Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt*

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Abstract

This paper proposes a dynamic politico-economic theory of debt, government finance and expenditure. Agents have preferences over a private and a government-provided public good, financed through labor taxation. Subsequent generations of voters choose taxation, government expenditure and debt accumulation through repeated elections. Debt introduces a conflict of interest between young and old voters: the young want more fiscal discipline as they are concerned with the ability of future governments to provide public goods. We characterize the Markov Perfect Equilibrium of the dynamic voting game. If taxes do not distort labor supply, the economy progressively depletes its resources through debt accumulation, leaving future generations "enslaved". However, if tax distortions are sufficiently large, the economy converges to a stationary debt level which is bounded away from the endogenous debt limit. We extend the analysis to redistributive policies and political shocks. Consistent with the empirical evidence, our theory predicts government debt to be mean reverting and debt growth to be larger under right-wing than under left-wing governments.

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1 Introduction

There are large differences in fiscal policies and government debt across countries and over time. Budgetary policy is a divisive issue, and different governments pursue diverse debt strategies.[1] In spite of this, there is still a limited theoretical understanding of the politico-economic forces determining public debt. Debt breaks the link between taxation and expenditure, allowing governments to shift the fiscal burden to future generations. In a world where Ricardian equivalence does not hold, this introduces a conflict of interest between current and future generations. As future generations are naturally under-represented in democratic decision making, there is a politico-economic force for debt accumulation. A fundamental question is, then: what prevents the current generations from passing the entire bill for current spending to future generations?

Financial markets could be part of the explanation; markets must believe that government liabilities will be honored. Yet, debt remains significantly below levels threatening solvency in industrialized countries. Moreover, despite the large cross-country heterogeneity in debt-GDP ratios, local interest rates respond little to the size of debt, at least among OECD countries.[2] In this paper, we abstract from effects working through changes in interest rates, and focus on the dynamic game over the provision of public goods in a small open economy where fiscal policy is set through repeated elections.

The model economy is populated by two-period-lived overlapping generations of agents who work when young and consume a private and a government-provided public good in both periods of their lives. The government can issue debt up to the natural borrowing limit and is committed to repay it. Every period old and young agents vote on public good provision, distortive labor taxation, and debt accumulation. The intergenerational conflict plays out as follows. Old voters support a large public expenditure financed by a budget deficit. Young voters, however, are wary of debt, because it crowds out future government spending. The political process, represented as a probabilistic-voting model *à la* Lindbeck and Weibull (1987), generates a compromise between the policies desired by the two groups.

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[1] For instance, under the Republican administrations of Reagan and Bush senior, the debt-GDP ratio in the US grew uninterruptedly from 26% to 49%. Clinton's administrations reversed this trend, and brought the ratio down to 35%. Thereafter, the debt has been rising under George W. Bush.

[2] For instance, the interest rate is almost uniform within the Euro area, although the debt ratios are very different across member countries (from less than 30% in Ireland, to more than 100% in Greece and Italy). In the same vein, Japan has been the OECD country with the highest debt-to-GDP ratio and the lowest interest rate for more than a decade.

Forward-looking voting is key. When voting over the current budget the young contemplate its implications on future public good provision. Leaving a large debt forces the next generation to make some fiscal adjustments: they must increase taxes, reduce expenditure, or expand debt further. When the lion's share of the future government's response is a cut in expenditure, young voters support a disciplined fiscal policy. Conversely, the more the future government responds by increasing taxes and debt, the less the young are opposed to debt expansion. Thus, expectations about the future fiscal policy shape the current fiscal policy. We embed such expectations into a dynamic voting game, and focus on Markov Perfect Equilibria (MPE) where the strategies of current voters are conditioned only on pay-off-relevant state variables. In our model, the only such state variable is the debt level, which greatly simplifies the analysis. The equilibrium response of future governments depends on the size of tax distortions. Intuitively, the more distortionary future taxation, the less future governments will be tempted to increase taxes, and the more they will instead cut public good provision in response to inherited debt. Therefore, the fiscal discipline becomes stronger when taxes are more distortionary, i.e., when the Laffer curve is more concave.[3]

We show that, in the absence of labor supply distortions, the economy would deplete resources through progressive debt accumulation. In the long run, future generations are "enslaved", i.e., their labor earnings are fully taxed away to service outstanding debt, and their consumption, both private and public, falls to zero. Instead, if tax distortions are sufficiently large, the economy converges to an "interior" steady state where debt is bounded away from the natural borrowing limit, and both private and public good consumption are positive. Thus, tax distortions help protect future generations from selfish fiscal policies of their ancestors.[4] The fiscal discipline hinges on the lack of commitment. In a Ramsey allocation where the first generation of voters can commit the entire future fiscal policy, debt is systematically larger than under repeated voting and converges asymptotically to the natural borrowing limit. Thus, future generations are better off in the political equilibrium than under commitment. In this sense, the time inconsistency has a *benign* nature.[5]

In the second part of the paper, we incorporate intra-generational conflict in our theory. We

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[3] For example, suppose that, at some high level of taxation, labor supply becomes infinitely elastic due to international tax competition. Then, future governments cannot increase taxes, and any increase in debt must be matched by a future reduction in expenditure. In this case, tax competition strengthens fiscal discipline.

[4] The point that in the presence of commitment problems government expenditure may be higher when the tax base is more elastic echoes the argument of Krusell, Quadrini and Rios-Rull (1997).

[5] The definition of a benign social planner's objective in OLG models is ambiguous. It is common in the literature to assume that the planner only attaches a positive weight to the welfare of the first generations, while future generations enter her preferences indirectly through the altruism of the first generation. For an exception, see Farhi and Werning (2005) where the planner attaches a positive weight on the welfare of all generations, resulting in an effective social discount factors exceeding the private one.

assume cross-sectional wage heterogeneity across dynasties, implying that agents belonging to high-productivity dynasties supply more labor to market activity, and hence bear a larger share of the tax burden of public good provision. Since public good provision entails a redistributive component, the poor want more government expenditure than the rich. Moreover, the poor are more averse to debt than the rich. The reason is that if more debt is issued today, then parts of the increased revenue will be used to finance a tax break in the current period at the expense of crowding out future public good provision. The poor and rich face a different tradeoff in this respect, as the poor gain less from reducing current taxes.[6] In equilibrium, the fiscal policy outcome depends on the political clout of the poor relative to the rich, which is assumed to change stochastically over time. Governments that attach a higher weight to the interests of poor voters ("left-wing governments") will choose higher government expenditure, lower debt growth and (in the short run) higher taxes than "right-wing governments". The predictions of our theory conform with the empirical evidence: in a panel of OECD countries right-wing governments are associated with more debt growth and less government expenditure growth.

Our paper contributes to the politico-economic literature that studies the determinants of government debt. Two important forebears are Persson and Svensson (1989) and Alesina and Tabellini (1990), who emphasized political conflict as a driving factor for public debt in two-period models without any intergenerational conflict.

In an influential article, Barro (1979) argued that governments should use debt to absorb fiscal shocks (e.g., a war), and spread the tax burden evenly over future periods in order to achieve tax smoothing. Thus, when the war is over, the increase in debt and taxes and reduction in public-good provision should persist. In contrast, our positive theory predicts a determinate debt level. In political equilibrium, a war is financed only partly through a short-term increase in debt. The rest is financed through a temporary increase in taxation and a temporary reduction in (non-military) public good provision. After the war, debt, taxes, and public goods revert back smoothly to their steady state levels. The data support this prediction of our theory. Bohn (1998) shows that a short-lived increase in US government expenditures implies an increase in debt with a subsequent reversion in debt. We find that this stylized fact holds up for a panel data set of OECD countries. Moreover, as noted by Barro (1986), non-military spending is crowded out during wars in the US – exactly as our model predicts.

Other recent papers predict autoregressive debt dynamics, albeit for different reasons. Aiya-

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[6] This point is reminiscent of the "starve-the-beast" argument, i.e., that budget deficits can be used to force reductions in future public expenditure.

gari *et al.* (2002) find that when the government has the ability to commit to future policies and only issues non-contingent debt, debt is stationary. Battaglini and Coate (2006) analyze fiscal policy and government debt with shocks to government policy in a legislative-bargaining model with no intergenerational conflict. In their model infinitely-lived agents would like to commit to large government savings when the value of the public good is low and debt accumulation and public-good provision when the value of the public good is high. However, legislators can also divert resources to pork-barrel transfers to geographically-defined districts. Due to this political conflict, legislators opt for inefficient transfers instead of government savings when the debt is low. Consequently, the equilibrium features too much debt, too little public-good provision, and stationary debt dynamics. Finally, Yared (2007) argues that debt should be persistent but stationary, due to voters trying to discipline a self-interested government. We view our paper as complementary to these papers, emphasizing a quite different mechanism for mean reversion of debt in the absence of commitment.

Our paper is also related to the growing politico-economic literature on dynamic fiscal policy, where agents vote repeatedly on redistribution and taxation. This literature includes Krusell *et al.* (1996), Krusell and Ríos-Rull (1999), Hassler *et al.* (2003), Hassler *et al.* (2005), Hassler, Storesletten and Zilibotti (2007), Song (2005a, 2005b), and Azzimonti Renzo (2005). These papers also focus on Markov-perfect equilibria (MPE), although they assume balanced government budget. One exception is Krusell *et al.* (2005), who investigate debt policies in a representative-agent Lucas-Stokey model without commitment. A common issue in this recent literature is that existence and uniqueness results are not in general available in dynamic games. In this paper, we exploit the methodology proposed by Judd (2004) to establish existence and uniqueness of a MPE in the neighborhood of particular cases to which contraction mapping arguments can be applied. In addition, we provide two examples for which full analytical solutions can be found.

Future pension liabilities are a form of government debt. Several authors have examined the political economy of pensions. The paper closest to ours is Tabellini (1990), who argues that pensions are driven by a coalition between young poor voters who want redistribution and retirees who want transfers. A large literature focuses on the politico-economic forces that would create and sustain the pension system.[7] These assume that there is no guarantee that the debt implicit in pension systems be honored. In contrast, we abstract from debt repudiation issues and focus on the intergenerational conflict about the timing of public-good

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[7]See, e.g., Chen and Song (2005) and Gonzalez Eiras and Niepelt (2004). Other authors focus on history-dependent (trigger) strategies (see e.g. Cooley and Soares (1999) and Boldrin and Rustichini (2000)).

consumption and taxation.

The paper is organized as follows. In section 2 we describe the model environment and characterize the commitment solution and the political equilibrium. Section 3 provides two tractable examples. Section 4 analyzes the general case. Section 5 introduces political shocks and discusses some empirical evidence. Section 6 concludes. The proofs of the Lemmas and Propositions are contained in an appendix.

2   Model Economy

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period. The population size is constant. Agents consume two goods: a private good (c) and a public good (g), provided by the government.

Private goods can be produced via two technologies – market and household production. Market production is subject to constant returns, and agents earn an hourly wage w. The household production technology is represented by the production function $y_H = F(1 - h)$, where the total time endowment is unity, h is the market labor supply, and $1 - h \geq 0$ is the time devoted to household production. The function F has the following properties;[8]

$$F'(\cdot) > 0, \ \ F'(1) < w, \ \ F''(\cdot) \leq 0, \ \ F'''(\cdot) \geq 0, \ \ \lim_{h \to 1} F'(1 - h) = \infty,$$

Since the government cannot tax household production, taxation distorts the time agents work in the market. Agents choose the allocation of their time so as to maximize total after-tax labor income, denoted by $A(\tau)$;

$$A(\tau) \equiv \max_{h \in [0,1]} \left\{ (1 - \tau) wh + F(1 - h) \right\}, \tag{1}$$

where $A'(\cdot) \leq 0$. This program defines the optimal market labor supply as a function of the tax rate, τ;

$$H(\tau) \equiv \arg \max_{h \in [0,1]} \left\{ (1 - \tau) wh + F(1 - h) \right\}, \tag{2}$$

where $H'(\cdot) \leq 0$ and $H''(\cdot) \leq 0$.

Consider the preferences of a young agent in dynasty i, born in period t;

$$U_{Y,i,t} = \log(c_{Y,i,t}) + \theta \log(g_t) + \beta \left(\log(c_{O,i,t+1}) + \theta \log(g_{t+1}) + \lambda U_{Y,i,t+1} \right), \tag{3}$$

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[8] In section 3, we analyze two limit cases where some of the assumptions do not hold. In example I, the household technology is not productive. In example II, the technology is linear, hence the Inada condition at $h = 1$ does not hold. Finally, in section 5 we assume that, for low-productivity agents, $F'(1) > w$. In all other cases, the assumptions above are assumed to hold.

where the subscript Y and O stand for "young" and "old", respectively. β is the discount factor, $\theta > 0$ is a parameter describing the intensity of preferences for public good consumption, and $\lambda \geq 0$ is the altruistic weight on the utility of the agent's child (denoted by $U_{Y,i,t+1}$). We assume λ to be small, consistent with our focus on imperfect altruism and on the ensuing failure of Ricardian equivalence. This assumption is key for the existence of an inter-generational conflict about the timing of taxation and public debt policy. In the rest of the paper, we omit time and dynasty subscripts when there is no source of confusion.

For the moment, we abstract from bequests. We will prove later that for λ sufficiently small – which is the case we focus upon – agents will not want to leave positive bequests even if they are allowed to. Abstracting from bequests simplifies the analysis, and is justified by the empirical observation, documented by a number of studies, that the bequest motive is modest and circumscribed to a limited fraction of the population (see, e.g., Hurd, 1989, Leitner and Ohlsson, 2001). Given labor supply $H(\tau)$ agents choose private consumption to maximize utility, (3), subject to their lifetime budget constraint;

$$c_{Y,i} + c_{O,i}/R = A(\tau), \tag{4}$$

where $R > 1$ is the gross interest rate and τ is the tax rate prevailing in the first period of the agent's life. This yields

$$c_{Y,i} = c_Y = \frac{A(\tau)}{1+\beta}, \ c_{O,i} = c_O = \frac{\beta R A(\tau)}{1+\beta}. \tag{5}$$

Fiscal policy is determined every period through repeated elections. Given an inherited debt b, the elected government chooses the tax rate (τ), the public good provision $(g \geq 0)$ and the debt accumulation (b'), subject to the following dynamic budget constraint;[9]

$$b' = g + Rb - \tau w H(\tau). \tag{6}$$

Both private agents and governments have access to an international capital market providing borrowing and lending at the constant gross interest rate R. The government is committed to not repudiate the debt. This implies that debt cannot exceed the present discounted value of the maximum tax revenue that can be collected;

$$b \leq \frac{\bar{\tau} w H(\bar{\tau})}{R-1} \equiv \bar{b}, \tag{7}$$

where $\bar{b}$ denotes the natural borrowing constraint and $\bar{\tau} \equiv \arg\max_\tau \tau \cdot H(\tau) > 0$ is the tax rate attaining the top of the Laffer curve. The constraint (7) rules out government Ponzi

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[9]Hereafter, we switch to a recursive notation with primes denoting next-period variables.

schemes. Throughout the paper we restrict attention to the range of taxes corresponding to the increasing portion of the Laffer curve, i.e., $\tau \leq \bar{\tau}$. This entails no loss of generality, since the political equilibrium would never select taxes above $\bar{\tau}$. This implies that in the relevant range $H(\cdot) > 0$, $H'(\cdot) < 0$ and $A'(\cdot) < 0$.

For technical reasons, we also impose a lower bound on debt, $b \geq \underline{b} > -\infty$. Note that $\underline{b}$ can be set arbitrarily small. Due to imperfect altruism, there is no motive in our economy for an unbounded surplus accumulation. Thus, the lower bound will never bind. Debt is then restricted to be in a compact set, $b \in [\underline{b}, \bar{b}]$. This, together with the government budget constraint (6), in turn implies that also g and τ are bounded : $g \in [0, \bar{g}]$ and $\tau \in [\underline{\tau}, \bar{\tau}]$, where $\underline{\tau}$ satisfies $-\underline{\tau}wH(\underline{\tau}) = \bar{b} - R\underline{b}$, and $\bar{g} = \bar{b} - R\underline{b} + \bar{\tau}wH(\bar{\tau})$.[10]

Having defined the domain of feasible fiscal policies, we can now establish that for sufficiently low λ agents do not want to leave positive bequests, which was assumed to derive the individual saving decision. The presence of a household technology ensures that for any parameter configuration, say Ω, there exists $\bar{\lambda}(\Omega) > 0$ such that, for all $\lambda \leq \bar{\lambda}(\Omega)$ the optimal bequest decision would yield a corner solution (zero bequests) for all feasible fiscal policies.[11]

Consider, next, the political process for the determination of fiscal policy. We model electoral competition as a two-candidate probabilistic voting model *à la* Lindbeck and Weibull (1987), which is extensively discussed in Persson and Tabellini (2000). In this model, agents cast their votes on one of two office-seeking candidates. Voters' preferences may differ not only over fiscal policy, but also over other orthogonal policy dimensions about which the candidates cannot make binding commitments. In a probabilistic voting equilibrium, both candidates propose the same fiscal policy, which turns out to maximize a weighted sum of individual utilities where the weights may differ between young and old agents.[12] Thus, the equilibrium policy maximizes a "political objective function" that is a weighted average utility for all voters. The first step to characterize the political equilibrium is to compute the indirect utility of young and old voters. In the case of the young, substituting (1) and (5) into (3), and ignoring

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[10] $\underline{\tau}$ is the maximum tax rebate that the government can pay if it runs the largest feasible deficit and provides zero public good, whereas $\bar{g}$ is the maximum public good provision attained by running the largest feasible deficit and collecting the maximum tax revenue.

[11] The marginal utility of private consumption is bounded (i) from above and (ii) away from zero. (i) Since $A(\tau) \geq F(1) > 0$, private consumption is strictly positive, implying that the marginal utility of private consumption is bounded from above. (ii) The marginal utility of private consumption is also bounded away from zero, since consumption is finite (taxes can be negative but not arbitrarily low due to the lower bound on debt). Suppose $\lambda = 0$. Then, old agents would strictly prefer to give negative bequests, and are therefore at a corner solution of the bequest decision. By continuity of the utility function, zero bequests must then also be optimal for λ sufficiently small, irrespective of fiscal policy.

[12] The weights can differ due to differences (between young and old) in their focus on fiscal policy relative to the orthogonal issues. The political clout of each group reflects the relative proportion of "swing voters", or the ability of the group to organize lobbies (see Persson and Tabellini, 2000).

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irrelevant constant terms yields:

$$U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1 + \beta) \log A(\tau) + \theta \log g + \beta \left(\theta \log g' + \lambda U_Y(\boldsymbol{b'}, \boldsymbol{\tau'}, \boldsymbol{g'}) \right), \tag{8}$$

where the primes denote next period's variables and boldface variables are vectors, defined as follows:

$$\boldsymbol{x} = \begin{bmatrix} x \\ x' \\ x'' \\ \dots \end{bmatrix} = \begin{bmatrix} x \\ \boldsymbol{x'} \end{bmatrix}.$$

Similarly the indirect utility of old voters can be expressed as

$$U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = \log(A(1 - \tau_{-1})) + \theta \log g + \lambda U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{9}$$

where τ_{-1} denotes the tax rate in the period when the current old were young and irrelevant constant terms are ignored.[13] Note that the old care about their children who are alive with them, so the children's utility, U_Y, is not discounted.

The equilibrium of a probabilistic voting model can be represented as the choice over time of τ, g and b' maximizing a weighted average indirect utility of young and old households, given b. We denote the weights of the old and young as ω and $1 - \omega$, respectively. Then the "political objective function" which is maximized every period by both political candidates is

$$U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1 - \omega) U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + \omega U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{10}$$

subject to (6) and (7).

2.1 The commitment solution

In this section, we characterize the (Ramsey) policy sequence that would be chosen by the first generation of voters if they could commit the entire future path of fiscal policy. It is useful to consider, first, the particular case in which $\omega = 1$, i.e., the old agents can dictate their preferred policy. Using equations (8)-(9), the problem admits the following recursive formulation;

$$V_O^{comm}(b) = \max_{\{\tau, g, b'\}} \left\{ v(\tau, g) + \beta \lambda V_O^{comm}(b') \right\} \tag{11}$$

subject to (6) and (7), where

$$v(\tau, g) \equiv (1 + \lambda) \theta \log g + (1 + \beta) \lambda \log A(\tau) \tag{12}$$

is the flow utility accruing to the initially old agents from the current public and private consumption, either directly or through their altruism for their children.

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[13] The term $A(1 - \tau_{-1})$ captures the wealth of the old. Note that due to logarithmic utility there is no interaction between the wealth of the old and any political choice variable. We focus on Markov equilibria so τ_{-1} should be irrelevant. With some abuse of notation, we therefore write $U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$ instead of $U_O(\boldsymbol{b}, \tau_{-1}, \boldsymbol{\tau}, \boldsymbol{g})$.

Lemma 1 *The program (11) subject to (6) and (7) is a contraction mapping. Hence, a solution exists and is unique.*

This is a standard recursive program, and its solution is time consistent, namely, it coincides with the allocation that would be chosen sequentially by successive generations of old agents. To solve the program, note that the intra-temporal first-order condition linking g and τ in problem (11) is;[14]

$$\frac{1+\beta}{\left(1+\frac{1}{\lambda}\right)\theta}g = A(\tau)(1-e(\tau)), \tag{13}$$

where $e(\tau) \equiv -(dH(\tau)/d\tau)(\tau/H(\tau))$ is the elasticity of labor supply. Note that $\tau \leq \bar{\tau}$ implies that $e(\tau) \leq 1$, and $F''' \geq 0$ implies that $e'(\tau) \geq 0$. Hence, along the equilibrium path, higher g is associated with lower τ and higher c. The intertemporal first-order condition leads to a standard Euler equation for public consumption;

$$\frac{g'}{g} = \beta\lambda R. \tag{14}$$

Next, we generalize the commitment solution to the case where the policy maximizes the weighted average discounted utility of all agents who are alive in the initial period (see equation (10)). In this case, a standard recursive formulation does not exist. However, the program admits a two-stage recursive formulation formalized in the following lemma;

Lemma 2 *The commitment problem admits a two-stage recursive formulation where;*
(i) In the initial period, policies are such that

$$\{\tau_0, g_0, b_1\} = \arg\max_{\{\tau_0, g_0, b_1\}} \left\{ v(\tau_0, g_0) - (1-\psi\lambda)\theta\log g_0 + \beta\lambda V_O^{comm}(b_1) \right\}, \tag{15}$$

subject to (6) and (7), where the function $V_O^{comm}(.)$ is given by (11), and the constant ψ is

$$\psi \equiv \frac{\omega}{1-\omega(1-\lambda)} \in \left(0, \frac{1}{\lambda}\right).$$

(ii) After the first period, the problem is equivalent to (11).

Existence and uniqueness results extend to the general case of $\omega < 1$. Three features are worth noting. First, comparing (15) with (11) shows that the influence of the young (i.e., low ω) reduces the weight on current public good provision (g_0) relative to current taxation (τ_0) and

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[14] The first-order conditions with respect to τ and g are;

$$\frac{(1+\beta)\lambda}{A(\tau)}\frac{A'(\tau)}{(wH(\tau)+\tau wH'(\tau))} = \beta\lambda\frac{\partial}{\partial b}\left(V_O^{comm}(b')\right) \quad \text{and} \quad \frac{(1+\lambda)\theta}{g} = -\beta\lambda\frac{\partial}{\partial b}\left(V_O^{comm}(b')\right),$$

These equations, plus the fact that $A'(\tau) = -wH(\tau)$, lead to (13).

debt accumulation (b_1) in the planner's objective function.[15] In the commitment problem, the "disciplining" role of the young, captured by the term $-(1-\psi\lambda)\,\theta\log g_0$, appears only in the first-period return function. The continuation utility is identical to the case in which $\omega=1$. We will see that a similar force operates every period in the dynamic game describing the politico-economic equilibrium. Second, when $\omega<1$ the commitment solution is time inconsistent. More precisely, if any future generation were allowed to re-optimize, it would deviate from the policy sequence dictated by the initial generation. Third, after the first period, the law of motion of the commitment solution is independent of ω: the government expenditure follows equation (14). Consistent with our focus on low altruism, we assume that $\beta\lambda R\leq 1$. When $\beta\lambda R<1$ (which will be our main case), public good provision declines asymptotically to zero, taxes converge to the top of the Laffer curve, and debt accumulates progressively, converging asymptotically to the natural limit, $\bar{b}$. If $\beta\lambda R=1$, the solution is stationary, so debt, taxes, and consumption remain constant at their initial levels, as in Barro (1979).

Proposition 1 *The "commitment" solution is such that (i) if $\beta\lambda R<1$, then $\lim_{t\to\infty} b_t=\bar{b}$ (hence, $\lim_{t\to\infty} g_t=0$ and $\lim_{t\to\infty}\tau_t=\bar{\tau}$); (ii) if $\beta\lambda R=1$, then $b_{t+1}=b_t$ for $t\geq 1$.*

2.2 The political equilibrium

We now characterize the political equilibrium without commitment, which is the main contribution of our paper. A political equilibrium is an equilibrium of a dynamic game between successive generations of voters. The set of equilibria is potentially large. We restrict attention to Markov Perfect Equilibria (MPE) where agents condition their voting strategies only on pay-off-relevant state variables. In principle, consecutive periods are linked by two state variables: the government debt, b, and the private wealth of the old. However, since preferences are separable between private and public goods consumption, the wealth of the old does not affect their preference over fiscal policies. Therefore, b is the only pay-off-relevant state variable. Our MPE thus feature policy rules as functions of b only.

Definition 1 *A Markov Perfect Political Equilibrium (MPPE) is defined as a 3-tuple of functions $\langle B,G,T\rangle$, where $B:\left[\underline{b},\bar{b}\right]\to\left[\underline{b},\bar{b}\right]$ is a debt rule, $b'=B(b)$, $G:\left[\underline{b},\bar{b}\right]\to[0,\bar{g}]$ is a*

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[15] Equation (15) is derived from writing the maximization problem as

$$\arg\max_{\{b_1,g_0,\tau_0\}}\left\{(1+\lambda\omega)\,\theta\log g_0+(1-\omega+\lambda\omega)\,(1+\beta)\log A\,(\tau_0)+(1-\omega+\lambda\omega)\,\beta V_O^{comm}\,(b_1)\right\}.$$

Here, both private consumption, $\log A\,(\tau_0)$, and the discounted continuation utility, $\beta V_O^{comm}\,(b_1)$, are weighted by $1-\omega$ (the weight of the young) plus $\lambda\omega$ (the altruistic preference of the old), whereas public-good consumption, $\theta\log g_0$, is weighted by unity (the sum of the weights of the young and of the old) plus $\lambda\omega$ (the altruistic preference of the old). Multiplying each term by $\lambda/\left(1-\omega+\lambda\omega\right)$, and rearranging terms, yields (15).

government expenditure rule, $g = G(b)$, and $T : [\underline{b}, \bar{b}] \to [\underline{\tau}, \bar{\tau}]$ is a tax rule, $\tau = T(b)$, such that:

1. $\langle B(b), G(b), T(b) \rangle = \arg\max_{\{b' \in [\underline{b}, \bar{b}], g \in [0, \bar{g}], \tau \in [\underline{\tau}, \bar{\tau}]\}} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$, *subject to (6) and (7), where*

$$
\boldsymbol{\tau} = \begin{bmatrix} \tau \\ T(b') \\ T(B(b')) \\ T(B(B(b'))) \\ \dots \end{bmatrix}, \boldsymbol{g} = \begin{bmatrix} g \\ G(b') \\ G(B(b')) \\ G(B(B(b'))) \\ \dots \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} b \\ b' \\ B(b') \\ B(B(b')) \\ \dots \end{bmatrix}
$$

and $U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$ is defined as in (10), given (8)-(9).

2. *The government budget constrained is satisfied:*

$$
B(b) = G(b) + Rb - T(b) \cdot w \cdot H(T(b)) \tag{16}
$$

Definition 2 *A MPPE is said to be Differentiable (DMPPE) if the equilibrium functions $\langle B, G, T \rangle$ are continuously differentiable on the interior of their domain, $(\underline{b}, \bar{b})$.*

In words, the government chooses the current fiscal policy (taxation, expenditure and debt accumulation) subject to the budget constraint, under the expectation that future fiscal policies will follow the equilibrium policy rules, $\langle B(b), G(b), T(b) \rangle$. Furthermore, the vector of policy functions must be a fixed point of the system of functional equations in part 1 and 2 of the definition, where part 2 requires the equilibrium policies to be consistent with the resource constraint.

The following Lemma is a useful step to characterize the MPPE.

Lemma 3 *Part 1 of Definition 1 admits the following two-stage recursive formulation:*

$$
\langle B(b), G(b), T(b) \rangle = \arg\max_{\{b' \in [\underline{b}, \bar{b}], g \in [0, \bar{g}], \tau \in [\underline{\tau}, \bar{\tau}]\}} \left\{ v(\tau, g) - (1 - \psi\lambda)\theta\log g + \beta\lambda V_O(b') \right\}, \tag{17}
$$

where $v(.)$ is defined as in (12), subject to (6) and (7), and where V_O satisfies the following functional equation;

$$
V_O(b') = v(T(b'), G(b')) + \beta\lambda V_O(B(b')). \tag{18}
$$

The difference between the commitment solution and the political equilibrium can be seen by comparing (11) with (18). In the political equilibrium, the first generation of voters cannot choose the entire future policy sequence, but take the mapping from the state variable into

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the (future) policy choices as given. For this reason, there is no maximization operator in the definition of V_O. The two programs are identical if and only if $\omega = 1$ (only the old vote), i.e., when the commitment solution is time consistent.

What is the source of the difference between the commitment solution and the MPPE? When $\omega < 1$, the young, who care directly (i.e., not only through their altruism) about next-period public expenditure, demand more public savings (and lower taxation) than the old, so the children are more fiscally disciplined than the parents. In the commitment solution, the intergenerational conflict affects only the first-period fiscal policy, since the altruistic preferences of parents and children in the first generation are aligned. In contrast, the conflict is persistent in the MPPE, as a new generation of young voters enters the stage in each election. As a result, the political equilibrium delivers, as we shall see, less debt accumulation.

The intra-temporal first-order condition linking g and τ in problem (17) is;[16]

$$\frac{1+\beta}{(1+\psi)\theta}g = A(\tau)(1-e(\tau)). \tag{19}$$

The only difference between (19) and (13) lies in the denominator of the term on the left-hand side, where λ^{-1} is replaced by $\psi \leq \lambda^{-1}$. Conditional on τ, public expenditure is lower (hence, there is less debt accumulation) in the MPPE than in the commitment solution. If a differentiable political equilibrium exists, it can be characterized by applying standard recursive methods to the first-order conditions of (17)-(18), together with (19):

Proposition 2 *A DMPPE is fully characterized by a system of three functional equations:*

1. The Generalized Euler Equation (GEE) for public good consumption:

$$\frac{G(B(b))}{G(b)} = \beta\lambda R - \underbrace{\beta\lambda G'(B(b))\left(\frac{1+\lambda^{-1}}{1+\psi}-1\right)}_{the\ disciplining\ effect}, \tag{20}$$

2. The intra-temporal first-order condition

$$\frac{1+\beta}{(1+\psi)\theta}G(b) = A(T(b))(1-e(T(b))). \tag{21}$$

3. The government budget constraint, given by equation (16).

The GEE is the key equation of Proposition 2. It expresses the growth rate of public good consumption as the sum of two terms. The first is a standard Euler-equation term capturing

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[16] The formal derivation of equation (19) can be found in the proof of Proposition 2.

the intertemporal trade off between current and future consumption, as in the commitment solution, (14). The second term (the *disciplining effect*) captures the dynamic-game element of the DMPPE. This term, which is absent in the commitment solution, hinges on the forward-looking voting of the young, and vanishes when the old have all political power (recall that $\omega = 1$, implies $\psi = \lambda^{-1}$). Young voters anticipate that an increase in the debt left to the future generation $(B(b))$ will prompt a future fiscal adjustment. The effect of such adjustment on next period's government expenditure is captured by the derivative $G'(B(b))$. Although a global characterization of $G'(.)$ is not available – except in particular cases discussed later –, it is easy to establish that $G'(b) < 0$ in the neighborhood of any steady state.[17]

$G'(.) < 0$ means, quite intuitively, that higher debt is associated with lower public spending. Since $\psi \leq 1/\lambda$, the disciplining effect is then positive: the growth rate of public expenditure $(G(B(b))/G(b))$ is an increasing function of the disciplining effect and, hence, of the political weight of the young. As in a standard Euler equation, high growth of g is attained by reducing expenditure and increasing public savings today.

The commitment solution coincides with the MPPE when only the old have political influence. In this case, the existence and uniqueness of the MPPE follows immediately from Lemma 1:[18]

Lemma 4 *Assume that $\omega = 1$. Then, the MPPE induces the same allocation as the commitment solution. Consequently, the MPPE exists and is unique.*

In the appendix, we provide sufficient conditions for the equilibrium policy functions to be continuous and differentiable, namely, for the equilibrium in the $\omega = 1$ case to be a DMPPE (see Lemma 5 in the appendix). The crux is to impose restriction on preferences and on the household technology that guarantee the concavity of the return function in the contraction mapping. Although the exact analytical condition is involved, it is easy to verify it numerically.

Extending the proof of existence and uniqueness of the MPPE to the general case of $\omega < 1$ is not straightforward. This is often difficult in dynamic games, as these problems generally do not admit a contraction-mapping formulation. However, Judd (2004) provides a strategy for proving local existence and uniqueness in such environments. He proposes to perturb the

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[17]To see why, observe that in steady state the GEE, (20), reduces to $G'(b^*) = -(1+\psi)(1-\beta\lambda R)((\beta(1-\lambda\psi)) < 0$. Thus, G' is independent of b^*. It is also easy to prove that G is concave around steady state, as long as B converges monotonically to the steady state (details are available upon request).

[18]The proof of the Lemma is immediate. Note that the problem (17) can be rewritten as $V_O(b) = \max_{\{b' \in [\underline{b}, \bar{b}], g \in [0, \bar{g}], \tau \in [\underline{\tau}, \bar{\tau}]\}} \{v(\tau, g) + \beta\lambda V_O(b')\}$, subject to (6) and (7), which is identical to the problem (11).

GEE in the neighborhood of a particular parameter configuration for which the problem can be shown to be a contraction mapping.[19] Here, we exploit the same strategy, by perturbing the equilibrium around the $\omega = 1$ case. The following proposition establishes local existence and uniqueness of the DMPPE.

Proposition 3 *Let $\left\langle \bar{B}\left(b\right), \bar{G}\left(b\right), \bar{T}\left(b\right)\right\rangle$ denote equilibrium policies when $\omega = 1$. Assume that $\bar{B}\left(b\right), \bar{G}\left(b\right), \bar{T}\left(b\right)$ are continuously differentiable. If*

$$\left| \beta \lambda R - \bar{G}'\left(b\right) \left(1 - \frac{1+\beta}{(1+\psi)\,\theta} \frac{H\left(\bar{T}\left(b\right)\right)\left(1 - e\left(\bar{T}\left(b\right)\right)\right)}{H\left(\bar{T}\left(b\right)\right)\left(1 - e\left(\bar{T}\left(b\right)\right)\right) - A\left(\bar{T}\left(b\right)\right)e'\left(\bar{T}\left(b\right)\right)} \right) \right| > 1, \quad (22)$$

then, for ω close to unity, there exists a unique DMPPE.

Note that condition (22) is imposed on the equilibrium functions of the case with $\omega = 1$, for which existence and uniqueness have been established in Lemma 4. Thus, condition (22) can be verified either analytically in special cases admitting closed-form solutions (see section 3.1), or numerically. Numerical results are reliable since the contraction mapping theorem applies.

3 Two Analytical Examples

In this section we study two special cases that can be solved analytically. We parameterize the household production technology as follows:

$$F\left(1-h\right) = X \cdot \xi \cdot \left(1-h\right)^{\xi},$$

where $\xi \in [0,1]$. This parameterization is maintained in the rest of the paper.

In the first example we set $\xi = 0$, implying that household activity has zero productivity. Then, under logarithmic preferences, labor taxation does not distort labor supply. In this case an analytical solution of the DMPPE can be found. The equilibrium functions $\left\langle B\left(b\right), G\left(b\right), T\left(b\right)\right\rangle$ are linear. Moreover, debt converges asymptotically to the debt limit as in the commitment solution (see Proposition 1). This property does not extend to the general case analyzed in section 4.

In the second example, we set $\xi = 1$ and $X < w$. This implies that market hours are supplied inelastically as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$. However, if taxation were to exceed $\bar{\tau}$, market hours and tax revenue would fall to zero. Under some additional parameter restrictions, a MPPE

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[19] In particular, Judd (2003) considers a model in which agents discount hyperbolically. His model has dynamic-games properties, and encompasses geometric discounting as a special case. Under geometric discounting, the solution is time consistent and standard contraction-mapping results apply. Using perturbation methods, Judd (2003) establishes the existence and uniqueness of equilibrium in the neighborhood of the geometric discounting case.

can be characterized analytically, even though the policy functions are neither continuous nor differentiable. This example provides an analytical characterization of the comparative statics in the neighborhood of a stable interior steady state with positive public good provision. The qualitative debt dynamics of this example carry over to the general case of section 4.

3.1 Example I: $\xi = 0$

In the first example, market labor supply is fixed at $H = 1$, irrespective of taxes. Hence, $A(\tau) = (1 - \tau) w$, $e(\tau) = 0$, $\bar{\tau} = 1$, and $\bar{b} = w/(R - 1)$. The intra-temporal FOC, (19), can be expressed as

$$1 - \tau = \frac{1 + \beta}{(1 + \psi) \theta w} g. \tag{23}$$

Substituting (23) into the government budget constraint (6) yields;

$$b' = \left(1 + \frac{1 + \beta}{\theta (1 + \psi)}\right) g + Rb - w. \tag{24}$$

To obtain a solution, we guess that G is linear; $G(b) = \gamma(\bar{b} - b)$. Then, the GEE, (20), yields:

$$\frac{\gamma(\bar{b} - B(b))}{\gamma(\bar{b} - b)} = \beta \lambda R + \beta \lambda \gamma \left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1\right). \tag{25}$$

Next, using (25), the budget constraint, (24), the equilibrium condition $b' = B(b)$, and the expression for $\bar{b}$ given above, yield the following solution for γ;

$$\gamma = \frac{(1 - \beta \lambda)\theta(1 + \psi)R}{(1 + \theta)(1 + \beta) + (1 - \beta \lambda)\theta \psi}.$$

Finally, substituting g by its equilibrium expression into (23) and (24) yields a complete analytical characterization, summarized in the following Proposition (proof in the text).[20]

Proposition 4 *Assume that $\xi = 0$. Then, there exists a DMPPE characterized by the following policy functions:*

$$\tau = T(b) = 1 - \frac{1}{w} \frac{(1 - \beta \lambda)(1 + \beta)R}{(1 + \theta)(1 + \beta) + (1 - \beta \lambda)\theta \psi} (\bar{b} - b), \tag{26}$$

$$g = G(b) = \frac{(1 - \beta \lambda)\theta(1 + \psi)R}{(1 + \theta)(1 + \beta) + (1 - \beta \lambda)\theta \psi} (\bar{b} - b), \tag{27}$$

---

[20] This example illustrates the proof of local uniqueness around $\omega = 1$ of the DMPPE (see Proposition 3). When $\xi = 0$, condition (22) simplifies to

$$\beta \lambda R - G'(b) \left(1 - T'(b)\right)\big|_{\omega=1} > 1$$

Using (26)-(27), and rearranging terms shows that this condition holds when $R > 1$. Therefore, in the neighborhood of $\omega = 1$ the DMPPE is unique.

$$b' = B(b) = \bar{b} - \frac{\theta + \lambda(1 + \beta + \theta)}{(1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi} \beta R (\bar{b} - b), \tag{28}$$

where $\bar{b} \equiv w/(R-1)$.

Note that $G'(.) = -\gamma < 0$. Thus, in this case we can verify globally that the disciplining effect (see equation (20)) increases the growth rate of public spending and decreases the growth rate of debt. However, since the equilibrium dynamics are linear, the disciplining effect does not change with the debt level. This implies that the growth rate of government expenditure remains constant along the equilibrium path and there is no stable interior steady state. If $\beta\lambda R$ is sufficiently small, the economy converges asymptotically to the maximum debt level $\bar{b}$ as in the commitment solution.[21] However, debt accumulation is slower in the DMPPE than under commitment. This illustrates that future generations benefit from their political empowerment.

FIGURE 1 HERE

Figure 1 illustrates the political equilibrium: Panel a shows that the equilibrium tax rate increases linearly with debt. Panel b shows that the equilibrium public good provision declines linearly with debt. Finally, Panel c shows the law of motion of debt converging to $\bar{b}$ (in the figure, the parameters imply that $\bar{b} = 0.7$). Panel d shows the time path of b starting out with $b_0 = 0$. As the figure shows, the economy gradually depletes its resources. Generation after generation, private and public consumption are progressively crowded out by debt repayment to foreign lenders. Note, however, that the debt build up is gradual, and the first generation refrains from depleting all resources by setting $b' = \bar{b}$. One reason is altruism. However, debt accumulates gradually even when $\lambda = 0$ (in contrast, under commitment and $\lambda = 0$ debt would converge to $\bar{b}$ in only two periods). In fact, the old would like to set $b' = \bar{b}$, but they meet opposition from the young who are concerned about public expenditure in their old age. The concern for public good consumption is crucial to prevent an immediate resource depletion: if $\theta = 0$, all voters would agree to set $b = \bar{b}$, and the young would secure their private consumption in old age through savings.

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[21] Otherwise, the government accumulates an ever growing surplus. In fact, surplus accumulation can occur for a range of economies where $\beta\lambda R < 1$. These economies would accumulate debt under commitment. Although an ever-growing surplus is inconsistent with the assumption that $b \in [\underline{b}, \bar{b}]$, in the particular case analyzed in this section it is not necessary to impose a lower bound on debt, since an analytical solution exists and we do not have to rely on numerical methods..

3.2 Example II: $\xi = 1$

In the second example, taxation does not distort labor supply as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$. However, if $\tau > \bar{\tau}$, agents stop working in the market, and the tax revenue falls to zero. When the interest rate is sufficiently high (though not so high that it induces perpetual surplus accumulation), the equilibrium is qualitatively different from that of section 3.1; an economy starting from low initial debt converges in finite time to a steady state where $\tau = \bar{\tau}$, $b < \bar{b}$ and $g > 0$.[22] In Appendix C (available upon request), we provide a full characterization of the equilibrium which involves discontinuous policy functions and multiple steady states.[23] In a neighborhood of the lowest steady state, the equilibrium policy rules are:

$$b' = B(b) = b^* \equiv \bar{b}\left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)}\right) \tag{29}$$

$$\tau = T(b) = \begin{cases} \bar{\tau} - \frac{R(1+\beta)}{w(1+\beta+\theta(1+\psi))}(b^* - b) & \text{if } b \in [\underline{b}, b^*) \\ \bar{\tau} & \text{otherwise} \end{cases} \tag{30}$$

$$g = G(b) = \begin{cases} \frac{w\theta(1+\psi)(1-\bar{\tau})}{1+\beta} + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)}(b^* - b) & \text{if } b \in [\underline{b}, b^*) \\ b^* + \bar{\tau}w - Rb & \text{otherwise} \end{cases} \tag{31}$$

FIGURE 2 HERE

Figure 2 plots the equilibrium in the neighborhood of the lowest steady state ($b = b^*$). Panel a shows the equilibrium tax policy: taxes increase linearly with the debt level as long as $b \leq b^*$. Thereafter, T is flat at $\tau = \bar{\tau}$. Panel b shows the equilibrium expenditure: public good provision declines linearly with the debt level as long as $b \leq b^*$. To the right of b^*, the government cannot increases taxes further, and must adjust entirely on the expenditure side. Thus, the government expenditure function becomes steeper. Panel c shows that the debt policy is flat around b^*. Therefore, if the initial debt level is sufficiently close to b^*, debt converges to b^* in one period and remains constant thereafter. Finally, panel d shows an example of the time path of b showing that convergence occurs in the first period.

We now discuss the intuition for the dynamics in the neighborhood of b^* focusing, for simplicity, on the case of no altruism ($\lambda = 0$). In the linear equilibrium of example I, the

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[22] We have also characterized equilibrium when the interest rate is low. In this case, the equilibrium is qualitatively similar to that of of section 3.1. Debt converges asymptotically to $\bar{b} = \bar{\tau}w/(R-1)$, and the economy features public poverty in the long run, i.e. $\lim_{t\to\infty} g_t = 0$. However, since taxes cannot go above by $\bar{\tau}$, private consumption does not fall to zero.

[23] Multiple steady states is a fragile aspect of this example in the sense that they vanish when the labor supply distortion becomes smooth (i.e., $\xi < 1$). However, the robust features of this equilibrium are captured by the local dynamics around the lowest steady state, plotted in Figure 2.

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concern of young voters for next period's public good provision was insufficient to prevent ever-growing debt accumulation. The reason was that along the linear equilibrium path, the next government would respond to a larger debt not only by cutting expenditure, but also by increasing taxes and debt proportionally. As a result, each generation of voters passed the bill to the next generation and only suffered a partial sacrifice of public good consumption. Passing the bill to future generations becomes harder, however, when taxation is increasingly distortionary. In example II, this effect is particularly strong. As debt approaches b^* and taxes approach $\bar{\tau}$, voters anticipate that future generations will not be able to set taxes higher than $\bar{\tau}$. The adjustment can only be made on government expenditure, and the disciplining effect of the young therefore becomes very large at the steady state.

4 The General Case: $\xi \in (0, 1)$

The intuition behind the result of example II carries over to the general case $\xi \in (0, 1)$, with smooth labor supply distortions. In this case, however, an analytical characterization of the equilibrium is not available. Instead, we perform numerical analysis, using a standard projection method with Chebyshev collocation (Judd, 1992) to approximate T and G, exploiting the first-order conditions (19) and (20).

4.1 Calibrated equilibrium

We calibrate the parameters as follows. Since agents live for two periods, we let a period correspond to thirty years. Accordingly, we set $\beta = 0.98^{30}$ and $R = 1.025^{30}$, implying a 2% annual discount rate and a 2.5% annual interest rate. This value of β is standard in the macroeconomics literature, and the value of R is consistent with the average real long-term U.S. government bond yields (2.5%) between 1960 and 1990. We do not have a strong prior on ω, so we simply assume equal political weights on the young and old ($\omega = 0.5$).[24] The wage is set equal to unity (a normalization).

Four parameters remain to be calibrated; θ, λ, ξ, and X. We calibrate these parameters to match four empirical observations:[25]

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[24] Our proof of existence and uniqueness (Proposition 3) applies in the neighborhood of $\omega = 1$ and is not guaranteed to hold for $\omega = 0.5$. We have run a large number of simulations holding constant the parameters of Table 1 and varying ω between 0.5 and 1. The equilibrium policy functions appear to change "with continuity", namely, small changes in ω leads to small changes in the equilibrium functions. See Figure A2 in Appendix C, available from our webpages.

[25] Given H, τ, and the labor elasticity, the expressions for labor supply and the Frisch elasticity pin down the parameters ξ and X. θ and λ are then jointly determined by the debt-to-output ratio and the tax-to-output ratio.

1. The ratio of hours worked in the market to hours worked at home is on average 2 in the US (Aguiar and Hurst, 2006), which implies a steady-state labor input of $H = 2/3$.

2. In the US, the ratio of explicit federal debt to GDP has been around 40% over the last decades. However, the government has also significant pension liabilities. The estimated size of the pension liabilities that have already accrued is 60-90% (van den Noord and Herd, 1993). This puts the total US debt-output ratio to 100-130%. One period in our model corresponds to 30 years. Our notion of aggregate production abstracts from capital. With an empirical labor's share of output of, say, 2/3, our notion of "output" should be 30*2/3=20 times larger than the empirical annual GDP. Therefore, a plausible quantitative target is a steady-state level of b/wH equal to 120%/20=6%, which implies $b^* = 4\%$.

3. The average tax on labor income in the US in the last two decades has been about 27%.[26] So we set $\tau^* = 0.27$.

4. The elasticity of the tax revenue to changes in the after-tax rate, $\chi(\tau) \equiv \frac{\partial(wH)}{\partial(1-\tau)} \frac{1-\tau}{wH}$ is set equal to 0.6 in the steady-state. In our model $\chi(\tau)$ coincides with the Frisch elasticity of labor supply. The estimates of these elasticities have a wide range. Micro estimates of the Frisch elasticity along the intensive margin – based on people who remain employed – indicate an elasticity close to zero for men and somewhat higher for women (see e.g. Altonji, 1986). Macro estimates tend to be higher, as they include adjustments along the extensive margin. For example, the Real-Business-Cycle literature often assumes an elasticity of unity (Cooley and Prescott, 1995). However, in our stylized model labor supply is the only margin of distortion, and in order to make our main theoretical points, it is the size of the fiscal distortion rather than its channel (labor supply) that matters. Estimates of the elasticity of the total tax revenue to changes in the after-tax rate vary, again, over a wide range. For instance, Feldstein (1987) argues that the elasticity is between one and two. In contrast, a micro literature based on the marginal-cost-of-funds approach shows that the elasticity is significantly lower (see e.g. Ballard and Fullerton, 1992, and Kleven and Kreiner, 2006). Given the lack of consensus, we choose an intermediate value ($\chi(\tau^*) = 0.6$). This yields a marginal cost of funds of about two, slightly above the preferred estimate of Browning (1987). We discuss robustness to

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[26] In the period 1979-2004, the average personal income tax as percentage of the gross earnings in the US was 18.7%. However, this increases to 26.1% and 31.4% if one adds, respectively, the employees' social security contributions (net of transfer payments), and in addition the employer's social security contributions (see Source OECD). Klein and Ríos-Rull (2003) report an average income tax rate of 24% for the period 1947-90.

changes of this elasticity below.

Table 1 summarizes the parameters.

Table 1: Calibration

Target observation		Parameter	
Annual discount rate	2%	β	0.98^{30}
Annual interest rate	2.5%	R	1.025^{30}
Average tax on labor	27%	θ	0.09
Market-household hours ratio	2	X	10.48
Elasticity of the tax revenue to changes in the after-tax rate	0.6	ξ	0.17
Debt-GDP ratio (including Social Security liabilities)	120%	λ	0.79
Relative political weight young-old	equal	ω	0.5

Figure 3 plots the equilibrium functions of our calibrated economy.[27] As in example I of section 3.1, taxes are increasing in b (panel a) and public expenditure is decreasing in b (panel b). The debt policy, however, is now a strictly convex function of b which crosses the 45-degree line twice: first at an interior steady-state level ($b = 0.04$), and then at the maximum debt. Only the interior steady-state is stable. Thus, for any initial debt level $b < \bar{b}$, the economy converges to the internal steady state (see panel d). The steady-state level of government expenditure is $g^* = 0.14$, implying a ratio of public expenditure to private market consumption of 21%. Panel d provides information about the speed of convergence of debt towards the steady-state. For example, it takes about four periods (i.e., 120 years) to get from $b_0 = 0$ to $b = 0.02$, i.e., to close half the gap between zero debt and the steady state, with an implied annual rate of convergence of 0.6%. Namely, debt is mean reverting, but with a high persistence. We will show that this is also a feature of the data.

FIGURE 3 (FOUR PANELS) HERE

To gain intuition for our main result – the internal stable steady state – it is useful to compare the calibrated economy with the analytical examples. In all cases, the tax function

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[27]Although we cannot claim formally that these Markov equilibria are unique – our existence and uniqueness results are local in nature –, we have run many simulations and never found more than one equilibrium for each parameter configuration. All the equilibria that we found are qualitatively similar to those displayed in the figure.

is non-decreasing and concave (strictly concave if $\xi > 0$), while the expenditure function is decreasing and concave (strictly concave if $\xi > 0$). In example I ($\xi = 0$), where taxation is not distortionary, an increase in debt causes a proportional increase in taxation and cut in expenditure, so as to keep c/g constant. In example II, the policy functions are piece-wise linear with a kink at the steady state. This is because taxation is non-distortionary to the left of $\bar{\tau}$ and infinitely distortionary to the right of it. Accordingly, the c/g ratio is constant for $b \leq b^*$, and increasing thereafter. In the general case of $\xi \in (0, 1)$, as b increases, the tax function, $T(b)$ becomes less steep, whereas the expenditure function, $G(b)$, becomes steeper. Namely, at high debt levels, the government responds to debt accumulation by cutting expenditure more than by increasing taxes. Hence, the ratio of public-to-private consumption falls as b increases. This fall in relative government expenditure is what deters young voters from demanding debt increases in steady state.

The qualitative findings of an internal steady state are robust to a large range of all parameter values. The most critical one is χ. Clearly, an internal steady-state hinges on the presence of significant tax distortions. In the calibrated economy any tax elasticity χ larger than 0.52 is consistent with an internal steady state as in Figure 3, when all other calibration targets are held constant. The range can be expanded if we allow a larger labor supply. For example, $\chi = 0.2$ and $H = 0.85$ will still generate an internal steady state. As far as altruism is concerned, it is important that λ be not too small (in particular $\lambda > 0.66$), or else the $B(b)$ function continues to be strictly convex, but only crosses the 45-degree line at $\bar{b}$.[28] In this case, the economy converges to the maximum debt. An equilibrium with an interior steady state can be sustained even for economies with no altruism ($\lambda = 0$). However, this requires either a higher interest rate, or a higher tax elasticity. Finally, the qualitative findings are robust to changes in ω. For example, increasing the political weight on the old to $\omega = 0.7$ renders the debt rule qualitatively as in Figure 3.

Commitment vs. Markov Equilibrium: It is interesting to compare the MPPE with the commitment solution in the calibrated economy. Consider the (Ramsey) allocation that would obtain if agents were to vote over the entire fiscal policy sequence in period zero, assuming that $\omega = 0.5$, as in the equilibrium of Figure 3. We have already shown in Proposition 2 that under commitment debt converges to $\bar{b}$ and g converges to zero. In Figure 4, we compare the transitional dynamics for economies starting with zero debt.

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[28] We have checked that the altruism in the calibrated economy ($\lambda = 0.75$) is sufficiently low to ensure that agents do not want to bequeath to their children.

FIGURE 4 (THREE PANELS) HERE

In the first period, the commitment solution features slightly lower taxes (τ_0) and slightly higher government spending (g_0) than the MPPE. Consequently, b_1 is higher under commitment. Government expenditure is significantly larger in the second period (g_1) than in the first period (g_0). This comes at the expense of a larger increase in the debt left to agents born in period two. Thereafter, debt accumulates at a higher rate in the commitment solution converging to $\bar{b}$ (panel a), whereas taxes and spending converge, respectively, to the top of the Laffer curve (panel b) and to zero (panel c). All generations born in period two or after are strictly worse off in the commitment solution, while the agents who are alive at the time of the initial vote are better off. The reason is intuitive, as in the commitment allocation the first generation dictates the entire fiscal policy and passes the bill of its high private and public consumption to the future generations. In contrast, in the political equilibrium all future generations are sequentially empowered and the young discipline period-by-period the fiscal policy. The difference in the long-run outcome is striking: even generations that can only exercise their political power in the far future inherit low debt and enjoy public good consumption.

Fiscal Shocks: Consider the effect of fiscal shocks in the MPPE. Suppose that an economy is hit by a one-period surprise war requiring an exogenous spending of Z units. During the war, the government's budget constraint changes to $b' = g + Rb - \tau wH(\tau) + Z$, and then, as peace returns, it reverts to (6). The shock occurs at the beginning of the period, before the government sets g, τ and b'. Suppose the economy is initially in the steady state b^* (see Figure 3). The shock is equivalent to an exogenous increase in debt from b^* to $b^* + Z/R$. Thus, the government reacts by increasing τ and decreasing in g in wartime. The time path of the fiscal adjustment is shown in Figure 5: the fiscal shock is absorbed by a combination of cuts in non-war expenditure and increases in debt and taxation. After the war debt, taxes and expenditure revert slowly to their original steady-state.[29] These results stand in contrast with the implications of the tax-smoothing model of Barro (1979), as well as the commitment version of our model when $\beta\lambda R = 1$. There, the lion's share of the current cost of the war would be financed by debt and, following the principle of tax smoothing, taxes and non-war expenditure would only be adjusted so as to guarantee a smooth repayment of the excess debt.

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[29] We have extended the analysis to recurrent wars, assuming that the state of the economy (war or peace) evolves following a first-order stationary Markov process. The results are similar to those of a surprise war. However, the positive probability of future wars induces an additional precautionary motive for public savings during peacetime. Interestingly, such motive is also present in the commitment solution, and it turns out that with recurrent wars, even the commitment solution features mean-reverting debt dynamics (as in Ayagari et al., 2002). Details are available upon request.

Therefore, the war induces small but permanent changes on taxes and government expenditure.

FIGURE 5 (THREE PANELS) HERE

The prediction of our theory are consistent with the empirical evidence of Bohn (1998). He analyzed the effects of short-lived increases in US government expenditures on the debt-to-output ratio. He found the debt-to-output ratio to be highly persistent but mean reverting in the US. Namely, a fiscal shock induces an increase in the debt-to-output ratio on impact and a subsequent reversion towards its initial level. We document below that the same pattern holds in a panel of 21 OECD countries.

5 Political Shifts and Fiscal Policy Dynamics

So far, we have developed a politico-economic theory of government debt. In the rest of the paper we extend this theory by introducing within-cohort heterogeneity and stochastic shifts in political outcomes. To motivate the analysis, we document some salient empirical features of debt and expenditure dynamics. We adopt a specification based on the analysis of Bohn (1998) who studied the effects of short-lived increases in US government expenditures on the debt-to-output ratio. We extend Bohn's analysis to a panel data set of 21 OECD countries over the period 1948-2005 and augment his specification to include political variables as detailed below. We run two sets of regressions, with the growth rate of debt-GDP ratio and government expenditure-GDP ratio as the dependent variables, respectively:[30]

$$\Delta d_{ct+1} = \alpha_{0c} + \alpha_{0t} + \alpha_1 POL_{ct} + \alpha_2 d_{ct} + \alpha_3 \Delta U_{ct} + \alpha_4 Z_{ct} + \varepsilon_{ct}^d \qquad (32)$$

$$\Delta g_{ct} = \beta_{0c} + \beta_{0t} + \beta_1 POL_{ct} + \beta_2 d_{ct} + \beta_3 \Delta U_{ct} + \beta_4 Z_{ct} + \varepsilon_{ct}^g \qquad (33)$$

where $(\alpha_{0c}, \beta_{0c})$ and $(\alpha_{0t}, \beta_{0t})$ are country and time fixed effects, respectively. POL_{ct} is a political indicator reflecting the ideology of the executive on a left-right scale. We use two alternative indicators. The first is an updated and simplified version of the index constructed by Woldendorp, Keman and Budge (1998). The second is an index available from the database built by Franzese (2002). Both indexes range between -1 (right) and +1 (left). Details about the construction of these indexes as well as the other variables are provided in Appendix A.

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[30] In our theoretical model, the government chooses current expenditure and budget deficit. This justifies the timing of the dependent variables in the two regressions. We also run regression (33) including the lagged expenditure-GDP ratio among the regressors. The results are qualitatively unchanged.

ΔU_{ct} is the annual change in the unemployment rate, intended to filter out variations in fiscal policy associated with short-term macroeconomic fluctuations, independent of political factors. Z_{ct} is a vector of control variables including GDP per capita, inflation and two measures of the age structure of the population (proportion below 14 and above 65). In all regressions we exclude non-democratic governments. We are primarily interested in the coefficients α_1 and β_1.[31] The purpose of our data analysis is to document correlations that motivate our theoretical investigation. Addressing causal relationship would be interesting but is outside the scope of our analysis in this section.

We first analyze whether debt policy is correlated with governments' ideology. One observation that motivated the work of Persson and Svensson (1989) was that Republican US administrations in the 1980's tended to accumulate more debt.[32] We find that the growth in the debt-to-output ratio is also significantly correlated with the ideology of the government in our panel of OECD countries for the post-war period. The estimated coefficient α_1 is consistently negative and significant across different specifications and different political measures. This shows that right-wing governments on average tend to accumulate larger debt. For instance, with the political measure POL_PK, a shift from a left-wing (+1) to a right-wing (-1) government increases the debt-GDP ratio by ca. 0.7 percentage points per year.[33][34] The estimates of α_2 is also interesting. Bohn (1998) found the debt-to-output ratio to be highly persistent but mean reverting in the US. We find the same pattern in our international panel: the estimates of the coefficient α_2 (row 2 of Table 2) are negative in all but one cases, and are highly significant in two cases.[35]

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[31] We estimate the regression equations (32)-(33) using a Least Square Dummy Variable (LSDV) estimator. As is well-known, this estimator produces biased estimates. However, for sample sizes of $T \geq 30$ and $N = 20$, the bias is small and the LSDV estimator generally perform better than the Arellano-Bond estimator or the Anderson-Hsiao estimator (see Judson and Owen, 1999).

[32] Interestingly, the finding that in the US Republican administration are more prone to debt extends to the period 1948-2005. Using our specification in Table 2, we find that the growth in the US debt-to-output ratio is 1.7 percentage point larger under Republican than under Democrat administrations.

[33] POL_FR (column 4 to 6) also ranges from -1 to +1. However, most observations are between -0.4 and 0.1. If one divides the estimated coefficient (-0.0138) by four, one obtains a quantitative effect which is similar to that obtained using POL_PK.

[34] There is an empirical literature focusing on strategic use of debt driven by ideological differences across parties (see e.g. Pettersson-Lidbom, 2001). Lambertini (2003) examines if the color of government affects the budget deficits in OECD countries but does not find significant effects. However, she uses a shorter sample than us and does not include the current level of debt as a control variable.

[35] Some qualifications are in order. When we use POL_PK, there is no evidence of mean reversion in columns (1) and (2). The apparent lack of mean reversion is driven by an outlier, Japan, whose debt has risen sharply in recent years, arguably due to temporary circumstances. When we introduce an interaction between d_t and a dummy variable for Japan (namely, we allow the autoregressive coefficient of Japan to be different), the process is significantly mean reverting (see column 3), and the Japanese dummy is positive and highly significant.

TABLE 2 HERE

The results of the regression analysis for government expenditure are summarized in Table 3. The growth rate of government expenditure is larger under left-wing governments than under right-wing governments: β_1 is always positive and in most cases significant – albeit the result are stronger with POL_FR than with POL_PK.[36] The results conform with the existing empirical studies. For example, Perotti and Kontopoulos (2002) find that changes in transfers and changes in expenditure are higher under left-wing governments in OECD countries, although the effects are not always significant. Pettersson-Lidbom (2003) find that both taxes and spending are significantly higher for left-wing municipal governments in a panel of Swedish municipalities. We also note that β_2 is negative and in most cases significant: consistently with our theory, the growth rate of government expenditure is negatively correlated with the debt level.

TABLE 3 HERE

In conclusion, our analysis documents four facts:

1. the growth rate of the debt-to-output ratio is negatively correlated with the debt level,

2. the growth rate of the expenditure-to-output ratio is negatively correlated with the debt level,

3. the growth rate of the debt-to-output ratio is negatively correlated with left-wing parties being in government.

4. the growth rate of the expenditure-to-output ratio is positively correlated with left-wing parties being in government.

Facts 1 and 2 are consistent with the theory discussed in section 4. Facts 3 and 4 motivate us to extend our model to include political and intra-generational conflict.

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[36] We have also run with the regression (33) with lagged government expenditure as an additional control variable. In this case the coefficient β_1 drops from 0.0020 to 0.0017 and becomes marginally insignificant when using the indicator POL_PK. However, β_1 remains significant with the POL_FR indicator.

5.1 Political Shocks and Intra-generational Conflict

In the theory discussed so far, there was political conflict only between generations. In this section, we introduce cross-sectional wage heterogeneity and intra-generational conflict. The purpose of the extension is to show that our theory can account for the stylized fact documented above.

Suppose that there are two types of dynasties, *rich* and *poor*. The former are more productive than the latter in market activity, whereas all agents have the same productivity in household production. For simplicity, we assume that the wage of low-productivity agents is so low that, for any feasible tax rate, they find it optimal not to participate in any market activity. Consequently, poor agents do not pay taxes, and their labor earnings are independent of τ.[37] Each cohort consists of a unit measure of rich and of a measure $\tilde{p}$ of poor. Again for simplicity, we assume productivity to be perfectly correlated within dynasties. Clearly, this is not crucial: we only need *some* degree of inter-generational persistence in income. The labor income process of the rich is as described in section 2, and hence the indirect utilities of the old and poor rich are given by (8) and (9). The private consumption of the poor is constant and independent of τ. Thus, the indirect utility of the young and old poor can be written, respectively, as

$$U_{YP}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right) = (1+\beta)\log\left(F\left(1\right)\right) + \theta\log\left(g\right) + \beta\theta\log\left(g'\right) + \beta\lambda U_{YP}\left(\boldsymbol{b'},\boldsymbol{\tau'},\boldsymbol{g'}\right),$$

$$U_{OP}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right) = \log\left(F\left(1\right)\right) + \theta\log\left(g\right) + \lambda U_{YP}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right).$$

Assume, as before, probabilistic voting. Denote by p the political weight of poor dynasties (if the poor and the rich were to have the same clout, p would be $p = \tilde{p}$). The political objective function can then be written as

$$U\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g};p\right) = (1-\omega)\left(pU_{YP}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right) + U_{YR}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right)\right) + \omega\left(pU_{OP}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right) + U_{OR}\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right)\right)$$

$$(34)$$

The proof of Proposition 5 in Appendix B shows that a version of Lemma 3 applies to this model, with the only modification that the weight on public good consumption in (17) is $\theta\left(1+p\right)$ instead of θ.[38] Namely, the elected government's taste for public good consumption increases with the political clout of the poor. All the results of the benchmark model of section 2 carry over to this extension up to a reinterpretation of the parameter θ.

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[37] The assumption that the poor abstain from market activity is inessential: lower productivity implies lower market labor supply, and hence a lower responsiveness of consumption $(A\left(\tau\right))$ to taxation.

The theory would deliver qualitatively similar predictions if we assumed that the government were to spend the tax revenue on transfers to the poor instead of on public good provision. The key feature is that the poor like public expenditure (dislike taxes) more (less) than the rich do.

[38] More precisely, equation (17) is replaced by

Next, we introduce political uncertainty. We assume that the political clout of the poor, p, changes stochastically over time. Formally, we let p follow a two-state first-order Markov process, with realizations $p \in \{p_r, p_l\}$, where $p_r < p_l$ (R and L stand for right-wing and left-wing, respectively).[39] The leftist wave of the 1960's and the neoconservative movement of the 1980's are examples of such political shifts. We denote by π_{ij} the probability that, conditional on the current state being j, next-period state will be i. The definitions of MPPE and DMPPE must be generalized to include p as an additional state variable. We denote by $T(b, p)$, $G(b, p)$, and $B(b, p)$ the equilibrium policy functions. The GEE in Proposition 2 can be generalized as follows;

Proposition 5 *In the model with political shocks, a DMPPE satisfies the following stochastic GEE*

$$\frac{1}{G(b,p)} = \beta \lambda R \cdot E_p \frac{1}{G(B(b,p),p')} - \beta \lambda \left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1 \right) E_p \frac{G'(B(b,p),p')}{G(B(b,p),p')}. \quad (35)$$

where E_p is a conditional expectation operator (e.g., $E_{p_l} G(B(b,p_l),p') = \pi_{ll} G(B(b,p_l),p_l) + \pi_{rl} G(B(b,p_l),p_r))$.

The stochastic GEE has a similar interpretation to (20) in the deterministic model. The first term on the right hand-side is the standard Euler-equation term. The second term arises from dynamic voting and captures the disciplining effect of the young voters.

The analysis of example I in section 3.1 under no altruism ($\lambda = 0$) is particularly simple and illuminating. The equilibrium policy functions are identical to (26)-(27)-(28) after replacing the parameter θ by $\theta(1 + p)$ in all expressions.[40] For instance,

$$B(b,p) = \bar{b} - \frac{(1 - \omega)\theta(1 + p)\beta R}{\omega\theta(1 + p) + (1 - \omega)(1 + \theta(1 + p))(1 + \beta)} \left(\bar{b} - b \right).$$

According to this result, a shift to the right ($p_l \rightarrow p_r$) has the same effect as a decrease in θ in the linear equilibrium of Proposition 4. More formally, looking back at Figure 1, the shock causes a downward shift in the tax policy ($T(b, p_r) < T(b, p_l)$), a downward shift in the expenditure

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$$\langle B(b), G(b), T(b) \rangle = \arg \max_{\left\{ b' \leq \bar{b}, g \geq 0, \tau \in [0,1] \right\}} \left\{ v(\tau, g) - (1 - \psi\lambda)(1 + p)\theta \log g + \beta\lambda V_O(b') \right\},$$

where $v(\tau, g) \equiv (1 + \lambda)(1 + p)\theta \log g + (1 + \beta)\lambda \log A(\tau)$.

[39] In our probabilistic voting model, there are no explicit parties, and in equilibrium candidates converge to the same fiscal policy platform. However, in "leftist times", the winning platform would be more favorable to the poor. When comparing the theory with the evidence discussed above, we identify leftist times with leftist governments. In order to make this mapping more precise in the theory, one could introduce elements of imperfect commitment and partisan politics, such as, for instance, in the citizen-candidate model of Besley and Coate (1997).

[40] The formal statement of this result and its proof are in Appendix C, available upon request.

policy $(G(b, p_r) < G(b, p_l))$, and an upward shift in the debt policy $(B(b, p_r) > B(b, p_l))$. Intuitively, as rich voters become more influential, government expenditure falls and debt accumulation increases. The impulse-response of taxes are U-shaped: they fall upon impact, but increase thereafter due to faster debt accumulation. The rich are more favorable to debt accumulation than the poor for the following reason. Issuing debt increases the government's revenue, allowing more expenditure and lower taxes. While both groups benefit from public good provision, the poor do not benefit from lower taxes. Thus the trade-off is different for the two groups.

Note that the probabilities π_{ij} do not enter the equilibrium functions $T(.)$, $G(.)$ and $B(.)$: neither the variance nor the persistence of political shocks have any effect. For instance, a permanent shift to the right has the same impact effect as a temporary one. This surprising result hinges on the cancellation of an income and a substitution effect.[41] This observation is of independent interest. In an influential article, Persson and Svensson (1989) argued that when governments are subject to a positive probability of not being reelected, their debt policy is affected by strategic considerations. For instance, a right-wing government will issue more debt if it anticipates to be replaced by a left-wing government with a stronger taste for public expenditure. They derive their results in a two-period model. In our environment, the sign of the strategic effects is ambiguous, being exactly zero under logarithmic preferences, no altruism and non-distortionary taxation. This ambiguity may explain why the empirical literature has found mixed support for this prediction.

When extending the analysis to the general case with elastic labor supply $(\xi < 1)$ we obtain qualitatively similar results to the linear equilibrium discussed above. However, in the general case, political shifts change the steady-state levels of b, τ and g. To illustrate this point, consider again the calibrated economy of Table 1, letting in addition $p_l = 0.05$ and $p_r = 0$. In this example, the preferences of the poor are totally ignored in the right-wing regime. We consider three alternative degrees of persistence of political shocks: i.i.d. shocks $(\pi_{ll} = \pi_{rr} = 0.5)$, persistent shocks $(\pi_{ll} = \pi_{rr} = 0.9)$ and permanent shocks $(\pi_{ll} = \pi_{rr} = 1)$. Figure 6 plots the equilibrium policy rules and the debt dynamics for the two realizations of the shock $(p_l$ and $p_r)$ in the case of zero persistence. To aid the visualization, we zoom in the region of the state space where $b \in [0, 0.1]$. Dotted lines are for the left-wing regime,

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[41] Suppose that in a leftist period voters anticipate a shift to the right. On the one hand, a disciplined fiscal policy today has a lower return to the poor since next period the rich will have more power and spend a smaller share of $\bar{b} - b'$ on public goods. Therefore, the substitution effect increases the desire of the poor to accumulate debt. On the other hand, the marginal utility of future government expenditure is larger to them precisely because the next government has a lower propensity to spend. Thus, the income effect induces more fiscal discipline from the leftist government. Under logarithmic preferences, non-distortionary taxation and no altruism the two effects cancel out.

whereas solid lines are for the right-wing regime. The figure shows that for a given level of debt a right-wing government delivers lower taxes and public good provision, and more debt accumulation.

FIGURES 6 HERE

Figure 7 plots the time-series dynamics of g, τ and b under the political regime shift in the three cases. By assumption the initial state (in period zero) has been preceded by a long sequence of left-wing governments. In period one, there is a political shift to the right that persists for the following thirty periods. The solid, dashed and dotted lines correspond to different expectations about the future transitions: permanent, persistent and i.i.d. shocks, respectively. In all cases, public spending decreases monotonically, public debt increases monotonically, while the tax rate falls in the first period, and increases thereafter. Note that the more persistent the shifts in political regime are perceived to be, the smaller is the fiscal effect of a political shift. For example, the increase in debt is largest in the i.i.d. case. Thus, like in Persson and Svensson (1989) right-wing (left-wing) governments accumulate more (less) debt when they face a lower probability of reelection.

FIGURES 7 HERE.

These dynamics are consistent with the empirical evidence documented above. First, debt is mean reverting. Second, right-wing governments accumulate more debt and spend less. Third, the debt level reduces government expenditure.

6   Conclusion

In this paper, we have proposed a positive theory of fiscal policy under repeated voting. In the Markov equilibrium of the dynamic voting game, the concern of voters for future public good provision can offset the desire of voters to pass the bill of their expenditure to future generations and discipline fiscal policy. This disciplining force can be strong enough to limit the expansion debt and generate an interior steady-state level. This result holds even for economies in which agents have no altruistic concern for future generations, local interest rates do not

respond to fiscal policy, and the commitment solution would converge to the endogenous debt limit with zero public-good consumption. Tax distortions are crucial for this result. Perhaps paradoxically, an increase in the elasticity of the tax base, due, e.g., to tax competition may ultimately increase public good provision in the long run.

We have extended the theory to an environment where fiscal policy entails a redistributive conflict between rich and poor dynasties, since public goods are financed by taxing the rich. An increase in the influence of the rich on the political process induces governments to cut taxes and spending on public goods and to increase debt. We document empirical support for this prediction.

Our analysis is subject to a number of caveats. The left-right distinction ignores other important dimensions of the political conflict. For instance, one might argue that both left-wing and right-wing populism can be interpreted as a decrease in the current voters' concern for future generations. In our analysis, however, altruism has been kept constant across political regimes.

While our analysis contributes to explain the effects of within-country shifts in fiscal policy, we have not attempted to explain cross-country differences (e.g., why Italy and Greece have larger debt than Switzerland or Sweden). A possible avenue for future research could be to extend the model to introduce cross-country heterogeneity in the efficiency of public good provision. For instance, differences in the propensity to indebtedness between Italy and Scandinavian countries might reflect differences in the efficiency of the public administration.

Finally, we have maintained throughout that governments are committed to repay their debt and ruled out government Ponzi schemes. The analysis could be enriched by endogenizing the incentive of government to repay debt. For instance, in equilibria with large debt there would be incentives for voters to support default. Integrating our analysis with the insights of the sovereign-debt literature may give rise to novel insights but requires non-trivial extensions which are also left to future research.

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A   Appendix: Empirical Analysis of Debt Dynamics

This appendix discusses the details of the empirical analysis of section 5.

We consider a panel of 21 OECD countries ranging from 1948 (or the earliest observation available) and 2006, excluding periods of dictatorships.[42] The dependent variables in the two sets of regressions (Table 2 and Table 3) are, respectively, the annual change in debt-GDP ratio (Δd_{ct+1}) and the annual change in government expenditure-GDP ratio (Δg_{ct}). To obtain the debt-GDP ratio we take the "Central Government Debt" from the Statistical Yearbook of OECD (various issues) for the period of 1980-2006, and divide it by the GDP from the International Financial Statistics (IFS) CD-ROM of November 2006. For the years before 1980, this ratio has been chained with the data compiled by Franzese (2002), based on different sources. The government expenditure-GDP ratio is constructed from the "Central Government Expenditure", again from the IFS CD-ROM. GDP per capita is from the Penn World Tables 6.2. The data of unemployment (used to construct the unemployment change, ΔU_{ct}) are also from IFS for the period 1997-2005 and have been chained with Franzese (2002) for earlier years. Population over 65 and under 14 is from United Nations - Demographic Yearbook Issues 1994-2004 for the period 1996-2004, with missing observations filled by interpolation. Data for the previous years are chained with Franzese (2002). We calculate the inflation rate by CPI from the IFS CD-ROM.

The most serious data issue concerns measuring the governments' ideology across countries and over time. Problems of cross-country comparability between governments' political ideologies are avoided by including country-specific fixed effects in the regressions. Our first political measure (POL_{PK}) is from Woldendorp, Keman and Budge (WKB, 1998), which assign scores for government and parliament ranging from 1 ("right-wing dominance") to 5 ("left-wing dominance"). The criterion for "dominance" is set by the share of seats in government and parliament. Since WKB is only available until 1997, we complete more recent years using the indicator provided by the World Bank Database of Political Institutions (see Beck et al. 2001). The latter (which is only available after 1975) is based on the classification of the political inclination of the chief executive's party. Since the World Bank Database of Political Institutions has only three levels (-1, 0, +1), we reduced the WKB index to a three-level indicator: -1 for RIGHT (≤ 2 in WKB), 0 for CENTER ($2 < x < 4$ in WKB) and 1 for LEFT (≥ 4 in WKB), to be consistent with the World Bank measure.

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[42] The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Switzerland, Sweden, United Kingdom, United States.

The alternative political measure (POL_FR) is based on Franzese (2002), who codes all parties in government from 1948 to 1997 from far left (value 0) to far right (value 10). This coding takes into account not only the color of the government according to its nominal affiliation (socialist, christian-democrat, etc.), but also factors in the opinion of experts on the government's ideology (see Franzese (2002) for a more detailed description of the index). For consistency with the other measure, we re-scaled Franzese's variable so that it ranges between -1 (far right) to +1 (far left). We did not extend this measure after 1997 since the criteria for extending the measure are complex and possibly controversial.

B   Appendix: Proofs of Lemmas and Propositions

B.1   Proof of Lemma 1

Consider the intra-temporal first order condition, (13), that is derived in the text. We can rewrite the government budget constraint as

$$b' - Rb = \Lambda(\tau) \equiv \frac{\left(1 + \frac{1}{\lambda}\right)\theta}{1 + \beta} A(\tau)(1 - e(\tau)) - \tau w H(\tau),$$

where $\Lambda : [\underline{\tau}, \bar{\tau}] \rightarrow \left[-\bar{\tau} w H(\bar{\tau}), \frac{(1+\psi)\theta}{1+\beta} A(\underline{\tau})\right]$ is a monotonic mapping. Therefore, $\tau = \Lambda^{-1}(b' - Rb)$. Then, (11) can be rewritten as

$$V_O^{comm}(b) = \max_{b' \in \Gamma(b)} \left\{ \hat{v}(b' - Rb) + \beta\lambda V_O^{comm}(b') \right\}, \tag{36}$$

where

$$\hat{v}(b' - Rb) \equiv (1 + \lambda)\theta \log\left(1 - e\left(\Lambda^{-1}(b' - Rb)\right)\right) + ((1 + \beta)\lambda + (1 + \lambda)\theta)\log A\left(\Lambda^{-1}(b' - Rb)\right).$$

Since the correspondence Γ is non-empty, compact-valued and continuous, the function $\hat{v}$ is bounded and continuous, and $\beta\lambda < 1$, Theorem 4.6 in Stokey and Lucas (1989) establishes that (36) has a unique fixed point.

B.2   Proof of Lemma 2

We rewrite the planner's objective function, (10), as:

$$\frac{\lambda}{1 - \omega + \omega\lambda} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) =$$

$$\frac{\lambda}{1 - \omega + \omega\lambda}\left((1 - \omega) U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + \omega\left(\theta\log(g_0) + \lambda U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right)\right) =$$

$$\frac{\lambda}{1 - \omega + \omega\lambda}\left(\omega\theta\log(g_0) + (1 - \omega + \lambda\omega) U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\right) =$$

$$\frac{\omega\lambda}{1-\omega+\omega\lambda}\theta\log(g_0)+\lambda\sum_{t=0}^{\infty}(\lambda\beta)^t\left((1+\beta)\log(A(\tau_t))+\theta\log(g_t)+\beta\theta\log(g_{t+1})\right)=$$

$$\lambda(1+\beta)\log(A(\tau_0))+(1+\psi)\lambda\theta\log(g_0)+\sum_{t=1}^{\infty}(\lambda\beta)^t\left((1+\beta)\lambda\log(A(\tau_t))+(1+\lambda)\theta\log(g_t)\right)=$$

$$v(\tau_0,g_0)-(1-\psi\lambda)\theta\log g_0+\sum_{t=1}^{\infty}(\lambda\beta)^t v(g_t,\tau_t),$$

where $v(g_t,\tau_t)$ is defined as in (12) and ψ is defined in the text. Then, the choice of $\{\tau_0,g_0,b_1\}$ maximizing $U(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g})$, given b_0, can be written as:

$$\{\tau_0,g_0,b_1\}=\arg\max_{\{\tau_0,g_0,b_1\}}\left\{v(\tau_0,g_0)-(1-\psi\lambda)\theta\log g_0+\right.$$

$$\left.\max_{\{\tau_t,g_t,b_{t+1}\}_{t=1}^{\infty}|b_1}\left\{\sum_{t=1}^{\infty}(\lambda\beta)^t v(g_t,\tau_t)\right\}|_{b_1}|_{b_0}\right\}=$$

$$\arg\max_{\{\tau_0,g_0,b_1\}}\left\{v(\tau_0,g_0)-(1-\psi\lambda)\theta\log g_0+\beta\lambda V_O^{comm}(b_1)\right\}|_{b_0}, \tag{37}$$

where the maximization is subject to (6), and the last step follows from equation (11). This proves part (i) of the Lemma. Part (ii) of the Lemma follows immediately from equations (11)-(14) in the text.

B.3 Proof Proposition 1

The intertemporal government budget constraint after the first period can be written as:

$$Rb_1+\sum_{t=1}^{\infty}\frac{g_t}{R^{t-1}}=\sum_{t=1}^{\infty}\frac{w\tau_t H(\tau_t)}{R^{t-1}}. \tag{38}$$

First, consider the case of $\beta\lambda R=1$. Then, (14) implies that g is constant. Hence, (13) implies that τ must also be constant. Therefore, (38) establishes that

$$(R-1)b_1=\tau^* wH(\tau^*)-g^*, \tag{39}$$

where τ^* and g^* denote constant solutions of τ and g, respectively. Substituting (39) into (6), we obtain $b_t=b_1$ for $t\geq 2$.

Next, consider the case of $\beta\lambda R<1$. Then, (14) implies that $\lim_{t\to\infty}g_t=0$. This, together with (13), implies in turn that $\lim_{t\to\infty}e(\tau_t)=1$. Hence, $\lim_{t\to\infty}\tau_t=\bar{\tau}$. These two facts, together with (6), establish that $b_t\to\bar{b}$ as $t\to\infty$.

B.4 Proof Lemma 3

Following the same steps as in the proof of Lemma 2 establishes that

$$\frac{\lambda}{1-\omega+\omega\lambda}U\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right)=v\left(\tau_0,g_0\right)-\left(1-\psi\lambda\right)\theta\log g_0+\sum_{t=1}^{\infty}\left(\lambda\beta\right)^t v\left(g_t,\tau_t\right).$$

Along the equilibrium path $\langle g_t,\tau_t\rangle=\left\langle G\left(B^t\left(b\right)\right),T\left(B^t\left(b\right)\right)\right\rangle$. Given the policy rules $T\left(b\right)$, $G\left(b\right)$ and $B\left(b\right)$, the discounted utility of the old can then be written as

$$V_O\left(b\right)\equiv\sum_{t=0}^{\infty}\left(\lambda\beta\right)^t v\left(G\left(B^t\left(b\right)\right),T\left(B^t\left(b\right)\right)\right).$$

Clearly, $V_O\left(b\right)$ satisfies the following functional equation:

$$V_O\left(b\right)=v\left(G\left(b\right),T\left(b\right)\right)+\beta\lambda V_O\left(B\left(b\right)\right). \tag{40}$$

Therefore, part 1 of the Definition 1 of MPPE can be rewritten as:

$$\left\langle B\left(b\right),G\left(b\right),T\left(b\right)\right\rangle=\arg\max_{\left\{b'\in[\underline{b},\bar{b}],g\in[0,\bar{g}],\tau\in[\underline{\tau},\bar{\tau}]\right\}}\left\{v\left(\tau,g\right)-\left(1-\psi\lambda\right)\theta\log g+\beta\lambda V_O\left(b'\right)\right\},$$

subject to (6), and the function $V_O\left(b'\right)$ solving (40).

B.5 Proof of Proposition 2

The FOCs of the program (17) with respect to τ and g (after substituting away b' using (6)) yield:

$$\frac{\left(1+\beta\right)\lambda A'\left(\tau\right)}{A\left(\tau\right)}-\beta\lambda V_O'\left(b'\right)\left(wH\left(\tau\right)+\tau wH'\left(\tau\right)\right)=0,$$

$$\frac{\lambda\theta\left(1+\psi\right)}{g}+\beta\lambda V_O'\left(b'\right)=0,$$

Using the definition of $e\left(\tau\right)$ and the fact that, from the program (1) and the envelope condition $A'\left(\tau\right)=-wH\left(\tau\right)$, the FOCs can be rewritten as:

$$-\frac{\left(1+\beta\right)\lambda}{A\left(\tau\right)}-\beta\lambda V_O'\left(b'\right)\left(1-e\left(\tau\right)\right)=0, \tag{41}$$

$$\frac{\lambda\theta\left(1+\psi\right)}{g}+\beta\lambda V_O'\left(b'\right)=0. \tag{42}$$

Combining the two FOCs yields equation (19). Then we can rewrite (18) as:

$$V_O\left(b\right)=\left(\left(1+\beta\right)\lambda+\left(1+\lambda\right)\theta\right)\log\left(G\left(b\right)\right)-\left(1+\beta\right)\lambda\log\left(1-e\left(T\left(b\right)\right)\right)+\beta\lambda V_O\left(B\left(b\right)\right).$$

Differentiating $V_O(b)$ yields

$$V'_O(b) = ((1+\beta)\lambda + (1+\lambda)\theta)\frac{G'(b)}{G(b)} - \frac{-(1+\beta)\lambda e'(T(b))T'(b)}{1-e(T(b))} + \beta\lambda V'_O(B(b))B'(b). \quad (43)$$

Differentiating $B(b)$ from equation (16) yields:

$$\begin{aligned}
B'(b) &= G'(b) + R - T'(b)wH(T(b))(1-e(T(b))) \\
&= \left(1 + \frac{1+\beta}{\theta(1+\psi)}\right)G'(b) + R + e'(T(b))T'(b)A(T(b)). \quad (44)
\end{aligned}$$

The last equality follows from the fact that

$$-T'(b)wH(T(b))(1-e(T(b))) = \frac{1+\beta}{\theta(1+\psi)}G'(b) + e'(T(b))T'(b)A(T(b)),$$

as implied by (19) and $A'(\tau) = -wH(\tau)$. Leading by one period equation (43) yields an expression for $V'_O(b')$ which can be used, together with (42), to eliminate $V'_O(b')$ and $V'_O(B(b))$. The resulting expression is:

$$\frac{1}{G(b)} = \frac{\beta\lambda}{G(B(b))}\left(B'(B(b)) - \frac{1+\beta+\left(1+\frac{1}{\lambda}\right)\theta}{\theta(1+\psi)}G'(B(b)) - \frac{(1+\beta)G(B(b))e'(T(B(b)))T'(B(b))}{\theta(1+\psi)(1-e(T(B(b))))}\right).$$

Next, using (44) to eliminate $B'(B(b))$ leads to:

$$\begin{aligned}
\frac{1}{G(b)} &= \frac{\beta\lambda}{G(B(b))}\left(R + \left(1 - \frac{1+\frac{1}{\lambda}}{1+\psi}\right)G'(B(b)) + e'(T(B(b)))T'(B(b))A(T(B(b))) -\right. \\
&\quad \left.\frac{(1+\beta)G(B(b))e'(T(B(b)))T'(B(b))}{\theta(1+\psi)(1-e(T(B(b))))}\right)
\end{aligned}$$

Finally, note the FOC (19) implies equation (21). Then, the GEE simplifies to:

$$\frac{1}{G(b)} = \frac{\beta\lambda R}{G(B(b))} - \frac{\beta\lambda G'(B(b))}{G(B(b))}\left(\frac{1+\frac{1}{\lambda}}{1+\psi} - 1\right),$$

as in equation (20).

B.6 Statement and Proof of Lemma 5

Lemma 5 *Assume that $\lambda > 0$, and*

$$(1+\lambda)\theta\left(\left((\Lambda^{-1})'\right)^2\left(-e''(1-e) - (e')^2\right) - (1-e)e'(\Lambda^{-1})''\right)$$

$$+ ((1+\beta)\lambda + (1+\lambda)\theta)\left(\left((\Lambda^{-1})'\right)^2\left(A''A - (A')^2\right) + AA'(\Lambda^{-1})''\right) < 0, \quad (45)$$

where Λ is defined in the proof of Lemma 1. Then, the unique MPPE of Lemma 1 is a DMPPE.

The proof is an the application of Theorem 2.1 in Santos (1991). The theorem states that the policy functions are differentiable if (i) the return function $\hat{v}$ is strictly concave and (ii) optimal paths are strictly interior. Consider the formulation of the problem used in the proof of Lemma 1. Standard differentiation shows that the function $\hat{v}$ is strictly concave if and only if assumption (45) holds.

We must show that the optimal paths are interior; i.e., that for any $b \in \left(\underline{b}, \bar{b}\right)$,

$$B\left(b\right) \in \left(\underline{b}, \bar{b}\right).$$

Since setting $B\left(b\right) = \bar{b}$ would imply zero public expenditure in the next period, then $\lambda > 0$ ensures that $B\left(b\right) < \bar{b}$. It remains to prove that $B\left(b\right) > \underline{b}$ for any $b \in \left(\underline{b}, \bar{b}\right)$. Suppose instead that there exists a $\hat{b} \in \left(\underline{b}, \bar{b}\right)$ such that $B\left(\hat{b}\right) = \underline{b}$. The Euler equation must then be (recall that $\hat{v}' < 0$);

$$\hat{v}'\left(\underline{b} - R\hat{b}\right) \leq \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right). \tag{46}$$

By concavity of $\hat{v}$, $\hat{b} > \underline{b}$ implies $\hat{v}'\left(\underline{b} - R\underline{b}\right) < \hat{v}'\left(\underline{b} - R\hat{b}\right)$. Equation (46) then implies

$$\hat{v}'\left(\underline{b} - R\underline{b}\right) < \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right) \tag{47}$$

Thus, if the agent in constrained for some $\hat{b} > b$, she must be constrained for $b = \underline{b}$. Hence, $B\left(\underline{b}\right) = \underline{b}$. However, $\beta\lambda R \leq 1$ implies $\hat{v}'\left(\underline{b} - R\underline{b}\right) \geq \beta\lambda R\hat{v}'\left(B\left(\underline{b}\right) - R\underline{b}\right)$, which contradicts (47). This concludes the proof.

B.7 Proof of Proposition 3

The strategy of the proof is based on Judd (2004). Let $\left\langle \bar{B}\left(b\right), \bar{G}\left(b\right), \bar{T}\left(b\right)\right\rangle$, denote the equilibrium policies when $\omega = 1$. Time subscripts will denote partial derivatives. We rewrite the equilibrium conditions (16), (20) and (21) in the following form:

$$\frac{G\left(\hat{B}\left(G\left(b\right), b\right)\right)}{G\left(b\right)} = \beta\lambda R - \beta\lambda G_1\left(\hat{B}\left(G\left(b\right), b\right)\right)\left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1\right), \tag{48}$$

$$\hat{B}\left(G\left(b\right), b\right) = G\left(b\right) + Rb - \hat{T}\left(G\left(b\right)\right) \cdot w \cdot H\left(\hat{T}\left(G\left(b\right)\right)\right), \tag{49}$$

$$\frac{1 + \beta}{\left(1 + \psi\right)\theta}G\left(b\right) = A\left(\hat{T}\left(G\left(b\right)\right)\right)\left(1 - e\left(\hat{T}\left(G\left(b\right)\right)\right)\right). \tag{50}$$

Let $\varepsilon \equiv \beta\lambda\left(\left(1 + 1/\lambda\right)/\left(1 + \psi\right) - 1\right)$ where $\lim_{\omega \to 1} \varepsilon = 0$. We prove that in the neighborhood of $\varepsilon = 0$ there exists a unique policy function $G\left(b, \varepsilon\right)$ that solves the GEE, (48). Note that $G\left(b, \varepsilon\right)$ involves some slight abuse of notation. We plug-in the candidate equilibrium function

$G\left(b,\varepsilon\right)$ into (48), obtaining

$$\Pi\left(\varepsilon,G\left(b,\varepsilon\right)\right)\equiv\frac{G\left(\hat{B}\left(G\left(b,\varepsilon\right),b\right),\varepsilon\right)}{G\left(b,\varepsilon\right)}-\beta\lambda R+\varepsilon G_1\left(\hat{B}\left(G\left(b,\varepsilon\right),b\right),\varepsilon\right)=0,\qquad(51)$$

where we define

$$\hat{B}\left(G\left(b,\varepsilon\right),b\right)=G\left(b,\varepsilon\right)+Rb-T\left(G\left(b,\varepsilon\right)\right)H\left(T\left(G\left(b,\varepsilon\right)\right)\right).\qquad(52)$$

Next, we differentiate (51) with respect to ε, and evaluate the resulting expression at $\varepsilon=0$ (recalling that $G\left(b,0\right)=\bar{G}\left(b\right)$ and $G_1\left(b,0\right)=\bar{G}_1\left(b\right)$). After rearranging terms, we obtain:

$$\begin{aligned}&\frac{G_2\left(\hat{B}(\bar{G}\left(b\right),b),0\right)}{\bar{G}\left(b\right)}\\&+\left(\frac{\bar{G}'\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)\cdot\hat{B}_1\left(\bar{G}\left(b\right),b\right)}{\bar{G}\left(b\right)}-\frac{\bar{G}\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)}{\left(\bar{G}\left(b\right)\right)^2}\right)G_2\left(b,0\right)\qquad(53)\\&+\bar{G}_1\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)=0\end{aligned}$$

Recall that the GEE implies that $\bar{G}\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)/\bar{G}\left(b\right)=\beta\lambda R$. Therefore, (53) implies that:

$$G_2\left(b,0\right)=J\left(b\right)\cdot G_2\left(\hat{B}\left(\bar{G}\left(b\right),b\right),0\right)+Z\left(b\right),\qquad(54)$$

where

$$\begin{aligned}J\left(b\right)&=\left(\beta\lambda R-\bar{G}'\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)\cdot\hat{B}_1\left(\bar{G}\left(b\right),b\right)\right)^{-1}\\Z\left(b\right)&=J\left(b\right)\cdot\bar{G}'\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)\cdot\bar{G}\left(b\right)\end{aligned}$$

Note that (54) has an iterative nature. Define the mapping:

$$\left(\Upsilon\,G_2\left(b,0\right)\right)\left(b\right)\equiv J\left(b\right)\cdot G_2\left(\hat{B}\left(\bar{G}\left(b\right),b\right),0\right)+Z\left(b\right).$$

If $\left|J\left(b\right)\right|<1$, then Υ is a contraction mapping. We now show $\left|J\left(b\right)\right|<1$ if and only if assumption (22) holds. Differentiating equation (49) leads to

$$\hat{B}_1\left(G\left(b\right),b\right)=1-\frac{1+\beta}{\left(1+\psi\right)\theta}\frac{H\left(T\left(b\right)\right)\left(1-e\left(T\left(b\right)\right)\right)}{H\left(\tau\right)\left(1-e\left(T\left(b\right)\right)\right)-A\left(T\left(b\right)\right)e'\left(T\left(b\right)\right)},$$

hence

$$\left|J\left(b\right)\right|=\left|\left(\beta\lambda R-\bar{G}'\left(\hat{B}\left(\bar{G}\left(b\right),b\right)\right)\cdot\hat{B}_1\left(\bar{G}\left(b\right),b\right)\right)^{-1}\right|<1,$$

by assumption (22). Hence, Υ is a contraction mapping. Therefore, in the neighbuorhood of $\omega=1$, there exists a unique derivative $G_2\left(b,0\right).$

Finally, we must show that the existence of a unique derivative $G_2(b, 0)$ implies that there exists a unique equilibrium policy function, $G(b, \varepsilon)$ that satisfies the GEE. Differentiating the functional equation (51) with respect to ε and evaluating the result at $\varepsilon = 0$ lead to the linear operator equation

$$\Pi_1(0, G(b, 0)) + \Pi_2(0, G(b, 0)) G_2(b, 0) = 0.$$

The existence and the uniqueness of solution, $G_2(b, 0)$, imply that $\Pi_2(0, G(b, 0))$ is invertible at neighborhood of $\varepsilon = 0$. Therefore, we can apply implicit function theorem (Judd, 2004, pp. 10) to show that there are neighborhoods ε_0 of $\varepsilon = 0$ and for all $\varepsilon \in \varepsilon_0$, there is a unique $G(b, \varepsilon)$.

B.8   Proof of Proposition 5

We prove the Proposition in two steps. First, we characterize the equilibrium for constant (i.e., non-stochastic) $\dot{p}$. Then, we characterize the GEE in the case in which p follows a Markov process.

When p is constant, the model with wage heterogeneity yields the same equilibrium as in Lemma 3. To see why, note that the political objective function can be written as

$$
\begin{aligned}
&\frac{\lambda}{1 - \omega + \omega\lambda} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) \\
=~&\frac{\lambda}{1 - \omega + \omega\lambda}\left((1 - \omega)(pU_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})) + \omega(pU_{OP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{OR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}))\right) \\
=~&\frac{\lambda}{1 - \omega + \omega\lambda}\left(\omega(1 + p)\theta\log(g_0) + (1 - \omega + \lambda\omega)(pU_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}))\right) \\
=~&\frac{\omega\lambda(1 + p)}{1 - \omega + \omega\lambda}\theta\log(g_0) + \lambda\sum_{t=0}^{\infty}(\lambda\beta)^t\left((1 + \beta)\log(A(\tau_t)) + \theta(1 + p)\log(g_t) + \beta\theta(1 + p)\log(g_{t+1})\right),
\end{aligned}
$$

which is exactly the same as the political objective function in the proof of Lemma 2, with the modification that the taste for the public good becomes $\hat{\theta} \equiv (1 + p)\theta$. Therefore, the results in Lemma 2 and 3 carry over unchanged to the model with wage heterogeneity.

When p is stochastic and follows a Markov process, the state vector consists of the level of debt and the political state, p. It is straightforward to extend Lemma 3 to the stochastic case. The political objective function can be expressed as

$$
\begin{aligned}
&\frac{\lambda}{(1 - \omega + \omega\lambda)} E_{p_0} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}; p_0) \\
=~&\lambda\psi(1 + p_0)\theta\log g_0 + \lambda E_{p_0}\{p_0 U_{YP}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + U_{YR}(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})\} \\
=~&(1 + \beta)\lambda\log(A(\tau_0)) + \lambda(1 + \psi)(1 + p_0)\theta\log g_0 + \sum_{t=1}^{\infty}\sum_{p_t}(\lambda\beta)^t\pi_t(p_t, p_0) v(g_t, \tau_t; p_0),
\end{aligned}
$$

where p_t denotes the political weight at time t and $\pi_t(p_t, p_0)$ is the probability of p_t in period t, conditional on the initial state p_0, and

$$v(g_t, \tau_t; p_0) \equiv (1 + \beta)\lambda \log A(\tau_t) + (1 + \lambda)(1 + p_0)\theta \log(g_t).$$

Therefore, the equilibrium must satisfy:

$$\left\langle \begin{array}{c} B(b,p), \\ G(b,p), \\ T(b,p) \end{array} \right\rangle = \arg\max_{\{b',g,\tau\}} \left\{ \begin{array}{c} v(\tau, g; p) - (1 - \psi\lambda)(1 + p)\theta \log g \\ + \beta\lambda E_p V_O(B(b,p), p'; p) \end{array} \right\}, \tag{55}$$

subject to (6), (7), and

$$V_O(b, p; p) = v(G(b,p), T(b,p); p) + \beta\lambda E_p V_O(B(b,p), p'; p),$$

where E_p is a conditional expectation operator. The second argument of the function V_O stands for the current p, while the third argument refers to the initial p. These two arguments are identical for the initial period.

We now proceed to solve the program and to derive the GEE. If all policy functions are continuous and differentiable, the solution must satisfy the following First Order Conditions

$$-\frac{(1 + \beta)\lambda}{A(\tau)(1 - e(\tau))} = \beta\lambda E_p V_O'(B(b,p), p'; p) \tag{56}$$

$$-\frac{\lambda(1 + \psi)(1 + p)\theta}{g} = \beta\lambda E_p V_O'(B(b,p), p'; p) \tag{57}$$

where $V_O'(b', p'; p)$ denotes the derivative of V_O with respect to b'. The two equations, (56)-(57), together with the equilibrium conditions $g = G(b,p)$ and $\tau = T(b,p)$ imply, for all p, the intra-temporal condition

$$\frac{1 + \beta}{(1 + \psi)(1 + p)\theta} G(b,p) = A(\tau)(1 - e(T(b,p))), \tag{58}$$

which is the analogue of equation (19). This leads to

$$\begin{aligned} V_O(b, p; p) &= ((1 + \beta)\lambda + (1 + \lambda)(1 + p)\theta)\log(G(b,p)) - (1 + \beta)\lambda \log(1 - e(T(b,p))) \\ &\quad + \beta\lambda E_p V_O(B(b,p), p'; p), \end{aligned} \tag{59}$$

$$B(b,p) = G(b,p) + Rb - T(b,p)wH(T(b,p)) \tag{60}$$

Differentiating $V_O(b, p; p)$ and $B(b, p)$ with respect to b yields, then,

$$
\begin{aligned}
V_O'(b, p; p) &= ((1 + \beta)\lambda + (1 + p)\theta(1 + \lambda))\frac{G'(b, p)}{G(b, p)} + \frac{(1 + \beta)\lambda e'(T(b, p))T'(b, p)}{1 - e(T(b, p))} \\
&\quad + \beta\lambda \cdot E_p V_O'(B(b, p), p'; p) \cdot B'(b, p),
\end{aligned}
\tag{61}
$$

$$
\begin{aligned}
B'(b, p) &= G'(b, p) + R - T'(b, p)wH(T(b, p))(1 - e(T(b, p))) \\
&= \left(1 + \frac{1 + \beta}{\theta(1 + p)(1 + \psi)}\right)G'(b, p) + R + e'(T(b, p))T'(b, p)A(T(b, p)) \tag{62}
\end{aligned}
$$

where $B'(b, p)$ denotes the derivative of B with respect to b and the last equality is derived as in the proof of deterministic case. Recalling that the First Order Condition, (57), implies that

$$
-\frac{\lambda(1 + \psi)(1 + p)\theta}{\beta\lambda g} = E_p V_O'(B(b, p), p'; p),
\tag{63}
$$

we can rewrite (61) as

$$
\begin{aligned}
V_O'(b, p; p) &= ((1 + \beta)\lambda + \theta(1 + p)(1 + \lambda))\frac{G'(b, p)}{G(b, p)} + \frac{(1 + \beta)\lambda e'(T(b, p))T'(b, p)}{1 - e(T(b, p))} - \\
&\quad \frac{\lambda(1 + \psi)(1 + p)\theta}{g} \cdot \\
&\quad \left(\left(1 + \frac{1 + \beta}{(1 + p)\theta(1 + \psi)}\right)G'(b, p) + R + e'(T(b, p))T'(b, p)A(T(b, p))\right).
\end{aligned}
$$

Taking one-period lead expectations yields

$$
\begin{aligned}
E_p V_O'(b', p'; p) = E_p &\left[((1 + \beta)\lambda + (1 + p)\theta(1 + \lambda))\frac{G'(B(b, p), p')}{G(B(b, p), p')} + \right. \\
&\frac{(1 + \beta)\lambda e'(T(B(b, p), p'))T'(B(b, p), p')}{1 - e(T(B(b, p), p'))} - \\
&\frac{\lambda(1 + \psi)(1 + p)\theta}{G(B(b, p), p')} \cdot \left(\left(1 + \frac{1 + \beta}{\theta(1 + p)(1 + \psi)}\right)G'(B(b, p), p') + R + \right. \\
&\left.\left. e'(T(B(b, p), p'))T'(B(b, p), p')A(T(B(b, p), p'))\right)\right].
\end{aligned}
$$

Hence,

$$
\begin{aligned}
-\frac{\lambda(1 + \psi)(1 + p)\theta}{\beta\lambda G(b, p)} &= E_p((1 + \beta)\lambda + \theta(1 + p)(1 + \lambda))\frac{G'(B(b, p), p')}{G(B(b, p), p')} \\
&\quad - E_p\frac{\lambda(1 + \psi)(1 + p)\theta}{G(B(b, p), p')} \cdot \left(\left(1 + \frac{1 + \beta}{\theta(1 + p)(1 + \psi)}\right)G'(B(b, p), p') + R\right)
\end{aligned}
$$

where the term on the left-hand side has been replaced using (63), while the simplification on the right hand-side follows from (58). Finally, after rearranging terms, we obtain

$$
\frac{1}{G(b, p)} = \beta\lambda R \cdot E_p\frac{1}{G(B(b, p), p')} - \beta\lambda E_p\left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1\right)\frac{G'(B(b, p), p')}{G(B(b, p), p')},
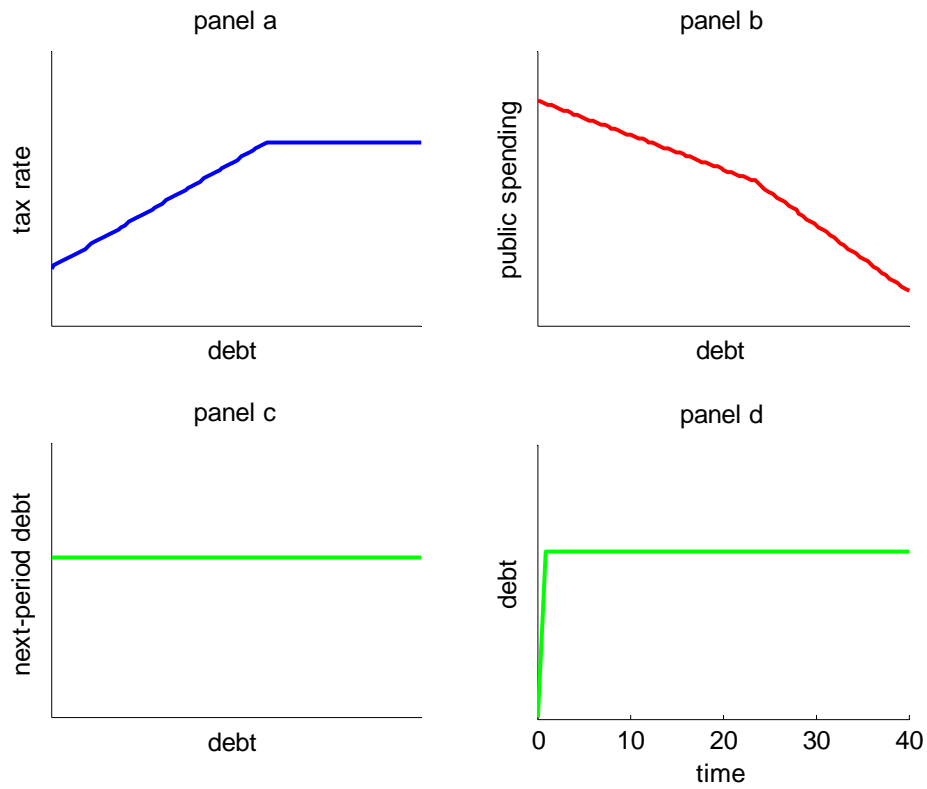$$

that is, the GEE (35) in the text. This concludes the proof of Proposition 5.

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Figure 1: Example I ($\xi=0$)
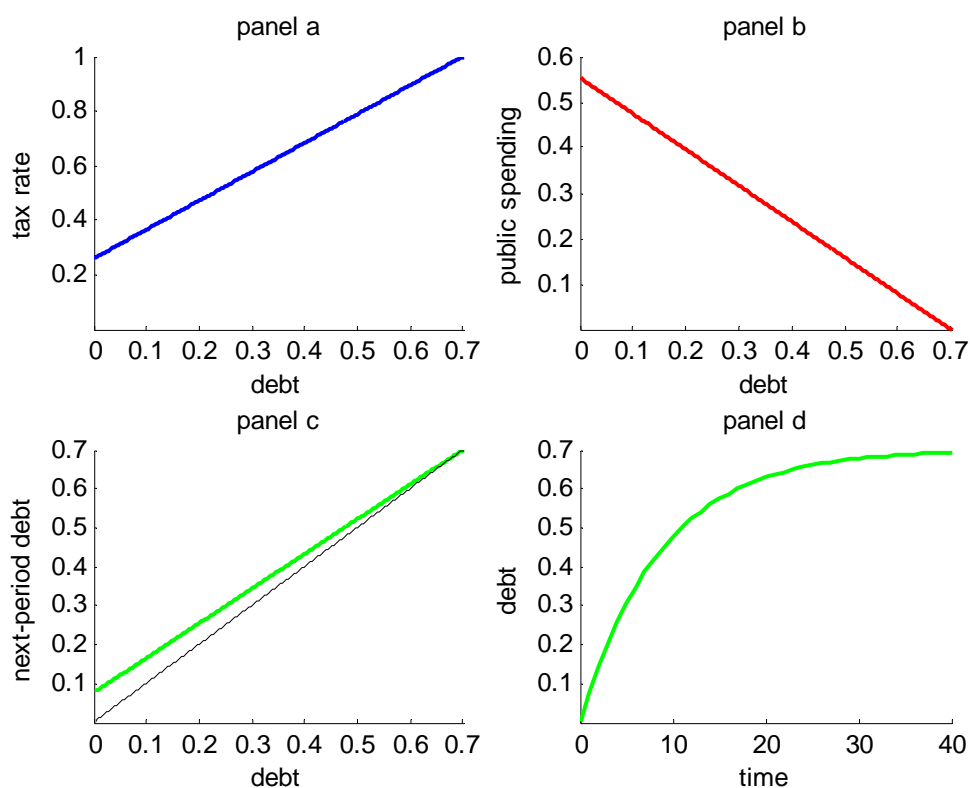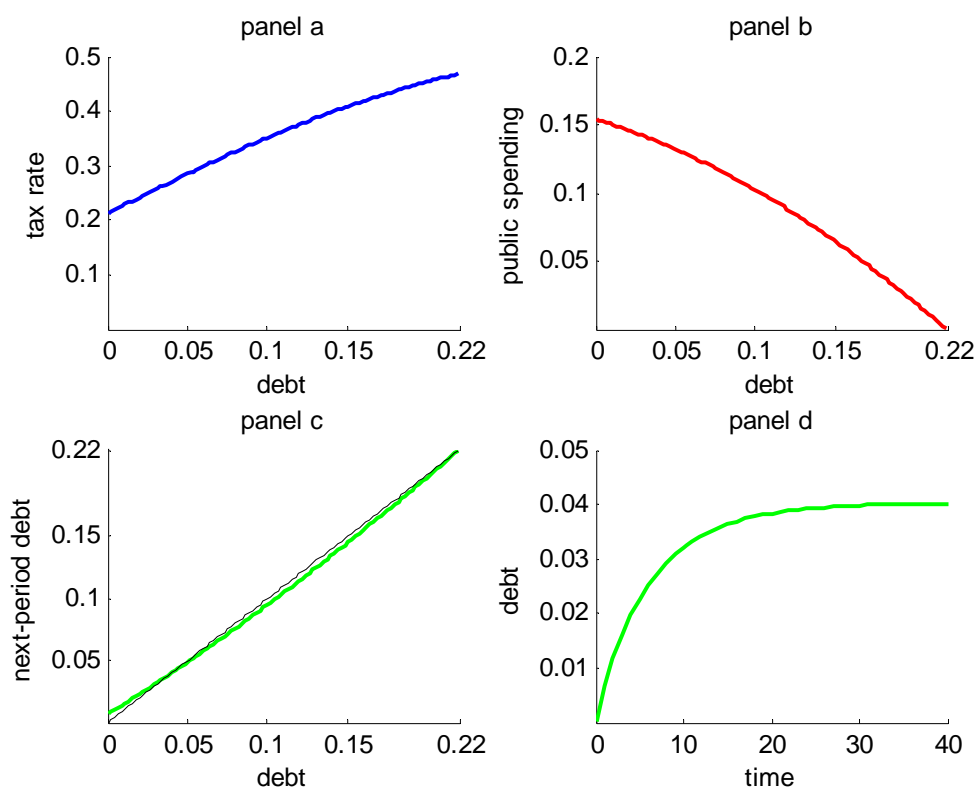

The figure shows equilibrium policy rules T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are $\beta = \lambda = 0.98^{30}$, $R = 1.03^{30}$, $\omega = 0.50$, $\theta = 1.00$ and $w = 1$. The maximum debt level is $\bar{b} = 0.70$.

Figure 2: Example II (ξ=1)

The figure shows equilibrium policy rules T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are $\beta = \lambda = 0.98^{30}$, $R = 1.03^{30}$, $\omega = 0.50$, $\theta = 1.00$, $w = 1$ and $\bar{\tau} = 0.60$.
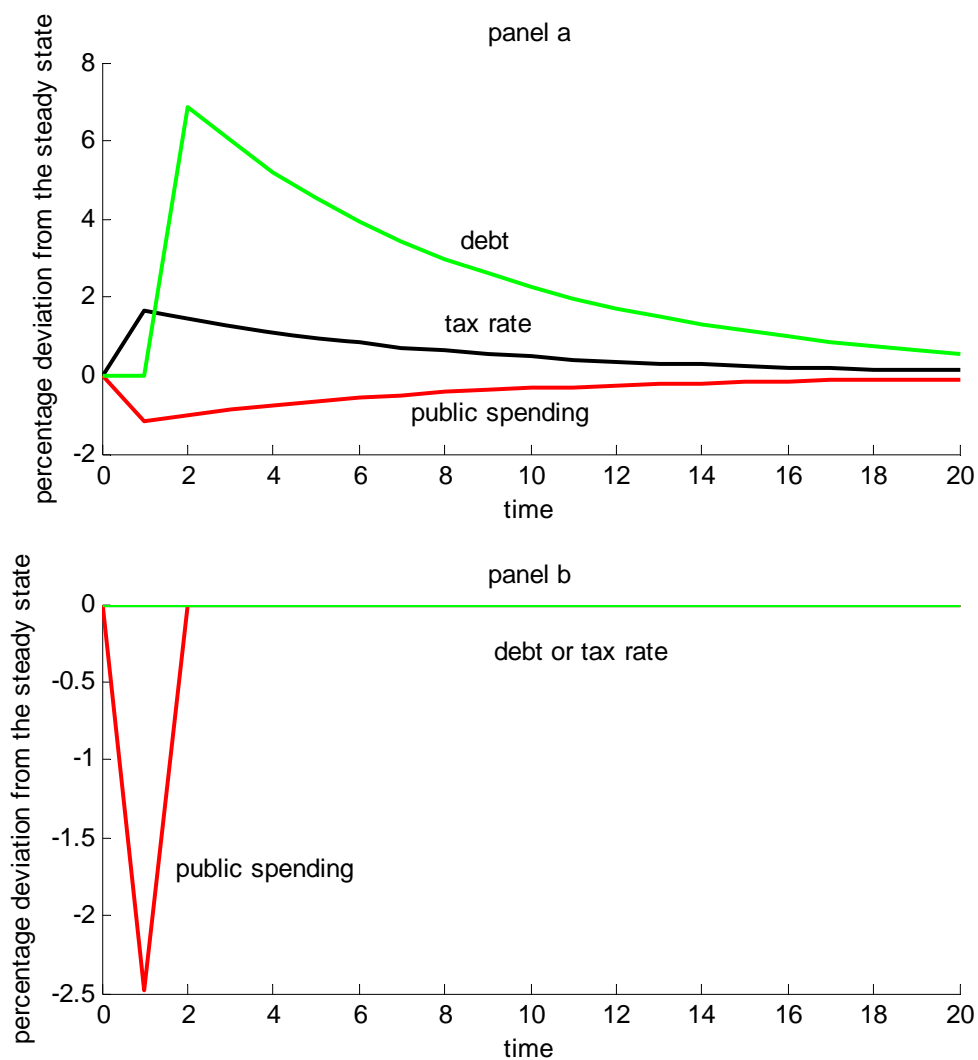
Figure 3: General case (0<ξ<1)

The figure shows equilibrium policy rules for the calibrated economy: T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are $\beta = 0.98^{30}$, $\lambda = 0.79$, $R = 1.025^{30}$, $\omega = 0.50$, $\theta = 0.09$, $\xi = 0.17$, $X = 1.75$, and $w = 1$. The maximum debt level is $\bar{b} = 0.22$. The steady state levels are $\tau = 0.27, g = 0.14, b = 0.04$.

Figure 4: Ramsey versus Markov
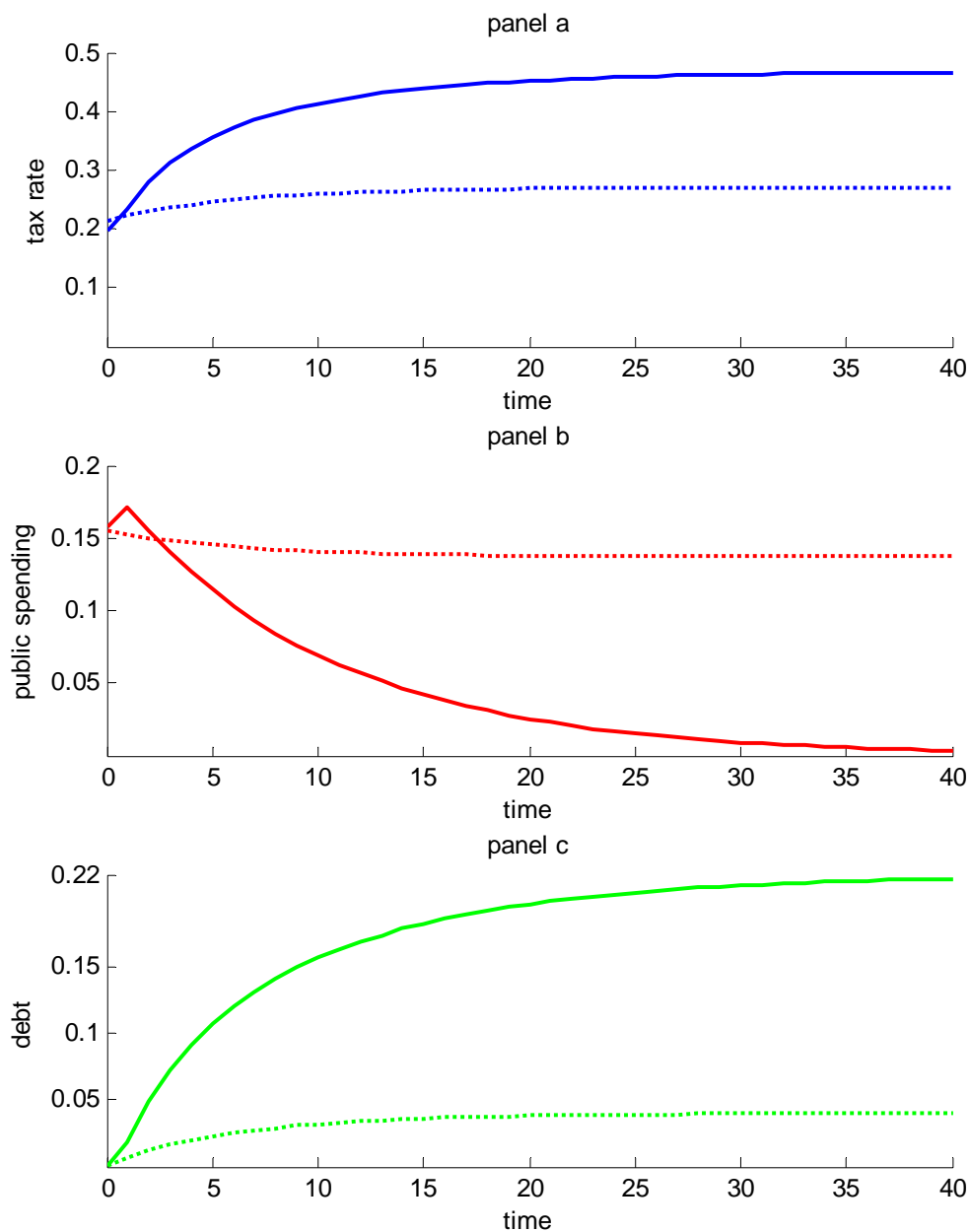
The figure shows the Ramsey paths (solid lines) and Markov equilibrium paths (dotted lines) of taxes (panel a), public spending (panel b) and debt (panel c). All parameter values are as in Figure 3 (calibrated economy).

**Figure 5: Impulse Response Functions for an Unanticipated War**

The figure shows the impulse-response functions of tax, government spending and debt for the calibrated economy (panel a), and for the economy in example II (panel b). Parameters are as in figure 3 and figure 2, respectively. The war expenditure Z is set equal to 1% of GDP.

Figure 6: Political Shocks
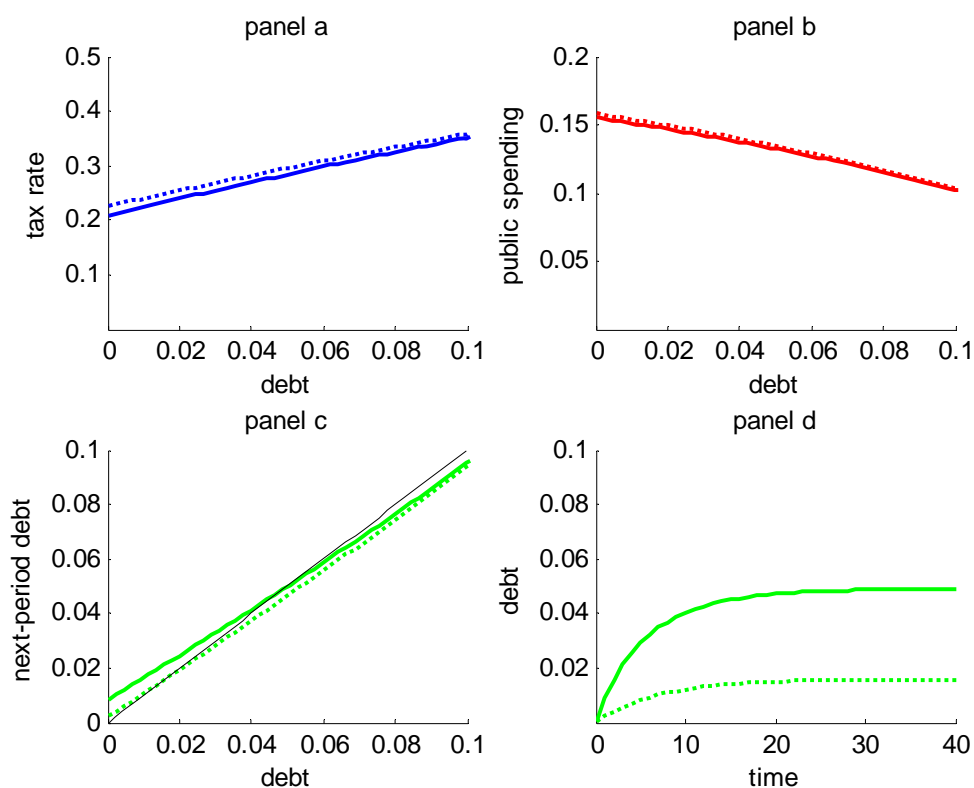
The figure shows equilibrium policy rules for the calibrated economy under left-wing (dotted lines) and right-wing (solid lines) governments: T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium path of b (panel d). Parameter values are: $p_l = 0.05, p_r = 0, \pi_{ll} = \pi_{rr} = 0.5$. The other parameter values are as in Figure 3 (calibrated economy). Panel d plots the evolution of debt under *perpetual* right- and left-wing governments.

Figure 7: Response to a Right-wing Shift
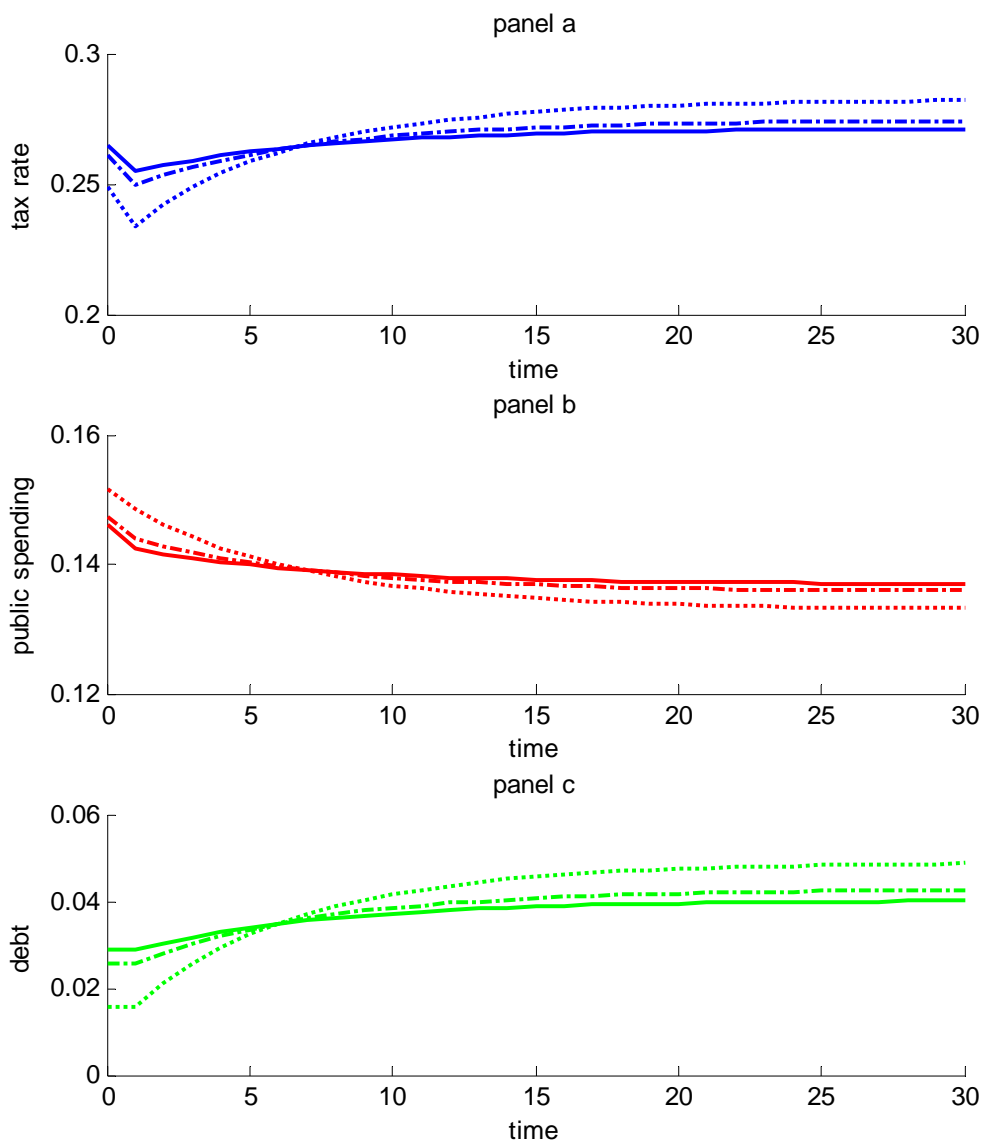
The figure shows the equilibrium paths of taxes (panel a), public spending (panel b) and debt (panel c) for economies which are initially in the left-wing steady states and experience a persistent shift to the right. The three lines in each panel represent economies with different persistence of political color: $\pi_{ll} = \pi_{rr} = 1.0$ (solid lines), $\pi_{ll} = \pi_{rr} = 0.9$ (dashed lines), and $\pi_{ll} = \pi_{rr} = 0.5$ (dotted lines). Parameter values are as in Figure 6.