat{p}(t)) + t\sum_{i=j}^{n} q_i'(\hat{p}(t))] \\
&= -(\hat{p}(t)-c)\hat{p}'(t)[\sum_{i=1}^{j-1} q_i''(\hat{p}(t)) + t\sum_{i=j}^{n} q_i''(\hat{p}(t))] \\
&-\sum_{i=j}^{n}\{(\hat{p}(t)-c)q_i'(\hat{p}(t) + q_i(\hat{p}(t)\}
\end{aligned} \tag{7.4}$$

$$\begin{aligned}
Q(t) &= \text{Aggregate demand at the equilibrium price vector of } OP_t \\
&= \sum_{i=1}^{j-1} q_i(\hat{p}(t)) + t\sum_{i=j}^{n} q_i(\hat{p}(t)) + (1-t)\sum_{i=j}^{n} q_i(p_i^m)
\end{aligned}$$

$$\begin{aligned}
\frac{dQ}{dt} &= \sum_{i=1}^{j-1} q_i'(\hat{p}(t))\,\hat{p}'(t) + t\sum_{i=j}^{n} q_i'(\hat{p}(t))\,\hat{p}'(t) + \sum_{i=j}^{n}\{q_i(\hat{p}(t)) - q_i(p_i^m)\} \\
&= -\frac{1}{2}(\hat{p}(t)-c)\hat{p}'(t)[\sum_{i=1}^{j-1} q_i''(\hat{p}(t)) + t\sum_{i=j}^{n} q_i''(\hat{p}(t))] \\
&-\frac{1}{2}\sum_{i=j}^{n}\{(\hat{p}(t)-c)q_i'(\hat{p}(t)) + q_i(\hat{p}(t))\} + \sum_{i=j}^{n}\{q_i(\hat{p}(t)) - q_i(p_i^m)\}
\end{aligned}$$

■

Derivation of equation (4.8)**:**

Proof.

$$W(t)=\sum_{i=1}^{j-1}\left\{\int_{\hat p(t)}^{\infty}q_i(v)dv+(\hat p(t)-c)q_i(\hat p(t))\right\}$$

$$+\sum_{i=j}^{n}\left[t\left\{\int_{\hat p(t)}^{\infty}q_i(v)dv+(\hat p(t)-c)q_i(\hat p(t))\right\}+(1-t)\left\{\int_{p_i^m}^{\infty}q_i(v)dv+(p_i^m-c)q_i(p_i^m)\right\}\right]$$

$$=\sum_{i=1}^{j-1}\left\{\int_{\hat p(t)}^{\infty}q_i(v)dv+(\hat p(t)-c)q_i(\hat p(t))\right\}$$

$$+t\sum_{i=j}^{n}\left\{\int_{\hat p(t)}^{p_i^m}q_i(v)dv+(\hat p(t)-c)q_i(\hat p(t))-(p_i^m-c)q_i(p_i^m)\right\}$$

$$+\text{a term independent of } t$$

$$\begin{aligned}
\frac{dW}{dt} &= \sum_{i=1}^{j-1}\left\{-q_i(\hat p(t))\hat p'(t)+q_i(\hat p(t))\hat p'(t)+(\hat p(t)-c)q_i'(\hat p(t))\hat p'(t)\right\}\\
&\quad +t\sum_{i=j}^{n}\{-q_i(\hat p(t))\hat p'(t)+q_i(\hat p(t))\hat p'(t)+(\hat p(t)-c)q_i'(\hat p(t))\hat p'(t)\}\\
&\quad +\sum_{i=j}^{n}\{\int_{\hat p(t)}^{p_i^m}q_i(v)dv+(\hat p(t)-c)q_i(\hat p(t))-(p_i^m-c)q_i(p_i^m)\}\\
&= \sum_{i=1}^{j-1}(\hat p(t)-c)q_i'(\hat p(t))\hat p'(t)+t\sum_{i=j}^{n}(\hat p(t)-c)q_i'(\hat p(t))\hat p'(t)\\
&\quad +\sum_{i=j}^{n}\{\int_{\hat p(t)}^{p_i^m}q_i(v)dv+(\hat p(t)-c)\{q_i(\hat p(t))-q_i(p_i^m)\}-(p_i^m-\hat p(t))q_i(p_i^m)\}\\
&= \sum_{i=1}^{j-1}(\hat p(t)-c)q_i'(\hat p(t))\hat p'(t)+t\sum_{i=j}^{n}(\hat p(t)-c)q_i'(\hat p(t))\hat p'(t)\\
&\quad +\sum_{i=j}^{n}(\hat p(t)-c)\{q_i(\hat p(t))-q_i(p_i^m)\}+\sum_{i=j}^{n}\{\int_{\hat p(t)}^{p_i^m}\{q_i(v)-q_i(p_i^m)\}dv\\
&= (\hat p(t)-c)[\sum_{i=1}^{j-1}q_i'(\hat p(t))\hat p'(t)+t\sum_{i=j}^{n}q_i'(\hat p(t))\hat p'(t)+\sum_{i=j}^{n}\{q_i(\hat p(t))-q_i(p_i^m)\}]\\
&\quad +\sum_{i=j}^{n}\{\int_{\hat p(t)}^{p_i^m}\{q_i(v)-q_i(p_i^m)\}dv\\
&\quad \text{by (4.7)}\\
&= (\hat p(t)-c)\frac{dQ}{dt}+\sum_{i=j}^{n}\{\int_{\hat p(t)}^{p_i^m}\{q_i(v)-q_i(p_i^m)\}dv
\end{aligned}$$

■

Derivation of equation (4.19)**:**

Proof. Applying the argument, which we used to derive (4.14), to the constructed optimization problem (4.16), we see that for any $\tau \in (\tau_{j-1}, \tau_j)$, $\hat{p}(\tau)$ solves

$$\frac{d}{dp}[\sum_{i=1}^{j-1}(p-c)\{q_i(p)+(1-\tau)m_i\}+\sum_{i=1}^{n}(p-c)\tau m_i]=0,$$
$$\text{or, } \frac{d}{dp}\left[(p-c)\left\{\sum_{i=1}^{j-1}(q_i(p)+m_i)+\sum_{i=j}^{n}\tau m_i\right\}\right]=0.$$

Hence,

$$\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i'(\hat{p}(\tau))+\left[\sum_{i=1}^{j-1}q_i(\hat{p}(\tau))+\sum_{i=1}^{j-1}m_i+\sum_{i=j}^{n}\tau m_i\right]=0$$

Differentiating with respect to τ,

$$\sum_{i=1}^{j-1}\{(\hat{p}(\tau)-c)q_i''(\hat{p}(\tau))\,\hat{p}'(\tau)+2q_i'(\hat{p}(\tau))\,\hat{p}'(\tau)\}+\sum_{i=j}^{n}m_i=0,$$
$$or,\ 2\sum_{i=1}^{j-1}q_i'(\hat{p}(\tau))\,\hat{p}'(\tau)=-\sum_{i=j}^{n}m_i-\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i''(\hat{p}(\tau))\,\hat{p}'(\tau) \tag{7.5}$$

For $\tau \in (\tau_{j-1}, \tau_j)$, $p_i^*(\tau),\ i=j,\ldots,n$, solves

$$\frac{d}{dp}[(p-c)\{q_i(p)+(1-\tau)m_i\}] = 0,$$
$$or,\ (p-c)q_i'(p)+q_i(p)+(1-\tau)m_i = 0.$$

Hence,

$$(p_i^*(\tau)-c)q_i'(p_i^*(\tau))+q_i(p_i^*(\tau))+(1-\tau)m_i=0.$$

Differentiating with respect to τ,

$$(p_i^*(\tau)-c)q_i''(p_i^*(\tau))\,p_i^{*\prime}(\tau)+2q_i'(p_i^*(\tau))\,p_i^{*\prime}(\tau)-m_i=0,$$
$$or,\ 2q_i'(p_i^*(\tau))\,p_i^{*\prime}(\tau)=m_i-(p_i^*(\tau)-c)q_i''(p_i^*(\tau))\,p_i^{*\prime}(\tau) \tag{7.6}$$

$$
\begin{aligned}
Q(\tau) &= \text{Aggregate demand at the equilibrium price vector of } OP_\tau \\
&= (1-\gamma)\left[\sum_{i=1}^{j-1} q_i(\hat{p}(\tau)) + \sum_{i=j}^{n} q_i(p_i^*(\tau)) + \sum_{i=1}^{n} m_i\right] \\
\frac{dQ}{d\tau} &= (1-\gamma)\left[\sum_{i=1}^{j-1} q_i'(\hat{p}(\tau))\,\hat{p}'(\tau) + \sum_{i=j}^{n} q_i'(p_i^*(\tau))\,p_i^{*\prime}(\tau)\right] \\
&= -\frac{1}{2}(1-\gamma)\left[\sum_{i=j}^{n} m_i + \sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i''(\hat{p}(\tau))\,\hat{p}'(\tau)\right] \\
&\quad +\frac{1}{2}(1-\gamma)\left[\sum_{i=j}^{n} m_i - \sum_{i=j}^{n}(p_i^*(\tau)-c)q_i''(p_i^*(\tau))\,p_i^{*\prime}(\tau)\right] \\
&= -\frac{1}{2}(1-\gamma)\left[\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i''(\hat{p}(\tau))\,\hat{p}'(\tau)\right] \\
&\quad -\frac{1}{2}(1-\gamma)\left[\sum_{i=j}^{n}(p_i^*(\tau)-c)q_i''(p_i^*(\tau))\,p_i^{*\prime}(\tau)\right]
\end{aligned}
$$

■

Derivation of equation (4.21)**:**

Proof.

$$
\begin{aligned}
\frac{dW}{dt} &= \frac{d}{dt}(1-\gamma)\left[\begin{array}{c} \sum_{i=1}^{n}\left(\bar{M}-c\right)m_i + \sum_{i=1}^{j-1}\{(\hat{p}(\tau)-c)q_i(\hat{p}(\tau)) + \int_{\hat{p}(\tau)}^{\infty} q_i(v)dv\} \\ + \sum_{i=j}^{n}\{(p_i^*(\tau)-c)q_i(p_i^*(\tau)) + \int_{p_i^*(\tau)}^{\infty} q_i(v)dv\} \end{array}\right] \\
&= (1-\gamma)\left[\frac{d}{dt}\sum_{i=1}^{j-1}\left\{(\hat{p}(\tau)-c)q_i(\hat{p}(\tau)) + \int_{\hat{p}(\tau)}^{\infty} q_i(v)dv\right\}\right] \\
&\quad + (1-\gamma)\left[\frac{d}{dt}\sum_{i=j}^{n}\{(p_i^*(\tau)-c)q_i(p_i^*(\tau)) + \int_{p_i^*(\tau)}^{\infty} q_i(v)dv\}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= (1-\gamma)\left[\sum_{i=1}^{j-1}\left\{(\hat{p}(\tau)-c)q_i'(\hat{p}(\tau))\hat{p}'(\tau)+q_i\left(\hat{p}(\tau)\right)\hat{p}'(\tau)-q_i\left(\hat{p}(\tau)\right)\hat{p}'(\tau)\right\}\right] \\
&\quad +(1-\gamma)\left[\sum_{i=j}^{n}\left\{(p_i^*(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)+q_i(p_i^*(\tau))p_i^{*\prime}(\tau)-q_i(p_i^*(\tau))p_i^{*\prime}(\tau)\right\}\right] \\
&= (1-\gamma)\left[\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i'(\hat{p}(\tau))\hat{p}'(\tau)+\sum_{i=j}^{n}(p_i^*(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)\right] \\
&= (1-\gamma)\left[\begin{array}{c}\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i'(\hat{p}(\tau))\hat{p}'(\tau)+\sum_{i=j}^{n}(\hat{p}(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau) \\ -\sum_{i=j}^{n}(\hat{p}(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)+\sum_{i=j}^{n}(p_i^*(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)\end{array}\right] \\
&= (1-\gamma)\left[\begin{array}{c}\sum_{i=1}^{j-1}(\hat{p}(\tau)-c)q_i'(\hat{p}(\tau))\hat{p}'(\tau)+\sum_{i=j}^{n}(\hat{p}(\tau)-c)q_i'(p_i^*(\tau))p_i^{*\prime}(\tau) \\ +\sum_{i=j}^{n}(p_i^*(\tau)-\hat{p}(\tau))q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)\end{array}\right] \\
&= (\hat{p}(\tau)-c)\frac{dQ}{d\tau}+(1-\gamma)\left[\sum_{i=j}^{n}(p_i^*(\tau)-\hat{p}(\tau))q_i'(p_i^*(\tau))p_i^{*\prime}(\tau)\right].
\end{aligned}
$$

■

Proof of Lemma 3. Note that $\frac{d}{dp_i}p_{q,\alpha}$ is strictly positive when price equals marginal cost in every market.

$$
\begin{aligned}
\frac{d}{dp_i}p_{q,\alpha}\left|_{p=(c,c,\ldots,c)}\right. &= (1-\alpha)\frac{d}{dp_i}p_q\left|_{p=(c,c,\ldots,c)}\right. \\
&= (1-\alpha)\frac{q_i\left(c\right)+(c-p_q\left|_{p=(c,c,\ldots,c)}\right.)q_i'\left(c\right)}{\sum_{j=1}^{n}q_j(c)} \\
&= (1-\alpha)\frac{q_i\left(c\right)}{\sum_{j=1}^{n}q_j(c)} \text{ since } p_q\left|_{p=(c,c,\ldots,c)}\right.=c \\
&> 0 \text{ for } i=1,2,\ldots,n.
\end{aligned}
$$

Therefore, (5.2) evaluated at $(c,c,\ldots,c)$ is strictly positive. Under Assumptions 2 and 3, problem (5.1) is globally concave, thus, (5.2) strictly positive at $(c,c,\ldots,c)$ implies $c<\hat{p}_i$ for $i=1,2,\ldots,n$. ■

Proof of Lemma 4. Since $p_i^m>c$ for all $i=1,2,\ldots,n$, we have $p_q\Big|_{p=\left(p_1^m,p_2^m,\ldots,p_n^m\right)}>c$. Since $q_i'\left(p_i^m\right)<0$ (if $q_i'\left(p_i^m\right)=0$ then p_i^m could not be profit maximizing, and so, by Assumption 2, could not satisfy the first order condition defining p_i^m), this implies $q_i\left(p_i^m\right)+(p_i^m-p_q\Big|_{p=\left(p_1^m,p_2^m,\ldots,p_n^m\right)})q_i'\left(p_i^m\right)>q_i\left(p_i^m\right)+(p_i^m-c)q_i'\left(p_i^m\right)=0$ for $i=1,2,\ldots,n$. Therefore,

$$
\frac{d}{dp_i}p_q\Big|_{p=\left(p_1^m,p_2^m,\ldots,p_n^m\right)}=\frac{q_i\left(p_i^m\right)+(p_i^m-p_q\Big|_{p=\left(p_1^m,p_2^m,\ldots,p_n^m\right)})q_i'\left(p_i^m\right)}{\sum_{j=1}^{n}q_j(p_j^m)}>0 \text{ for } i=1,2,\ldots,n.
$$

■

Proof of Lemma 5. For given $i\in\{1,2,\ldots,n\}$ and $p_{-i}=(p_1,\ldots p_{i-1},p_{i+1},\ldots,p_n)\in(c,\infty)^{n-1}$, define $\tilde{p}_i(p_{-i})\equiv\max\{r:\frac{d}{dp_i}p_q>0 \text{ for all } p_i\in(c,r)\}$. If $\frac{d}{dp_i}p_q>0$ for all $p_i>c$, define $\tilde{p}_i(p_{-i}) \equiv \infty$. Observe that

$$\frac{d}{dp_i} p_q \left|_{p=(c,p_{-i})}\right. = \frac{q_i(c) + (c - p_q \left|_{p=(c,p_{-i})}\right.) q_i'(c)}{q_i(c) + \sum_{j=1, j\neq i}^{n} q_j(p_j)}$$

and $\frac{d}{dp_i} p_q \left|_{p=(c,p_{-i})}\right.$ is strictly positive since, for $p_{-i} \in (c, \infty)^{n-1}$, $p_q(c, p_{-i})$ is strictly greater than c. As $\frac{d}{dp_i} p_q$ is continuous, either there exists a first $r > c$ such that $\frac{d}{dp_i} p_q \left|_{p=(r,p_{-i})}\right. = 0$ (and so $\tilde{p}_i(p_{-i}) = r$) or $\tilde{p}_i(p_{-i}) = \infty$. Thus, $\tilde{p}_i(p_{-i})$ is well defined.

Recall that $\hat{p}$ is the solution of (5.1). By Lemma 3, $\hat{p}_i > c$ for every i. Since $\frac{d}{dp_i} p_{q,\alpha} = (1-\alpha)\frac{d}{dp_i} p_q > 0$ for all $p_i \in (c, \tilde{p}_i(p_{-i}))$, to prove the result it is enough to show that $\hat{p}_i < \tilde{p}_i(\hat{p}_{-i})$ for each i. If $\tilde{p}_i(\hat{p}_{-i}) = \infty$, then we are done. Therefore, assume that $\tilde{p}_i(\hat{p}_{-i})$ is finite. Then, by definition of $\tilde{p}_i$ and continuity of $\frac{d}{dp_i} p_{q,\alpha}$, $\frac{d}{dp_i} p_{q,\alpha} = 0$ at $\tilde{p}_i(\hat{p}_{-i})$. Denoting the left-hand side of equation (5.2) by $F_i(p_i, p_{-i})$, this yields

$$F_i(\tilde{p}_i(\hat{p}_{-i}), \hat{p}_{-i}) = (1-\gamma)[(\tilde{p}_i(\hat{p}_{-i}) - c) q_i'(\tilde{p}_i(\hat{p}_{-i})) + q_i(\tilde{p}_i(\hat{p}_{-i}))].$$

As $\hat{p}_{-i} > c$ and $\tilde{p}_i(\hat{p}_{-i}) > c$, we have $p_q(\tilde{p}_i(\hat{p}_{-i}), \hat{p}_{-i}) > c$. Therefore,

$$\begin{aligned} F_i(\tilde{p}_i(\hat{p}_{-i}), \hat{p}_{-i}) &= (1-\gamma)[(\tilde{p}_i(\hat{p}_{-i}) - c) q_i'(\tilde{p}_i(\hat{p}_{-i})) + q_i(\tilde{p}_i(\hat{p}_{-i}))] \\ &< (1-\gamma)[(\tilde{p}_i(\hat{p}_{-i}) - p_q(\tilde{p}_i(\hat{p}_{-i}), \hat{p}_{-i})) q_i'(\tilde{p}_i(\hat{p}_{-i})) \\ &\quad + q_i(\tilde{p}_i(\hat{p}_{-i}))]. \end{aligned} \tag{7.7}$$

Since $\frac{d}{dp_i} p_{q,\alpha} = 0$ at $\tilde{p}_i(\hat{p}_{-i})$, $\tilde{p}_i(\hat{p}_{-i})$ satisfies

$$(\tilde{p}_i(\hat{p}_{-i}) - p_q(\tilde{p}_i(\hat{p}_{-i}), \hat{p}_{-i})) q_i'(\tilde{p}_i(\hat{p}_{-i})) + q_i(\tilde{p}_i(\hat{p}_{-i})) = 0,$$

and, from this and (7.7),

$$F_i(\tilde{p}_i(\hat{p}_{-i}), \hat{p}_{-i}) < 0.$$

Since $\hat{p}_i$ satisfies (5.2), $F_i(\hat{p}_i, \hat{p}_{-i}) = 0$. Strict concavity of the profit function under APP then implies $\hat{p}_i < \tilde{p}_i(\hat{p}_{-i})$. ■

Proof of Proposition 9. The optimal price $\hat{p}(\alpha) = (\hat{p}_1(\alpha), \hat{p}_2(\alpha), \ldots, \hat{p}_n(\alpha))$ under APP with discount parameter α solves the first order conditions in (5.2). Let $G_i(\alpha, p)$ denote the left-hand side of (5.2) as a function of the discount α and the price vector $p = (p_1, p_2, \ldots, p_n)$ for $i = 1, 2, \ldots, n$. Fix $0 \leq \alpha_1 < \alpha_2 < 1$. We know $G_i(\alpha_1, \hat{p}(\alpha_1)) = G_i(\alpha_2, \hat{p}(\alpha_2)) = 0$ for $i = 1, 2, \ldots, n$. Computing $G_i(\alpha, p)$ at $(\alpha_1, \hat{p}(\alpha_2))$, we find that

$$\begin{aligned} G_i(\alpha_1, \hat{p}(\alpha_2)) = (1-\gamma)[(\hat{p}_i(\alpha_2) - c) q_i'(\hat{p}_i(\alpha_2)) + q_i(\hat{p}_i(\alpha_2))] + \\ \gamma(\frac{d}{dp_i} \hat{p}(\alpha_2)_{q,\alpha_1})[\sum_{j=1}^{n} \{q_j(\hat{p}(\alpha_2)_{q,\alpha_1}) + (\hat{p}(\alpha_2)_{q,\alpha_1} - c) q_j'(\hat{p}(\alpha_2)_{q,\alpha_1})\}]. \end{aligned} \tag{7.8}$$

Since $G_i(\alpha_2, \hat{p}(\alpha_2)) = 0$, we have

$$(1-\gamma)[(\hat{p}_i(\alpha_2) - c)q_i'(\hat{p}_i(\alpha_2)) + q_i(\hat{p}_i(\alpha_2))] = \tag{7.9}$$
$$-\gamma(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2})[\sum_{j=1}^{n}\{q_j(\hat{p}(\alpha_2)_{q,\alpha_2}) + (\hat{p}(\alpha_2)_{q,\alpha_2} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_2})\}].$$

Therefore, (7.8) can be rewritten as

$$G_i(\alpha_1, \hat{p}(\alpha_2)) = \gamma(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_1})[\sum_{j=1}^{n}\{q_j(\hat{p}(\alpha_2)_{q,\alpha_1}) + (\hat{p}(\alpha_2)_{q,\alpha_1} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_1})\}] \tag{7.10}$$
$$-\gamma(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2})[\sum_{j=1}^{n}\{q_j(\hat{p}(\alpha_2)_{q,\alpha_2}) + (\hat{p}(\alpha_2)_{q,\alpha_2} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_2})\}].$$

Furthermore, $\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_i} = (1-\alpha_i)\frac{d}{dp_i}\hat{p}(\alpha_2)_q$. Hence,

$$G_i(\alpha_1, \hat{p}(\alpha_2)) = \gamma(\frac{d}{dp_i}\hat{p}(\alpha_2)_q)\left[\begin{array}{c} (1-\alpha_1)\left[\begin{array}{c}\sum_{j=1}^{n}\{q_j(\hat{p}(\alpha_2)_{q,\alpha 1}) \\ +(\hat{p}(\alpha_2)_{q,\alpha 1} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha 1})\}\end{array}\right] \\ -(1-\alpha_2)\left[\begin{array}{c}\sum_{j=1}^{n}\{q_j(\hat{p}(\alpha_2)_{q,\alpha_2}) \\ +(\hat{p}(\alpha_2)_{q,\alpha_2} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_2})\}\end{array}\right]\end{array}\right]$$

We now show that $G_i(\alpha_1, \hat{p}(\alpha_2)) < 0$ if $\alpha_2 < \alpha^*$. By Lemma 5, $\frac{d}{dp_i}\hat{p}(\alpha_2)_q \geq 0$. Therefore,

$$G_i(\alpha_1, \hat{p}(\alpha_2)) < 0 \text{ if and only if}$$
$$\left[\begin{array}{c} (1-\alpha_1)\left[\sum_{j=1}^{n}\left\{q_j(\hat{p}(\alpha_2)_{q,\alpha_1}) + (\hat{p}(\alpha_2)_{q,\alpha_1} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_1})\right\}\right] \\ -(1-\alpha_2)\left[\sum_{j=1}^{n}\left\{q_j(\hat{p}(\alpha_2)_{q,\alpha_2}) + (\hat{p}(\alpha_2)_{q,\alpha_2} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_2})\right\}\right]\end{array}\right] < 0$$

If $\alpha_2 < \alpha^*$, then, by Corollary 1, prices decrease under APP and, by Proposition 7,

$$\sum_{j=1}^{n}\left\{q_j(\hat{p}(\alpha_2)_{q,\alpha_2}) + (\hat{p}(\alpha_2)_{q,\alpha_2} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_2})\right\} < 0.$$

Since $\alpha_1 < \alpha_2$, $\hat{p}(\alpha_2)_{q,\alpha_1} > \hat{p}(\alpha_2)_{q,\alpha_2}$ and, $\sum_{j=1}^{n}\{q_j(k) + (k-c)q_j'(k)\}$ strictly decreasing in k (by Assumption 2),

$$(1-\alpha_1)\sum_{j=1}^{n}\left\{q_j(\hat{p}(\alpha_2)_{q,\alpha_1}) + (\hat{p}(\alpha_2)_{q,\alpha_1} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_1})\right\}$$
$$< (1-\alpha_2)\sum_{j=1}^{n}\left\{q_j(\hat{p}(\alpha_2)_{q,\alpha_2}) + (\hat{p}(\alpha_2)_{q,\alpha_2} - c)q_j'(\hat{p}(\alpha_2)_{q,\alpha_2})\right\}.$$

Therefore $\alpha_2 < \alpha^*$ implies $G_i(\alpha_1, \hat{p}(\alpha_2)) < 0$. This is useful because under Assumptions 2 and 3, the APP problem with discount parameter α_1 is globally concave in prices. Thus, $G_i(\alpha_1, \hat{p}(\alpha_2)) < 0$ implies $\hat{p}_i(\alpha_1) < \hat{p}_i(\alpha_2)$ and prices rise with α under APP in the region $\alpha < \alpha^*$. ■

Derivation of equation (5.3):

Proof.

$$\begin{aligned} Q(t) &= (1-t)\sum_{i=1}^{n} q_i(\hat{p}_i(t)) + t\sum_{i=1}^{n} q_i(\hat{p}_{q,\alpha}(t)) \\ \frac{dQ}{dt} &= \left[\begin{array}{c} -\sum_{i=1}^{n} q_i(\hat{p}_i(t)) + (1-t)\sum_{i=1}^{n} q_i'(\hat{p}_i(t))\,\hat{p}_i'(t) \\ +\sum_{i=1}^{n} q_i(\hat{p}_{q,\alpha}(t)) + t\sum_{i=1}^{n} q_i'(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) \end{array}\right] \\ &= \left[\sum_{i=1}^{n}\{q_i(\hat{p}_{q,\alpha}(t)) - q_i(\hat{p}_i(t))\}\right] + \left[(1-t)\sum_{i=1}^{n}\{q_i'(\hat{p}_i(t))\,\hat{p}_i'(t)\right] \\ &\quad + \left[t\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right)\sum_{i=1}^{n} q_i'(\hat{p}_{q,\alpha}(t))\right]. \end{aligned} \tag{7.11}$$

■

Derivation of equation (5.4):

Proof.

$$\begin{aligned} W(t) &= (1-t)\sum_{i=1}^{n}\left\{(\hat{p}_i(t)-c)q_i(\hat{p}_i(t)) + \int_{\hat{p}_i(t)}^{\infty} q_i(v)dv\right\} \\ &\quad + t\sum_{i=1}^{n}\left\{(\hat{p}_{q,\alpha}(t)-c)q_i(\hat{p}_{q,\alpha}(t)) + \int_{\hat{p}_{q,\alpha}(t)}^{\infty} q_i(v)dv\right\} \end{aligned}$$

$$\begin{aligned} \frac{dW}{dt} &= (1-t)\sum_{i=1}^{n}\left\{\begin{array}{c} (\hat{p}_i(t)-c)q_i'(\hat{p}_i(t))\,\hat{p}_i'(t) + q_i(\hat{p}_i(t))\,\hat{p}_i'(t) \\ -q_i(\hat{p}_i(t))\,\hat{p}_i'(t) \end{array}\right\} \\ &\quad - \sum_{i=1}^{n}\left\{(\hat{p}_i(t)-c)q_i(\hat{p}_i(t)) + \int_{\hat{p}_i(t)}^{\infty} q_i(v)dv\right\} \\ &\quad + t\sum_{i=1}^{n}\left\{\begin{array}{c} (\hat{p}_{q,\alpha}(t)-c)q_i'(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) + q_i(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) \\ -q_i(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) \end{array}\right\} \\ &\quad + \sum_{i=1}^{n}\left\{(\hat{p}_{q,\alpha}(t)-c)q_i(\hat{p}_{q,\alpha}(t)) + \int_{\hat{p}_{q,\alpha}(t)}^{\infty} q_i(v)dv\right\} \\ &= (1-t)\sum_{i=1}^{n}(\hat{p}_i(t)-c)q_i'(\hat{p}_i(t))\,\hat{p}_i'(t) \\ &\quad - \sum_{i=1}^{n}\left\{(\hat{p}_i(t)-c)q_i(\hat{p}_i(t)) + \int_{\hat{p}_i(t)}^{\infty} q_i(v)dv\right\} \\ &\quad + t\sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-c)q_i'(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) \\ &\quad + \sum_{i=1}^{n}\left\{(\hat{p}_{q,\alpha}(t)-c)q_i(\hat{p}_{q,\alpha}(t)) + \int_{\hat{p}_{q,\alpha}(t)}^{\infty} q_i(v)dv\right\} \end{aligned}$$

$$
\begin{aligned}
= \quad & (1-t)\sum_{i=1}^{n}(\hat{p}_i(t)-c)q_i'(\hat{p}_i(t))\,\hat{p}_i'(t) \\
& + t\sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-c)q_i'(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) \\
& + \sum_{i=1}^{n}\int_{\hat{p}_{q,\alpha}(t)}^{\hat{p}_i(t)} q_i(v)dv + \sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-c)q_i(\hat{p}_{q,\alpha}(t)) - \sum_{i=1}^{n}(\hat{p}_i(t)-c)q_i(\hat{p}_i(t)).
\end{aligned}
$$

Subtracting $(\hat{p}_{q,\alpha}(t)-c)\frac{dQ}{dt}$, we get

$$
\frac{dW}{dt}-(\hat{p}_{q,\alpha}(t)-c)\frac{dQ}{dt}
$$

$$
\begin{aligned}
&= (1-t)\sum_{i=1}^{n}(\hat{p}_i(t)-c)q_i'(\hat{p}_i(t))\,\hat{p}_i'(t) + t\sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-c)q_i'(\hat{p}_{q,\alpha}(t))\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right) \\
&\quad + \sum_{i=1}^{n}\int_{\hat{p}_{q,\alpha}(t)}^{\hat{p}_i(t)} q_i(v)dv + \sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-c)q_i(\hat{p}_{q,\alpha}(t)) - \sum_{i=1}^{n}(\hat{p}_i(t)-c)q_i(\hat{p}_i(t)) \\
&\quad -(\hat{p}_{q,\alpha}(t)-c)\left[\sum_{i=1}^{n}\{q_i(\hat{p}_{q,\alpha}(t))-q_i(\hat{p}_i(t))\}\right] \\
&\quad -(\hat{p}_{q,\alpha}(t)-c)\left[(1-t)\sum_{i=1}^{n}\{q_i'(\hat{p}_i(t))\,\hat{p}_i'(t)\right] \\
&\quad -\left[t\left(\frac{d}{dt}\hat{p}_{q,\alpha}(t)\right)\sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-c)q_i'(\hat{p}_{q,\alpha}(t))\right]
\end{aligned}
$$

$$
\begin{aligned}
&= (1-t)\sum_{i=1}^{n}(\hat{p}_i(t)-\hat{p}_{q,\alpha}(t))q_i'(\hat{p}_i(t))\,\hat{p}_i'(t) \\
&\quad -\sum_{i=1}^{n}(\hat{p}_i(t)-\hat{p}_{q,\alpha}(t))q_i(\hat{p}_i(t)) + \sum_{i=1}^{n}\int_{\hat{p}_{q,\alpha}(t)}^{\hat{p}_i(t)} q_i(v)dv \\
&= \sum_{i=1}^{n}\int_{\hat{p}_{q,\alpha}(t)}^{\hat{p}_i(t)}\{q_i(v)-q_i(\hat{p}_i(t))\}\,dv - (1-t)\sum_{i=1}^{n}(\hat{p}_{q,\alpha}(t)-\hat{p}_i(t))q_i'(\hat{p}_i(t))\,\hat{p}_i'(t)
\end{aligned}
$$

■

The following lemma, useful for proving Proposition 11, says that under APP, discounted quantity-weighted average price is increasing in the price in each market.

Lemma 6 *Suppose Assumptions 1, 2 and 4 hold. At the solution of (5.5), $\frac{d}{dp_i}p_{q,\alpha}$ is positive for all i.*

Proof. Similar to the proof of Lemma 5 and therefore omitted. ■

Proof of Proposition 11. The optimal price under APP with discount parameter α, denoted $\hat{p}(\alpha) = (\hat{p}_1(\alpha), \hat{p}_2(\alpha), \ldots, \hat{p}_n(\alpha))$, satisfies the first order conditions in (5.7). Let $G_i(\alpha, p)$ denote the left-hand side of (5.7) as a function of the discount α and the price vector

$p = (p_1, p_2, \ldots, p_n)$ for $i = 1, 2, \ldots, n$. Fix $0 \le \alpha_1 < \alpha_2 \le 1$. We know $G_i(\alpha_1, \hat{p}(\alpha_1)) = G_i(\alpha_2, \hat{p}(\alpha_2)) = 0$ for $i = 1, 2, \ldots, n$. Computing $G_i(\alpha, p)$ at $(\alpha_1, \hat{p}(\alpha_2))$, we find that

$$G_i(\alpha_1, \hat{p}(\alpha_2)) = [(\hat{p}_i(\alpha_2) - c)q_i'(\hat{p}_i(\alpha_2)) + q_i(\hat{p}_i(\alpha_2))] + (\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_1}) \sum_{j=1}^{n} m_j. \tag{7.12}$$

Since $G_i(\alpha_2, \hat{p}(\alpha_2)) = 0$, we have

$$[(\hat{p}_i(\alpha_2) - c)q_i'(\hat{p}_i(\alpha_2)) + q_i(\hat{p}_i(\alpha_2))] = -(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2}) \sum_{j=1}^{n} m_j.$$

Therefore, (7.12) can be rewritten as

$$G_i(\alpha_1, \hat{p}(\alpha_2)) = \left(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_1} - \frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2}\right) \sum_{j=1}^{n} m_j. \tag{7.13}$$

Furthermore, $\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_k} = (1 - \alpha_k)\frac{d}{dp_i}\hat{p}(\alpha_2)_q$. Hence,

$$\begin{aligned} G_i(\alpha_1, \hat{p}(\alpha_2)) &= \frac{d}{dp_i}\hat{p}(\alpha_2)_q [(1 - \alpha_1) - (1 - \alpha_2)] \sum_{j=1}^{n} m_j \\ &= (\alpha_2 - \alpha_1)\frac{d}{dp_i}\hat{p}(\alpha_2)_q \sum_{j=1}^{n} m_j. \end{aligned}$$

If $\alpha_2 < 1$, by Lemma 6, $\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2} > 0$, $\frac{d}{dp_i}\hat{p}(\alpha_2)_q = \frac{1}{1-\alpha_2}\left(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2}\right) > 0$, and therefore,
$G_i(\alpha_1, \hat{p}(\alpha_2)) > 0$. Similarly, if $\alpha_2 = 1$, $\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_2} = 0$ and $G_i(\alpha_1, \hat{p}(\alpha_2)) =$
$\left(\frac{d}{dp_i}\hat{p}(\alpha_2)_{q,\alpha_1}\right)\sum_{j=1}^{n} m_j > 0$. Under Assumptions 2 and 4, the APP problem with discount parameter α_1 is globally concave in prices. Thus, $G_i(\alpha_1, \hat{p}(\alpha_2)) > 0$ implies $\hat{p}_i(\alpha_1) > \hat{p}_i(\alpha_2)$ and prices fall with α. ■