King of the Hill

Positional Dynamics in Contests

Halvor Mehlum and Karl Moene[1]

Department of Economics, University of Oslo

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halvor.mehlum@econ.uio.no and k.o.moene@econ.uio.no

Abstract

In a contest with positional dynamics between an incumbent and a challenger i) inequality of power may magnify conflicts, ii) more severe conflicts can go together with lower turnover of incumbents, and iii) power can be self defeating as cost advantages can reduce pay-offs. These three propositions of our paper are contrary to the implications of static conflict models. They follow from incorporating positional dynamics into the standard static approach. Such positional dynamics are relevant for competition in battlefields, politics, and market places.

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1 Introduction

By knocking down the present "king of the hill", the winner can take the king's place and enjoy both the prestige and the fighting advantage that standing on the top of the hill entail. Many real power struggles are positional just like this - incumbents versus challengers, insiders versus outsiders, market leaders vs followers - but the structure is often less symmetric than the childrens game suggests. Usually contestants have different abilities to take advantage of incumbency. Some winners - in battlefields, politics, or market places - receive an even stronger position than their predecessors. They become the king, not just of a hill, but maybe of a whole mountain – raising the prestige of the position and making it even more difficult to overthrow them as the competition continues. Other winners hardly get a mound to start out from in the next period.

Below we explore such positional dynamics where fighting for power is both over rents and over positions and fighting edge in the subsequent fights. Like Esteban and Ray (1999) we focus on a situation where there is no collective decision rule and where groups with opposed interests are willing to spend resources to increase the chance of obtaining their preferred outcome. Unlike most other papers, however, we focus on alternating powers in lasting conflicts. While the immediate rent of a contestant is the difference in utilities between his own favored policy and that of his opponents, the advantages in future battles are captured by differences in the unit cost of influence. The effective prize of winning a battle contains both an immediate rent and the value of the incumbency cost advantage and will generally be different for the two groups.

We address three questions:

1. Inequality. Does leveling of the battlefield maximize conflict efforts?
2. Turnover. Do circumstances that lead to higher fighting intensities imply lower regime stability?
3. Power. Is more power always to the advantage of the powerful?

Applying traditional static contest models one would be lead to insist on an unambiguous yes to all three questions.[1] We show that these conclusions do not carry over to contests with positional dynamics.

The differences have clear policy implications. If the results from static contest models capture the essence of most conflicts, policies to reduce fighting and stabilize regimes should provide resources to the incumbent to deter the opposition from fighting. Such interventions are far from successful in all cases. Nato's intervention in Afghanistan after 2001 and Ethiopa's intervention in Somalia in 2006, demonstrate that a strengthening of the incumbent can be met by more fighting by the challengers – the Taliban in Afghanistan and the Union of Islamic Courts in Somalia.[2]

The word "feud" - a state of prolonged mutual hostility, typically between two families, clans or communities - captures the essential aspects of the conflicts that we are considering. European history has many examples. One old but prominent example is the struggle between the Guelphs and the Ghibellines, the two opposing factions in German and Italian politics in the 13th and 14th centuries.[3]

[1]Regarding 1), in static contest models, an edge to one player raises his chances of winning and induces the opponent to fight less. This reduction in the efforts of the opponent leads the more powerful to spend less in the conflict as well. Thus in static models, inequality of power dampens the struggle. The level of resources wasted in the fighting relative to the total prizes at stake is highest when the contestants are equally strong and fight for the same prize. Regarding 2), the average turnover must be highest when the chances of winning is fifty-fifty, implying that a leveling of the battlefield raises the rate of regime turnover. Thus, more fighting goes together with lower regime stability. Regarding 3), as long as the prizes are given independently of fighting efforts the powerful is bound to gain as he is more likely to obtain the prize for less effort. (For a review of models of static rent-seeking contests, see Nitzan 1994)

[2]Somalia is a particularly instructive case of lasting power struggles. Since the disintegration of the central government in 1991, the country has been wracked by internal clan warfare and a struggle with the Islamic movement. In June 2006, after months of fighting between Mogadishu's US-backed militia leaders and the Union of Islamic Courts, the Transitional Federal Government lost its power as the islamists took control of the capital city and appointed a hard-line Islamic leader to head its new legislature. With significant Ethiopian support, the Transitional Federal Government regained Mogadishu on 28 December 2006. Since then, the fighting has just escalated. The extra edge that the Ethiopian support provided, did far from deterr the fighting by the Islamic movement. In contrast to what static conflict models would predict, the extra incumbency edge induced harder opposition.

[3]The feuds consisted of a long series of battles over the control of cities like Bergamo, Ferrara, Florence, Lucca, Milan, Padua, Parma, Piacenza, Treviso, Verona and Vicenza. In each battle, the faction in power had a clear advantage over the challenger and the losing party was often exiled from the city. The two parties had diverging economic interests. The Guelphs were middle class merchants, artisans and burgers, the Ghibellines were landed aristocrats - implying that their opportunity costs of fighting were quite unequal. Once in power, each party favored different policies that the opposing party strongly disliked. The Guelphs favored mercantile liberties of urban communes and fought for the pope and against the Emperor's encroachment. The Ghibellines

Latin-America has similarly experienced repeated incumbency shifts in lasting power struggles between non-autocratic and autocratic rule. Argentina, for instance, had four episodes of moving into autocracy plus four episodes of moving out of autocracy from 1950 to 1990, Honduras and Bolivia had three plus three, while several others had two plus two (Przeworski et al. 1990). The incumbency advantages that the two types of regimes enjoyed was dramatically unequal. While authoritarian regimes had more support from the military, less authoritarian regimes had more legitimacy and more popular support.

An approach that incorporates unequal stakes and advantages with endogenous turnover of power positions is more in line with these observations. A sharper incumbency edge for one side may not only imply that this incumbency becomes more valuable, but also that the position of the challenger becomes worse. Thus, since the gain of winning is the value of not loosing, the stakes may go up for both the incumbent and the challenger, implying that the fighting intensity may go up when the strength of each side becomes more unequal. The long lasting consequences of present struggles are what distinguishes dynamic contests from static ones.

The unequal stakes may reflect differences in the supporting groups and in the access to resources. The immediate rents to a party in power may vary with ideological orientations, political capabilities and economic opportunities. Advantages in future contests may reflect varying capabilities to utilize incumbency to raise campaign contributions or, in more violent struggles, varying sympathies from the army, the police and from other parts of the state apparatus.

Fighting in the shadow of future conflicts implies that the relationships between resources spent and expected turnover of incumbents, and between power and pay-off can be less straight forward than the static models predict. Incorporating a simple but essential dynamic element, sheds new light on how conflicts affect political instability and how strength affects pay-offs.

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were traditionalists who fought against the temporal power of the pope. Dante Alighieri, himself a Gulph cavalryman, gives a poetic account of the reoccurring fights in the book Inferno in his Divine Comedy (Dante 2000).

The struggles can be over the control of natural resources (a diamond mine); over political representation; or over religion (sharia laws). The issues can be political, economic or institutional: colonial rule versus independence; democratic versus one party rule. The simple dynamics we incorporate do not exclude that the victory of one side can become an absorbing state while the struggling continues as long as the present incumbent is undefeated. In the bitter fighting struggles in South-Africa, for instance, the stakes for the apartheid regime were to prevent a permanent loss of their privileges. For the ANC, however, winning implied that the hated system was gone forever.

Our stylized set-up may also have other interpretations, such as patent races and competition over technological leadership. A case in point is the contest over mobile phone designs between Nokia and Sony-Ericsson, where winning the consumers over in one round implies a technological leadership that can help in future contests as well.

We build on an expanding literature where conflicts are seen as rent seeking contests, see Trygve Haavelmo (1954), Gordon Tullock (1980), Jack Hirschleifer (1991), Herschel Grossman (1994), Stergios Skaperdas (1992), Kai Konrad and Stergios Skaperdas (1998) and Derek Clark and Christian Riis (1998). We are also inspired by Daron Acemoglu and Jim Robinson's work (2001 and 2006) on political transition and elites. The three papers closest to ours are the contributions by Joan Esteban and Debraj Ray (1999), by William Rogerson (1982), and by Stergios Skaperdas and Constantinos Syropoulos (1996). While Esteban and Ray construct a general model of multi group conflicts with heterogenous prizes, Skaperdas and Syropoulos consider the problem of achieving cooperation when an early victory to one group improves the groups position in subsequent periods. The somewhat overlooked contribution by Rogerson (1982) focuses on insiders and outsiders in a symmetric lobbying game over a homogenous prize where a winning outsider becomes the new insider. We go beyond Rogerson, however, by focusing on an asymmetric cost structure, an asymmetric equilibria, and by incorporating a more general contest success function.

2 Contests with incumbency advantages

An incumbent and a challenger compete for power and use force to improve their chances of winning. We derive how the stakes at play depend i) on the polarization of preferences and ii) on the cost structure of the contestants.

There are two possible fighting constellations: the first with a as the incumbent and b as the challenger; the other with b as the incumbent and a as the challenger. The simple dynamics that we incorporate is the endogenous switches between the two fighting constellations: when a loses we move to the constellation where b is the incumbent, and so on. The probability of winning is positively related to the use of force in the contest. More specifically we define relative force S_i of an incumbent as

$$S_i = \frac{\text{force of } i}{\text{total force of } a \text{ and } b} \tag{1}$$

Throughout we use the convention that capital letters reflect incumbency position and lower case letters reflect a challenger position, such that

$$S_a + s_b = 1 \text{ and } s_a + S_b = 1 \tag{2}$$

This contest has many similarities to Cournot duopoly competition. In this analogy the parallel to "force of i" is supply of commodities from i while the parallel to S_i is the market share of i.

The antagonism between the two contestants is captured by the immediate consequences of being in power. When a is in power he implements his optimal incumbency behavior without commitment. This choice is valued as U_a for the incumbent a and as u_b for the challenger b. Thus the value of U_a reflects a's evaluation of a's own choice as incumbent while u_a reflects a's evaluation of the choice of b as incumbent. The difference between the two, D_a, is a's immediate gain of assuming

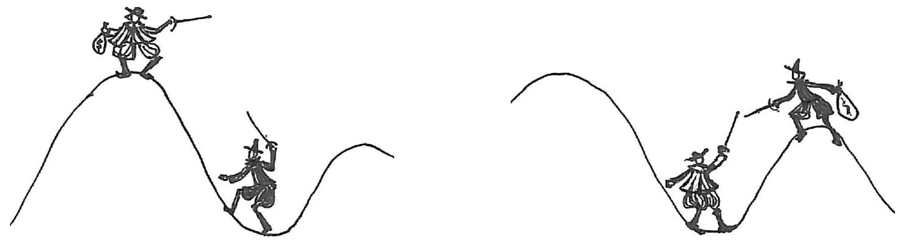


Figure 1: Example of the two fighting constellations

power - his incumbency rent. Similarly D_b is b's incumbency gain.[4]

$$D_a = [U_a - u_a] \tag{3}$$

$$D_b = [U_b - u_b] \tag{4}$$

The second aspect of the conflict concerns the contestants' ability to take power and remain in power. We assume that for each contestant, incumbency implies a cost advantage in the sense that the unit cost of the use of force is lower as incumbent than as challenger. This cost advantage yields an advantage over and above the immediate rent of being in power. In Figure 1 we have illustrated one example of the difference in incumbency advantage and the difference in incumbency rent in the two fighting constellations. The incumbency advantages are illustrated by the heights of the hill, while the size of the per period rent D_a and D_b is illustrated by the size of the purse. Thus, the figure illustrates the case where a (the white) has a strong incumbency advantage and collects a modest rent, while b (the black) has a smaller incumbency advantage but collects a higher rent.

For each contestant the immediate rent (of *assuming* power) plus the evaluation of the cost advantage (of *being in* power) constitute the prize that he fights for. These total prizes are denoted F_a and F_b.

Due to the competition, the contestants can never collect the whole of these

[4] In a multi-group context, Esteban and Ray (1999) use utility differences like these as an indication of intergroup distance. The value of $[U_a - u_a]$ measures the distance from group a to group b, the value of $[U_b - u_b]$ the distance from b to a. The larger these measures, the more antagonism there is between groups and the more polarized are the preferences.

prizes for free. For ease of exposition we first consider the case where the net rate of return for the contestants are known parameters H (for incumbents) and h (for challengers). Hence,

$$\text{net return for incumbent} = H_i F_i \tag{5}$$

$$\text{net return for challenger} = h_i F_i \tag{6}$$

Consider contestant a. If he wins, he starts the next contest as the incumbent and anticipates a net return equal to $H_a F_a$. If he loses he starts the next contest as the challenger with an anticipated return equal to $h_a F_a$. The difference $[H_a - h_a] F_a$ is the valuation of the cost advantage and equals the net gain from starting out as an incumbent rather than as a challenger. When adding the immediate rent D_a, we find the prizes from winning. Including a time index t and a discount factor $\delta < 1$, for both contestants we have

$$F_{a,t} = D_a + \delta [H_{a,t+1} - h_{a,t+1}] F_{a,t+1} \tag{7}$$

$$F_{b,t} = D_b + \delta [H_{b,t+1} - h_{b,t+1}] F_{b,t+1} \tag{8}$$

In the remainder of the paper we focus on stationary equilibria. In the appendix we discuss the possibility of multiple equilibria and of non stationarity. Solving (7) and (8) it follows that in a stationary equilibrium.

$$F_a = \frac{D_a}{1 - \delta [H_a - h_a]} \geq D_a \tag{9}$$

$$F_b = \frac{D_b}{1 - \delta [H_b - h_b]} \geq D_b \tag{10}$$

The contestants prizes differ in two dimensions: i) they fight for different incumbency rents $D_i = U_i - u_i$ and ii) the rents are enjoyed for different expected length and comes at different costs.

Clearly, the higher the antagonism between the two, the higher the prizes they are fighting for. Each of the two prizes also depends on the incumbency power of both contestants. The valuation of incumbency $(H_i - h_i)$ will be substantial if i-s incumbency position is strong (H_i large) or if i-s challenger position is weak (h_i small). As we will prove below, an incumbency advantage to at least one of the contestants implies that the evaluation of incumbency is strictly positive for both. The intuition is simple: if a has a strong incumbency advantage, then H_a is large; but if H_a is large then h_b is small. Or to put it differently: *the gain of winning is the value of not losing.*

The stakes of a contestant may be high because his incumbency benefits are high, or because his challenger benefits are low. The stakes may also be high because incumbency is expected to last, implying that the contestant can either, for a long period, enjoy the incumbency benefits or be squeezed as a challenger. A weak challenger can therefore fight hard for an advantageous incumbency position in future battles, or he can fight hard to eliminate the harmful consequences of a strong incumbent of today. In other words, a contestant may put in a lot of effort either because winning enables him to remain in power, or because winning prevents the opponents from an advantageous incumbency. To prevent the opponent from utilizing its insider edge can be equally important as to utilize it for oneself.

2.1 Micro motives

In this section we derive the contest efforts and net returns as a perfect Markov equilibrium. We assume that winning the contest requires the relative force, S, to be larger than an uncertain threshold - analogous to probabilistic voting models. Hence, we write the probability of winning as

$$\text{probability of winning} = \Psi(S)$$

In this generalization of the Tullock contest success function the probability of winning is homogeneous of degree zero in force. We have three additional requirement for the relationship $\Psi(.)$

1. Anonymity (i.e. symmetry): For all S we require that $\Psi(S) = 1 - \Psi(1-S)$

2. Force pays: $\Psi'(S) > 0$.

3. No force implies a sure loss: $\Psi(0) = 0$.

It follows from the first requirement that $\Psi'(S) = \Psi'(1-S)$ and $\Psi''(S) = -\Psi''(1-S)$. Clearly, the much used Tullock contest success function, $\Psi(S) = S$, is a special case of our set-up.[5]

How sensitive the winning probabilities are to relative force (spending) may reflect institutional arrangements. In a pure unrestricted power struggle the Ψ-function is more sensitive to changes in relative force than it would be with more checks and balances. In elections, for instance, where the opposing blocks are not allowed to buy political TV ads and instead have an equal share of time on public television to present their political programs, the Ψ- function is rather flat around $S = 1/2$.

Given the probability of winning $\Psi(S_a)$, when a is the incumbent and b is the the challenger, the present value of the pay-offs ($V_{a,t}$ for incumbent a and $v_{b,t}$ for challenger b) can be written as

$$V_{a,t} = \Psi\overbrace{\left(\frac{Y_a/C_a}{Y_a/C_a + y_b/c_b}\right)}^{S_a}\overbrace{\left[D_a + \delta\left(V_{a,t+1} - v_{a,t+1}\right)\right]}^{F_{a,t}} + \delta v_{a,t+1} - Y_{a,t} \tag{11}$$

$$v_{b,t} = \left[1 - \Psi\left(\frac{Y_a/C_a}{Y_a/C_a + y_b/c_b}\right)\right]\overbrace{\left[D_b + \delta\left(V_{b,t+1} - v_{b,t+1}\right)\right]}^{F_{b,t}} + \delta v_{b,t+1} - y_{b,t} \tag{12}$$

Here Y and y are incumbent's and challenger's costs while C and c are the unit costs of force. Hence, if a is the incumbent and a wins, he obtains the prize $F_{a,t}$, which is the utility difference D_a, plus the excess present value from incumbency, $\delta\left(V_{a,t+1} - v_{a,t+1}\right)$. Whether he loses or not he has to pay the costs Y_a. In addition

[5] See Skaperdas (1996) for a structured discussion of success functions.

he always receives the discounted challenger pay-off $\delta v_{a,t}$. The expressions for $V_{b,t}$ and $v_{a,t}$ follow symmetrically.

When a is incumbent, the first order conditions for the choice of efforts follows from (11) and (12) as

$$\frac{\partial V_a}{\partial Y_a} = 0 => S_a \left(1 - S_a\right) F_a \Psi' \left(S_a\right) = Y_a \tag{13}$$

$$\frac{\partial v_b}{\partial y_b} = 0 => S_a \left(1 - S_a\right) F_b \Psi' \left(S_a\right) = y_b \tag{14}$$

The second order conditions can be written as

$$-\frac{2}{S_a} < \frac{\Psi'' \left(S_a\right)}{\Psi' \left(S_a\right)} < \frac{2}{1 - S_a} \tag{15}$$

In addition, we impose participation constraints, assuring equilibrium in pure strategies with positive expected return - i.e. that the net rate of returns are positive:

$$H_a \equiv \Psi \left(S_a\right) F_a - Y_a > 0 \text{ and } h_b \equiv \left(1 - \Psi \left(S_a\right)\right) F_b - y_b > 0 \tag{16}$$

Using (2) it follows that in a Nash equilibrium where both (13) and (14) are satisfied, then relative force is

$$S_a = \frac{F_a/C_a}{F_a/C_a + F_b/c_b} \tag{17}$$

and by symmetry when b is incumbent, the relative force is

$$S_b = \frac{F_b/C_b}{F_a/C_a + F_b/c_b} \tag{18}$$

Thus, each contestant's prize relative to the unit cost of force determines the force of the two sides. A contestant with either a high stake or a low cost has a high equilibrium relative force.

From (13) and (14) and from the symmetry of Ψ it follows that the net rate of return H_a and h_b both are determined by the following function $h(.)$

$$h(S) = \Psi(S) - S(1-S)\Psi'(S) \ , \ \frac{\partial h(S)}{\partial S} > 0, \ h(0) = 0, \ h(1) = 1 \qquad (19)$$

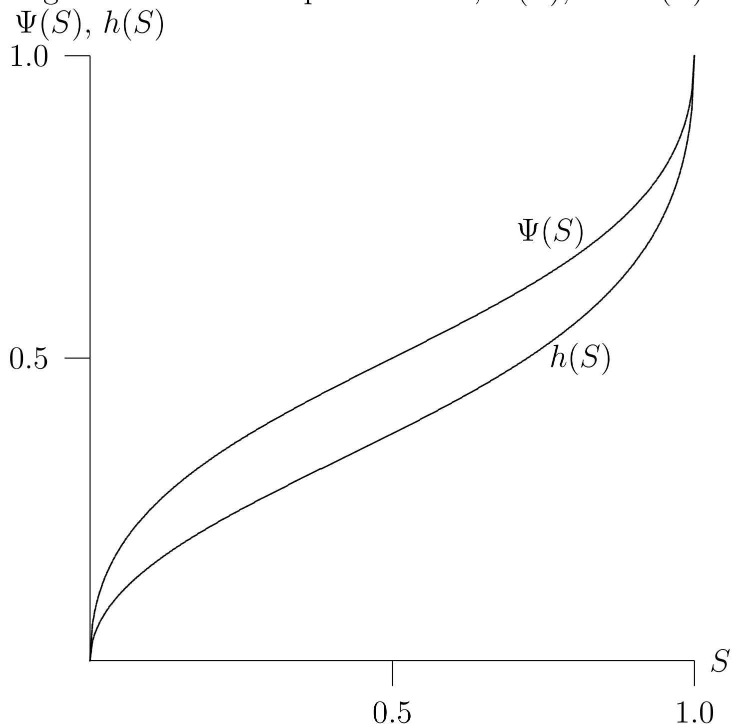
such that

$$H_a \equiv \frac{\Psi(S_a) F_a - Y_a}{F_a} = h(S_a) \qquad (20)$$

$$h_b \equiv \frac{[1 - \Psi(S_a)] F_b - y_b}{F_b} = h(s_b) = h(1 - S_a) \qquad (21)$$

One illustration of the relationship between S, Ψ, and h is given in Figure 2. By

Figure 2: Relationship between S, $\Psi(S)$, and $h(S)$



definition the h function is below Ψ for all S. The h function reaches unity when S is one, since the prize is then won with certainty and for free. The h function is zero when S is zero since the prize cannot be won and no efforts are wasted.[6]

[6] In the case where the prizes are fixed and equal to $F_a = F_b = F$, the sum of $h(S)$ and $h(1-S)$ is negatively associated with the social waste of the fight $Y_a + y_b$. Then we can define the total waste ratio as total resources spent on the conflict relative to the prize. The waste ratio function

We are now ready to prove the result anticipated in the previous section. By differentiating (9), (10), (17), and (18), using (20) and (21) it follows that

$$\frac{\mathrm{d}F_i}{F_i} = -S_a S_b \delta h'(S_i)\left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b}\right) \tag{22}$$

$$\mathrm{d}S_i = -S_a^2 S_b^2 \delta(h'(S_i) - h'(1-S_i))\left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b}\right) - S_a S_b \frac{\mathrm{d}C_i}{C_i} \tag{23}$$

Hence we have the following proposition

Proposition 1 *Compared to the case without incumbency advantage an incumbency edge to one contestant raises the prize to both sides of the struggle. The effects on prizes are the same irrespective of who gets the incumbency advantage.*

Proof. Follows from (22) when noting that incumbency advantage to one contestant implies that either $\mathrm{d}C_a/C_a$ or $\mathrm{d}C_b/C_b$ is negative. ■

An incumbency edge that favors one party only, makes the stakes higher for both. The reason is that the prize of winning is high both when the pay-off of becoming the incumbent is high and when the pay-off of becoming the challenger is low. The winning contestant obtains the *difference* between these as part of the prize.

If the utility structure is not too uneven an incumbency advantage to one contestant may increase efforts by both at the same time as regime stability increases:

Proposition 2 *If both sides are equally strong ($S_a = S_b = 1/2$) resource use goes up for both sides in each fighting constellation while regime stability goes up as one side gets an incumbency advantage. If one side (a) is particularly strong as incumbent ($S_a \approx 1$), resource use goes down for both sides as a gets a further incumbency advantage while it goes up for both sides if b gets an incumbency advantage.*

$\omega(S)$ is given by

$$\omega(S) = 1 - (h(S) + h(1-S)) = 2S(1-S)\Psi'(S) > 0 \text{ when } S \in \langle 0, 1\rangle$$

When Ψ'' is not too large, the waste ratio ω has its maximum at $S = 1/2$.

$\omega(S_a)$ has a local extrema when $\Psi''/\Psi' = (2S_a - 1)/\left(S_a - S_a^2\right)$.Given the symmetry of the Ψ-function $S_a = 1/2$ obviously satisfies the first order condition and if Ψ is S-shaped ($\Psi'' < 0$ to the right of $S_a = 1/2$ i.e. a strong decreasing returns to effort) $S_a = 1/2$ is a unique and global maximum. As it happens, the function $\Psi(S) = 1/2 - k/4(\ln(1-S) - \ln(S))$ (where $\Psi'(1/2) = k$) has exactly the curvature such that social waste is independent of S, as in this case $\omega(S) = k/2$ for all S.

Proof. When $S_a = S_b = 1/2$ it follows that $\Psi'' = 0$. It then follows from the first order conditions (13) and (14) that

$$\frac{\mathrm{d}Y_i}{Y_i} = \frac{\mathrm{d}y_i}{y_i} = \frac{\mathrm{d}F_i}{F_i}$$

Combined with the results from the previous proposition we know that resource use goes up. That stability goes up follows directly from (23) as $h'(S_i) - h'(1 - S_i) = 0$ when $S_a = S_b = 1/2$.

With uneven positions these results are altered. When $S_a \to 1$ it follows from (22) and (23) that the F-s and the S-s are unaffected by a change in the C-s. It thus follows from (17) and (18) that

$$\frac{\mathrm{d}Y_a}{Y_a} = \frac{\mathrm{d}y_b}{y_b} = \frac{\mathrm{d}C_a}{C_a} \text{ and } \frac{\mathrm{d}Y_b}{Y_b} = \frac{\mathrm{d}y_a}{y_a} = -\frac{\mathrm{d}C_b}{C_b}$$

■

The proposition establishes that more fighting from both sides can go together with higher regime stability. As both contestants raise their conflict spending equally much, the probability of winning for the contestant that obtains the edge increases and the average regime stability goes up. The main result is that an incumbency edge to one group raises the stakes for both groups since it is equally important to obtain the incumbency edge as it is to prevent the opponent from getting it. The prize of winning goes up for both groups. For the incumbency group it goes up as the pay-off of winning increases. For the challenger group it goes up as the pay-off of losing declines. With higher stakes both fight harder either to win the edge for future battles, or to prevent the opponent from winning it. As a result the amount of resources wasted in the conflict increases. Hence, more unequal strengths may imply that more resources are spoiled in the struggle even though the incumbent who obtains the edge wins the battle more often than the challenger.

The result that an incumbency advantage raises the prize for both contestants is not only a local phenomenon around $S_a = S_b = 1/2$. Let us define an absolute incumbency advantage where the unit cost of force approaches zero. Clearly, when a contestant with an absolute advantage becomes the incumbent, he stays forever. When one contestant gets an absolute incumbency advantage we get the following result:

Proposition 3 *Compared to the case without incumbency advantage, the introduction of an absolute incumbency advantage to one contestant raises the prizes to both sides, but most for the contestant who gets the advantage. In the limit case where both groups have absolute advantage the valuation of incumbency is the same for both sides and equal to* $1/(1-\delta)\,D_i$.

Proof. Consider the case where a gets an absolute incumbency advantage ($C_a \to 0$). It follows from (17) and (18) that $S_a = 1$ and $S_b < 1$. From (19) it in turn follows that

$$h(S_a) - h(1 - S_b) = 1 - h(1 - S_b) > h(S_b) - h(1 - S_a) = h(S_b)$$

When both get absolute incumbency power ($C_a \to 0$ and $C_b \to 0$) It follows from (17), (18) that $S_a = S_b = 1$ and that $\Psi(S_a) = \Psi(S_b) = 1$. From (19) it in turn follows that

$$h(S_a) - h(1 - S_b) = h(S_b) - h(1 - S_a) = 1 - 0$$

■

As long as only one contestant has an absolute incumbency advantage and he remains the challenger, fighting is hard as the stakes are high for both.

We have shown that (i) introducing a minor incumbency advantage to one contestant, and (ii) introducing an absolute incumbency advantage to one side or both, raises the prize for both sides in the conflict. But the prizes do not always increase as the unit cost of influence decreases for one contestant. To see this consider the simple case where $C_a = C_b$, $D_a = D_b$, and $\Psi(S) = S$: Then we can show that a reduction in the cost of influence for a as incumbent would lower the prize to b if the discount factor is high enough, since

$$\frac{\partial F_b}{\partial C_a} > 0 \iff C_a = C_b < 2\sqrt{\delta / (1+\delta)} - 1 \tag{24}$$

Therefore, if both parties have a strong incumbency advantage initially, a further advantage for one group will lower the prize for the other. This result is a combination of two effects. First when both sides have strong incumbency advantages, the challenger position is dismal for both. Hence, both v_a and v_b are low and cannot be much affected by a further reduction in the influence costs of the incumbents. Now, if party a gets an even stronger incumbency advantage, implying that C_a goes down, contestant a will fight harder as challenger, lowering V_b. If the future matters sufficiently for party a (δ high) the value of V_b will go so much down that F_b also declines. From condition (24) it is clear that $\delta > 1/3$ is an absolute requirement for this to be possible for any positive C.

2.2 Self-defeating power

Above we have seen that incumbency advantages can explain higher fighting efforts and higher conflict spending for the contestants. It is even possible that the cost associated with increased force makes the situation worse for the contestant who improves his incumbency advantage. A strengthening of the incumbency advantage for a present challenger may represent a serious threat for the present incumbent, and the challenger could as a result be met with a much more heavy resistance. The fact that the challenger actually may lose by getting the prospect of incumbency advantage could make it optimal for a challenger to try to commit to abstaining from using some of his incumbency power. More precisely:

Proposition 4 *Power can be self-defeating: An incumbency advantage to a weak contestant may induce so much fighting that the pay-off to the weak contestant goes down.*

Proof. Consider a marginal incumbency advantage for a weak group a ($S_a = s_a$ small). From (20), combined with the value functions (11) and (12) (where in the latter a and b change places and where time subscripts are ignored), it follows that

$$\begin{aligned}(1-\delta)\frac{\mathrm{d}v_a}{F_a} &= h(S_a)\frac{\mathrm{d}F_a}{F_a} - h'(S_a)\,\mathrm{d}S_b \\ \frac{\mathrm{d}V_a}{F_a} - \delta\frac{\mathrm{d}v_a}{F_a} &= h(S_a)\frac{\mathrm{d}F_a}{F_a} + h'(S_a)\,\mathrm{d}S_a\end{aligned}$$

It follows by combining with (22) and (23) that

$$\begin{aligned}\frac{\mathrm{d}v_a}{F_a} &= \frac{\delta}{1-\delta}h'(S_a)\,S_aS_b\left[\left(h'(S_b)-h'(S_a)\right)S_aS_b - h(S_a)\right]\frac{\mathrm{d}C_a}{C_a} \\ \frac{\mathrm{d}V_a}{F_a} &= \frac{\mathrm{d}v_a}{F_a} - h'(S_a)\,S_aS_b\frac{\mathrm{d}C_a}{C_a}\end{aligned}$$

Using (19) it follows that, when S_a is small

$$\frac{\mathrm{d}v_a}{F_a} \approx \frac{2\delta}{1-\delta}h'(S_a)\,S_a^2S_b\Psi'(0) > 0$$

$$\frac{\mathrm{d}V_a}{F_a} \approx \frac{1}{1-\delta}S_aS_bh'(S_a)\left[-1+\delta\left(2\Psi'(0)+1\right)\right]\frac{\mathrm{d}C_a}{C_a} \begin{matrix} > 0 \text{ when } \delta \text{ close to } 1 \\ < 0 \text{ when } \delta \text{ close to } 0 \end{matrix}$$

■

This result is in stark contrast to the result in static contests where the return unambiguously goes up when costs go down. The intuition is simple enough. As a challenger contestant a has no direct gain from his incumbency advantage. If the incumbency advantage increases, the probability of becoming the incumbent may go so much down that there is a net loss. This mechanism may even be the dominant one for an incumbent. A weak incumbent a will know that the larger part of the future periods will be played as challenger. The value V_a will largely be determined by v_a. Hence, if δ is large and S_a is low, an incumbency advantage that lowers v_a may indeed also lower V_a.

That a challenger may benefit from lowering his incumbency advantage has a clear relevance for present power struggles. In Turkey for example, the Islamist party only won when it promised to respect the secular state and the democratic principles. One way to interpret this is that it was seen less of a treat to the secular elite (including the army) when it had promised to be moderate in its use of power once in power.

3 Concluding remarks

Disparities play an essential role in lasting power struggles. They shape the dynamics of the conflicts. A specific incumbency advantage implies that a victory today may to some degree guarantee the victory also tomorrow, and as the expected outcome today depends on who is the incumbent, the victory of tomorrow actually depends on who was yesterday's winner. The past thus affects present fighting efforts, which again affects the future path of the struggle.

Thus the struggle between groups over the control of a country plays out differently in countries where the control entails access to a strong state apparatus compared to countries where the state apparatus is weak. In divided societies, a strong state may fuel conflicts rather that mitigate them. Control of the state apparatus makes the incumbent stronger, but a stronger incumbent makes the control of the state apparatus more valuable. As a result the struggle for state control is intense and the amount of resources wasted is high. Therefore, regimes may be long lasting without deterring fighting by opposing groups. The attractiveness of taking over a strong incumbency position may dominate the low odds of a short run success.

The end of the cold war represented a dramatic shift in many internal conflicts of the world. Many groups that had previously been supported by the East and the West respectively, now were left to cater for themselves. As a result, the incumbency advantage associated with international recognition and access to government resources became relatively more important. Our analysis may explain why the many civil wars continued with high intensity even after 1990. The reason could simply be that victory became more important when a higher incumbency advantage became part of the prize of winning.

References

Acemoglu, Daron and James A. Robinson (2001) "A Theory of Political Transition" *The American Economic Review*, 91(4), pp. 938-963.

Acemoglu, Daron and James A. Robinson (2006) *Persistence of Power, Elites and Institutions*, NBER workingpaper No. 12108.

Clark, Derek J. and Christian Riis (1998) "Competition over More Than One Prize" *American Economic Review* 88(1), pp. 276-89.

Dante Alighieri (2000) *The Divine Comedy,* Available from http://www.italianstudies.org/comedy/index.htm.

Esteban, Joan and Debraj Ray (1999) "Conflict and Distribution" *Journal of Economic Theory* 87, pp. 379-415.

Grossman, Herschel I. (1994) "Production, appropriation, and land reform" *American Economic Review*, 84 No. 3. 705–712.

Haavelmo, Trygve (1954) *A study in the theory of economic evolution*, North-Holland, Amsterdam.

Hirshleifer Jack. (1991) "The paradox of Power" *Economics and Politics* 4, pp. 177-200.

Konrad, Kai A. and Stergios Skaperdas (1998) "Extortion" *Economica*, 65, pp. 461-477.

Nitzan, Shmuel (1994) "Modelling Rent-Seeking Contests" *European Journal of Political Economy* 10(1), pp. 41-60.

Przeworski, Adam et al. (1990) *Democracy and Development- Political Institutions and Well-Being in the World, 1950-1990*, Cambridge University Press, Cambridge.

Rogerson, William P. (1982) “The Social Costs of Monopoly and Regulation: A Game-Theoretic Analysis” *The Bell Journal of Economics* Vol. 13, No. 2, pp. 391-401

Skaperdas, Stergios, (1996) “Contest success functions” *Economic Theory* 7, pp. 283-290.

Skaperdas, Stergios and Constantinos Syropoulos (1996) ’Can the Shadow of the Future Harm Cooperation?’, *Journal of Economic Behavior and Organization*, 29(3), 355-372.

Tullock, Gordon (1980) “Efficient Rent Seeking” in J. Buchanan R Tollison and G. Tullock, eds. *Toward a Theory of Rent Seeking Society.* Texas A&M University Press, pp. 97-112

A Appendix Non-stationary solutions

In our analysis we have assumed that we are in a stationary Markov equilibrium where (9) and (10) holds. In this section we will prove that there always exists at least one stationary Markov equilibrium. We will also discuss the possibility of non-stationary equilibria and the possibility of more than one stationary equilibrium.

A Markov equilibrium (not necessarily stationary) of the game is defined as time paths of F_a and F_b such that (7) and (8) holds hence

$$\begin{aligned} F_{a,t-1} &= F_a\left(F_{a,t}, F_{b,t}\right) \equiv D_a + \delta\left(h\left(S_{a,t}\right) - h\left(s_{a,t}\right)\right) F_{a,t} \\ F_{b,t-1} &= F_b\left(F_{a,t}, F_{b,t}\right) \equiv D_b + \delta\left(h\left(S_{b,t}\right) - h\left(s_{b,t}\right)\right) F_{b,t} \end{aligned} \tag{25}$$

where

$$S_{a,t} = \frac{F_{a,t}c_b}{F_{a,t}c_b + F_{b,t}C_a} \text{ and } S_{b,t} = \frac{F_{b,t}c_a}{F_{b,t}c_a + F_{a,t}C_b} \tag{26}$$

A fix point of the system (25) is defined as (F_a^{**}, F_b^{**}) such that

$$F_a^{**} = F_a\left(F_a^{**}, F_b^{**}\right) \text{ and } F_b^{**} = F_b\left(F_a^{**}, F_b^{**}\right) \tag{27}$$

In order to find the fix points we first find the fix point, F_a^*, for F_a given F_b and vice versa such that

$$\begin{aligned} F_a^* &= F_a\left(F_a^*, F_{b,t}\right) \\ F_b^* &= F_b\left(F_{a,t}, F_b^*\right) \end{aligned} \tag{28}$$

From (19) it follows that

$$0 \leq \left(h\left(S_i\right) - h\left(s_i\right)\right) \leq 1$$

therefore a fix point for F_i has to be larger than D_i and less than $D_i/\left(1-\delta\right)$. From (19), (15), (16) and (26) it follows that

$$\frac{\partial F_{i,t-1}}{\partial F_{i,t}} = \delta\left(\left(h_i + \frac{\partial h\left(S_{i,t}\right)}{\partial S_{i,t}}\frac{\partial S_{i,t}}{\partial F_{i,t}}F_{i,t}\right) - \left(h_i + \frac{\partial h\left(s_{i,t}\right)}{\partial s_{i,t}}\frac{\partial s_{i,t}}{\partial F_{i,t}}F_{i,t}\right)\right) < 1$$

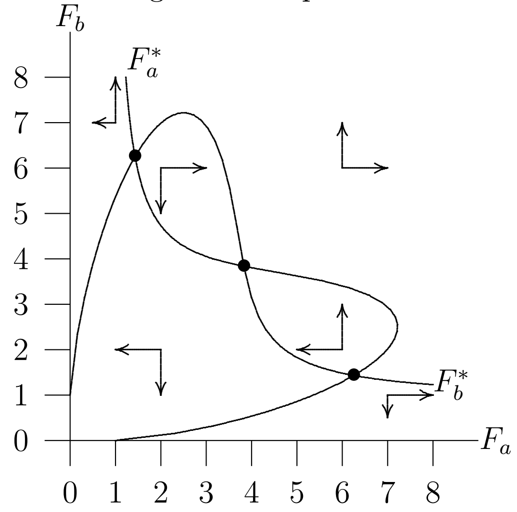
It therefore follows that (28) has unique solutions.

$$\begin{aligned} F_a^* &\in R_a \equiv \left[D_a, \frac{D_a}{(1-\delta)}\right] \\ F_b^* &\in R_b \equiv \left[D_b, \frac{D_b}{(1-\delta)}\right] \end{aligned}$$

The combined problem (27) therefore always has at least one solution. Moreover, all solutions are found within the rectangle R_a, R_b. A simple case is the one where δ is low or $c_i \approx C_i$ for both sides. Then F_i^* is close to D_i, and F_a^* and F_b^* will only cross once.

If there are strong incumbency advantages combined with a high discount factor δ, however, the fix point curves may get sufficient curvature to generate multiple stationary equilibria. One illustration of this possibility is provided in Figure (3) . Here, the parameter configurations are symmetric ($C_a = C_b \ll 1$ and $D_a = D_b = 1$). We see that one of the equilibria is symmetric, reflecting a situation where each party inherits the others behavior when they change status. The upper left equilibrium is a case where group a (caused by a low F_a) takes on a more passive role as

Figure 3: Fixpoints



incumbent causing a low F_a. Group b however (caused by the high F_b) takes on a more aggressive role as incumbent causing a high F_b. A similar skewed equilibrium exists down and to the right. These three points all satisfy the conditions for a stationary equilibrium.