Polarization, risk and welfare in general equilibrium

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Preliminary version

Abstract

This paper studies the determinants of inequality in an infinite-horizon general equilibrium model. Missing capital markets decreases motivations for capital accumulation among the poor, while uncertainty about future income leads to precautionary savings. Hence, the degree of polarization in the wealth distribution depends critically on the level of risk in the economy. With low risk, there are two distinct population groups: the poor and the rich. There is no mobility between groups, and the wealth distribution is history dependent. With high risk, there is mobility between groups and a unique steady state.

When comparing welfare across steady states with different parameters, the rich and the very poor prefer economies where risk is low. The middle class, on the other hand, is better off if risk is higher. The effects are stronger in societies with a larger poor population. This comparison offers a new perspective on why social security systems emerge even in societies where the poor have limited democratic voice.

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1 Introduction

This paper combines a general equilibrium model of increasing returns with one of idiosyncratic labor market risk, and investigates the interactions between labor market risk and the degree of polarization in the steady state of the economy.

When the poor have lower returns to saving than the rich, the poor tend to save less, leading to a polarized economy with distinct rich and poor groups. When individial labor income is uncertain, people — in particular the poor — will save more to alleviate the adverse effects of future income shortfalls. The degree of polarization in the economy will depend on these two effects. In volatile environments, the poor have greater incentive to do buffer-stock saving, and this can overturn the separating effect of differences in saving returns.

A polarized economy, in this sense, has a two-peaked wealth distribution; a clear distinction between "rich" and "poor". In the case of low risk, there will be no mobility between groups; even with an infinite run of good-luck productivity draws, the poor will never become rich. If the level of risk is higher, however, there is a possibility of both upward and downward mobility across groups. This means that the steady states with and without mobility are qualitatively different. When there is no between-group mobility because risk is low, there is history dependence; a range of steady states are supported, and the degree of inequality in the economy depends on history. When the level of risk is sufficiently high, between-group mobility ensures that the economy eventually converges to a unique steady state.

Analyzing the interactions between volatility and income mobility raises interesting welfare questions. If low risk means that the working class are stuck where they are, could higher risk be welfare improving? It is shown that when comparing steady states for economies with different risk parameters, the richer parts of the working class do indeed benefit from increased risk. On the other hand, if we consider one economy and the transition from a low-risk to a high-risk environment, this effect does not hold; everyone loses from risk.

These results follow in part from missing markets for capital; while the rich can save by producing with self-owned capital, the poor have to resort to being workers and utilizing inferior storage technology. With this setup, we can think of the mechanisms as applying to developing countries. We can envision applications both towards contemporary developing countries, or to the Western world before the full integration of the population into capital markets.

Two main economic effects can be identified. First, if capital markets are missing, the degree of income volatility determines whether there is history dependence in the wealth distribution. The numerical solutions show that the critical value for history dependence is quite high; for most parameter values, there is history dependence.

Second, the rich are worse off in steady states with higher risk, while those with intermediate wealth are better off. This second effect has an interesting analogy in nineteenth-century introductions of social welfare reforms in the Western world. The rich were often in favor of reforms decreasing income risk, while extension of the vote to the middle class decreased popular support for social insurance.

Literature

Regarding poverty and increasing returns to saving, Galor & Zeira (1993) show how convex returns may lead to inequality. Banerjee & Newman (1993) extend this to a continous wealth space, with a two-sector structure where modern and small-scale enterprise is qualitatively different. Banerjee & Moll (2010) integrate increasing returns to the literature on capital misallocation, and argue that in steady state, the only misallocation of capital will be on the "extensive margin" — that is, caused by increasing returns. Zimmerman & Carter (2003) and Carter & Barrett (2006) show the plausibility of applying macroeconomic models with increasing returns to present-day developing countries.

The precautionary savings model in this paper is based on Aiyagari (1994), who model the general equilibrium results of increased saving due to income risk. A survey of the literature on incomplete insurance is given by Heathcote *et al.* (2009). Carroll & Kimball (1996) show that the poor have, in a relative sense, higher precautionary savings than the rich, as they are closer to the point where insurance runs out. One of the few papers combining incomplete insurance with increasing returns is Buera & Shin (2011), who show that when entrepreneurs have to build up capital to invest, the degree of shock persistence is important for the dynamics of the economy, as high persistence will mean that more entrepreneurs overcome the minimum capital threshold sooner.

Welfare analysis of the model with precautionary savings is done by Heathcote *et al.* (2008), who provide an analytical solution to a model with endogenous labor supply and show that, given insurance and an exogenous level of capital, there can be gains from increasing income volatility. Davila *et al.* (2005) discuss the case with endogenous capital, and argue that the welfare implications of different capital levels depend crucially on the type of labor market risk agents face (unemployment vs. fluctuating productivity).

2 A model of risk and polarization

The setup builds on the neoclassical model with heterogeneous agents and idiosyncratic labor market risk. The convexity in investment returns arises from missing capital markets; agents do not have the ability to rent out or borrow capital.

2.1 Setup

There is a continuum of agents summing to 1. Agents are indexed by i, and maximize the expected discounted utility of consumption

$$U_{i,t} = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{(c_{i,\tau})^{1-\eta} - 1}{1-\eta} \tag{1}$$

where $\beta < 1$ is the discount rate and $\eta > 0$ is the inverse of the intertemporal elasticity of substitution. There is no utility of leisure.

There is one good. Production of this good takes place in a continuum of firms with identical technology. The inputs are capital k and labor ℓ. Firms are indexed by j and have the production function

$$y_{j,t} = k_{j,t}^{\alpha}\ell_{j,t}^{1-\alpha} \tag{2}$$

Each agent has a labor endowment $z_{i,t}$. The endowment is stochastic and cannot be insured against; it is drawn from a finite and discrete support Z. Realizations follow a Markov probability chain with transition matrix Π. The realizations are uncorrelated across agents, and z averages to 1. Each agent i may only work in one firm j, though a firm j can utilize labor from several laborers.

Agents hold one type of asset: physical capital. The holding of an agent in a given period is denoted $k_{i,t}$. Gross income is denoted $h_{i,t}$; this is allocated to present consumption or future capital.

$$c_{i,t} + k_{i,t+1} = h_{i,t} \tag{3}$$

The agents in the economy make up a distribution over capital and individual productivity, $\Phi_t(k,z)$. Total labor supply is 1, as the individual idiosyncratic productivity shocks average to zero.

The key friction of the model is the missing capital market: Capital cannot be rented. For an individual to utilize his own capital productively, he must work in the firm himself. Capital used in production depreciates at a rate δ. Capital that is not used productively can be stored, with a storage return $\nu \leq (1-\delta)$ per unit of capital.

Labor is paid its marginal product. All agents participate in the production process, and have a choice of either producing with their own capital, or working with someone else's capital (labor alone gives no output). For those holding large amounts of capital, it is most profitable to produce with self-owned capital: operating a firm. In addition to their own labor, these agents will hire labor from other people, paying the marginal product $\frac{\partial y}{\partial \ell}$ per efficiency unit of labor. This group will be denoted **capitalists**. Agents with intermediate levels of capital will also prefer to produce for themselves, but will not have sufficient amounts of capital to make it profitable to hire others. This group, who operate with just their own capital and labor, will be termed **independents**. Agents with low levels of capital will find it most productive to place their capital in storage — earning ν — and working in the firm of a capitalist. These will be denoted **workers**. There is a frictionless market for labor, with the wage w_t clearing the market each period.

The optimal occupation choice is increasing in individual capital k. There are two threshold wealth levels separating the three occupations, or social classes. These threshold levels depend on the individual productivity realization and the wage level. The lower threshold, separating workers and independents, is denoted $k_w(z_{i,t}, w_t)$, while the threshold separating independents and capitalists is $k_c(z_{i,t}, w_t)$.

Formally, the choice between renting out labor or producing in a separate firm can be written as

$$h_{i,t} = \max\left\{ (w_t z_{i,t} + \nu k_{i,t}) \, , \left(\max_{\hat{\ell}>0} k_{i,t}^{\alpha}(z_{i,t} + \hat{\ell})^{1-\alpha} - w_t \hat{\ell} + (1-\delta) k_{i,t} \right) \right\} \tag{4}$$

where ℓ is hired labor.[1] Optimizing over the amount of capital gives the income function for independents, where the constraint $\hat{\ell} > 0$ binds, and for capitalists, where it doesn't. Inserting for optimal labor demand, and using the threshold levels defined above, the income function for an agent with productivity level $z_{i,t}$ and capital level $k_{i,t}$ income $h_{i,t}$ is

[1] Above k_c, several capitalists with similar capital owners can pool their capital together to run a firm.

$$h_{i,t} = m(k_{i,t}, z_{i,t}, w_t) = \begin{cases} w_t z_{i,t} + \nu k_{i,t} & \text{if } k \leq k_w(z_{i,t}, w_t) \\ k_{i,t}^{\alpha} z_{i,t}^{1-\alpha} + (1-\delta) k_{i,t} & \text{if } k_w(z_{i,t}, w_t) < k < k_c(z_{i,t}, \\ (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha w_t^{-\frac{1-\alpha}{\alpha}} k_{i,t} + w_t z_{i,t} + (1-\delta) k_{i,t} & \text{if } k > k_c(z_{i,t}, w_t) \end{cases} \quad (5)$$

The three income functions, for a given wage and individual productivity level, are drawn in Figure 1.

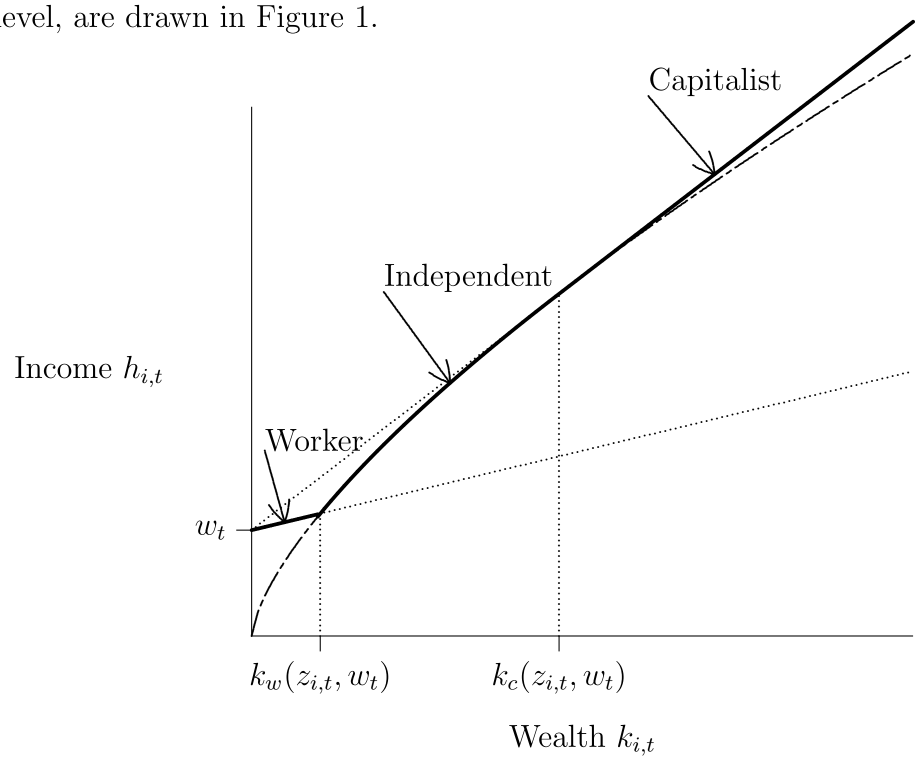

Figure 1: Income as a function of wealth (for given productivity)

Lemma 1 *With indivisible labor and missing capital markets, the income function is locally convex; over some range, there is increasing returns to saving.*

This is evident from the income function. For a given wage, agents increasing their wealth will see the return to saving increase sharply when they pass k_w, and then slowly decrease until the return is linear above k_c.

2.2 Wage determination

The price of the consumption good is set to 1; the only other market that clears is the labor market. The population mass in each group, defined by occupation and individual productivity, is denoted $L_t^{q,z}$; q indexes occupation as one of $\{W, I, C\}$, worker, independent or capitalist. Similarly, the total capital held by each group is denoted $K_t^{q,z}$. The demand for labor comes from the capitalists, who employ labor to go with their own capital and labor. The supply of labor is provided by the low-wealth workers. The independents do not participate directly in the labor market, though the wage does influence their decisions as it affects the threshold levels between the occupations. By summing labor demand and labor supply, and setting these equal to each other, the wage in a given period is found to be

$$w_t = (1-\alpha)\left(\frac{\sum_{e\in Z}\left[K_t^{C,e}\right]}{\sum_{e\in Z}\left[z^e\left(L_t^{W,e}+L_t^{C,e}\right)\right]}\right)^{\alpha} \tag{6}$$

This completes the description of the short-term equilibrium: for a given wealth-productivity distribution $\Phi(k_t, z_t)$, agents supply and demand labor, leading to the equilibrium wage w_t. With the right hand side of the budget constraint (3) determined, we now turn to the decision of capital accumulation.

2.3 Saving decisions

When deciding how much to save, agents maximize future utility (1) subject to (3) and (5). Less saving gives utility of consumption now; more saving gives utility of consumption in the future, and these concerns are traded off. Define the savings function $k_{i,t+1} = f(k_{i,t}, z_{i,t}, w_t)$ and consider the behavior for fixed z and w. From the concavity of the utility function and the increasing income function, we know that saving is increasing in wealth; rich people will save more than poor people, so $f(k)$ is an increasing function. But what is the curvature of this function?

We can separate the determinants of saving into two motivations. First, there is saving motivated by consumption smoothing; the saving that would have taken place in a deterministic environment. Second, there is precautionary saving due to the possibility of future income shortfalls.

The saving (as a share of income) that is induced by consumption smoothing is increasing in k over at least one interval; independents will save more than workers. Workers have the storage return ν, which is lower than the return of capitalists ($(1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha z_{i,t}^{\frac{1-\alpha}{\alpha}} w_t^{-\frac{1-\alpha}{\alpha}} + 1 - \delta$) and also that of independents. From the deterministic Euler equation

$$\beta \frac{\partial m(k_{i,t}, z_{i,t}, w_t)}{\partial k_{i,t}} \left(\frac{c_t}{c_{t+1}} \right)^{\eta} = 1 \text{ if } k_{i,t} > 0 \tag{7}$$

it is evident that agents with higher level of $\frac{\partial m}{\partial k}$ will choose lower $\frac{c_t}{c_{t+1}}$ ratios: save more. This means that absent risk, there is at least one convexity in $f(k)$.[2]

The saving induced by precautionary motives is decreasing in wealth, at least for poor agents. Poor agents are closer to the point where consumption cannot be smoothed, leading to higher precautionary saving. Moreover, because workers depend more heavily on labor income than independents or capitalists, there exists an additional motive for poor workers to save more.[3]

It follows that the shape of the savings function $k_{t+1} = f(k_t)$ — the degree and magnitude of local convexities — depends on the parameter values; in particular, on the level of risk in the economy. With low risk, the convexities from the consumption smoothing motives will to dominate, while with high risk, concavities from the precautionary motives become more important. To examine this in more detail, it is necessary to set values for the parameters of the model and examine the optmial behavior numerically.

[2]The result is explained in more detail in Modalsli (2011) and the mechanisms are similar to those in Galor & Zeira (1993) and Mookherjee & Ray (2003).

[3]That increasing savings are higher for the poor in general cases is proven analytically by Carroll & Kimball (1996) and Wang (2003).

2.4 Parameter values

The realization space of individual productivity Z has two realizations, $1-\epsilon$ and $1+\epsilon$, and the transition matrix is of the form

$$\Pi = \begin{bmatrix} \pi_{LL} & \pi_{LH} \\ \pi_{HL} & \pi_{HH} \end{bmatrix} = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix} \tag{8}$$

meaning that half the population is in each state at a given point in time.

Define as Θ a set of values for all the parameters of the model. As a reference point, the model is parameterized to values commonly used in the macroeconomic literature. The values used are shown in Table 1. In the later analysis, several values for the the parameter ϵ will be considered; for this reason, it is convenient to group the values for all the other parameters into θ, and we have $\Theta = \{\theta, \epsilon\}$ where the values in Table 1 are denoted $\Theta_0 = \{\theta_0, 0.2\}$.

Production		
Capital share α	1/3	Standard
Depreciation rate δ	0.06	Caselli (2005)
Storage return ν	0.94	$1-\delta$
Uncertainty		
Risk spread ϵ	0.2	Krueger *et al.* (2010)
Transition probabilities π	1/2	See text
Preferences		
Discount rate β	0.95	Target: Capital-output ratio of 3 (Caselli, 2005)
Elasticity parameter η	2	Standard

Table 1: Parameter values for the parameter set Θ_0

Setting the storage return ν to $1-\delta$ implies that capital deteriorates equally from storage and use, and that there are no other costs of storage such as uncertain property rights or risk of theft. As the model, with its missing capital markets, would best fit developing countries, the risk parameter is taken from the "least developed" of the six countries reported in Krueger *et al.* (2010, Table 7C), namely Russia. The implications of different levels of idiosyncratic risk will be examined in more detail later. To simplify the later welfare analysis, income shocks are assumed to be IID. The discount rate β is calibrated to a capital-output ratio of 3. This is one of the higher values reported in the cross-country study by Caselli (2005). As capital here could also include some forms of human capital or other unmeasured production resources, it seems right to pick one of the higher values.[4]

2.5 Steady state

The economy is in **steady state** when the wealth-productivity distribution Φ is stationary. An individual's place in next period's distribution will always be uncertain and depend on the individual productivity shock. However, in steady state, the aggregate distribution is still constant. From Equation (6), the wage does not change once a steady state is reached.

Local convexities in saving can lead the population to be split up into several groups in steady state; when the rich save more than the poor, wealth differences will increase over time. However, as shown above, the extent of these convexities depend on the level of precautionary saving. If the risk level is high, the convexities induced by the income function matter less. Moreover, there is an additional effect: higher risk gives the possibility of higher "income jumps" that overcome the convexities. This gives the preliminary result

Lemma 2 *There exists a threshold level of idiosyncratic risk $\tilde{\epsilon}$. For $\epsilon < \tilde{\epsilon}$, the steady state has several distinct population groups with no mobility between groups. For $\epsilon > \tilde{\epsilon}$, there is full mobility across the entire wealth distribution in steady state.*

The threshold level depends on the values of other parameters (θ) and will be solved for numerically. With high risk, there is one ergodic distribution in steady state where agents will, given long enough time, spend some time at all populated wealth-productivity levels. With low risk, there can be several distinct populations. This means that the very properties of the steady state

[4]Due to the time needed for computation, β is calibrated for the case of no uncertainty. From Modalsli (2011): $\beta = \left(\alpha \cdot X^{-1} + (1-\delta)\right)^{-1}$, where X is the desired K-Y ratio (for example 3). With $\alpha = 1/3$ and $\delta = .06$, this gives $\beta = .9514 \approx .95$.

depend on parameters; the level of risk affects whether there is a unique steady state and whether the steady state is history dependent.

2.6 Steady state with two groups

For the parameters in Table 1 (Θ_0) we have two distinct population groups in steady state. The size of the poor group will be denoted μ; there are $1-\mu$ people in the rich group. Increasing returns to saving makes large-scale accumulation too costly for the poor, regardless of the realization of the productivity shock. Similarly, becoming a poor worker is so undesirable for the rich group that they never dis-save below a certain point. While the stochastic income process gives mobility *within* each of the two groups, there is no mobility *between* the groups.

The numerical solution method for the steady state is described in the Appendix. A search for stable distributions (steady states) shows that a range of values for μ are supported.

Lemma 3 *When the level of idiosyncratic risk is low, $\epsilon < \tilde{\epsilon}$, there is a continuum of feasible steady states. The steady states differ in wealth distributions and wage levels; distributions with larger shares of poor agents correspond to a lower steady state wage.*

Figure 2 shows the set of steady states for Θ_0. The steady states differ in the steady state wage level w and the share of poor agents μ, as well as in within-group distributions.[5] There is a continuum of steady states; all are locally stable. Consider an exogenous, small wealth gain by one of the agents in the rich group. This will increase aggregate productive capital A_C, increasing the wage. A higher wage means lower return to capital for the rich, reducing saving among the capitalists until aggregate capital is back at A_C. Transfers between groups, however, would cause the economy to move from one steady state to the other.

[5]The exact bounds of the ranges of steady states (the end points of the line in Figure 2) depend on the accuracy of the numerical solution; potentially the range could be longer than that shown. Extending the line would not affect the following analysis.

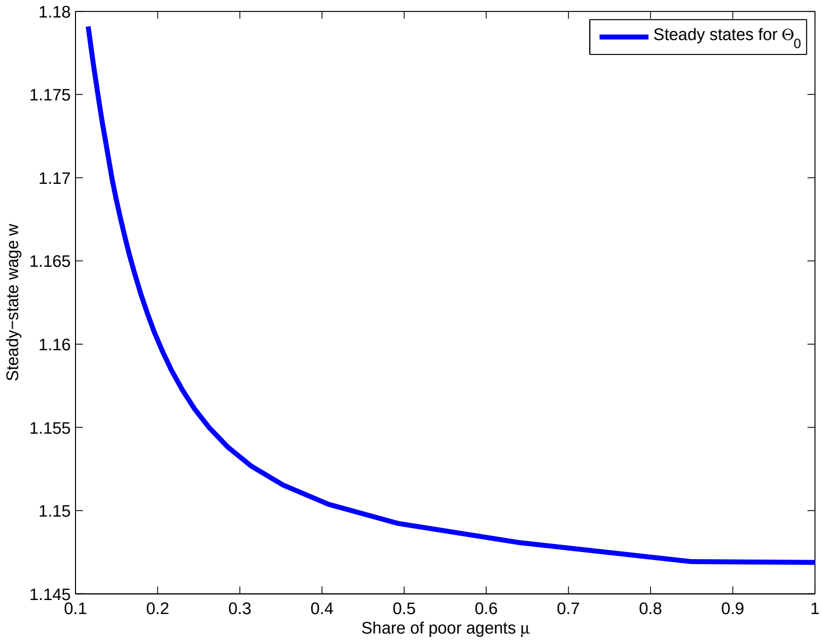

Figure 2: Range of steady states for the parameters in Table 1 (Θ_0)

As there are no forces driving the distribution away from the steady state, the wealth distribution and wage is history dependent.

Proposition 1 *When the level of idiosyncratic risk is sufficiently low, $\epsilon < \tilde{\epsilon}$, there is strong history dependence and the wealth distribution and wage in steady state depends on previous wealth distributions.*

This follows from the stability discussed above; for most values of μ, there is an associated wage and the steady state is stable. Hence, we can refer to steady states with several groups as *history dependent steady states.*

Examples of the wealth distributions for three of the steady states are shown in Figure 3. The two peaks of the wealth distribution are evident, as is the empty region that follows from no mobility.

Low values of μ are associated with high wages. When there are many people in the rich group, the aggregate savings of this groups will be higher. General equilibrium effects limit the saving that is due to consumption smoothing — the wage increases when the aggregate capital held by capitalists increases. However, there is still precautionary savings, and when there are

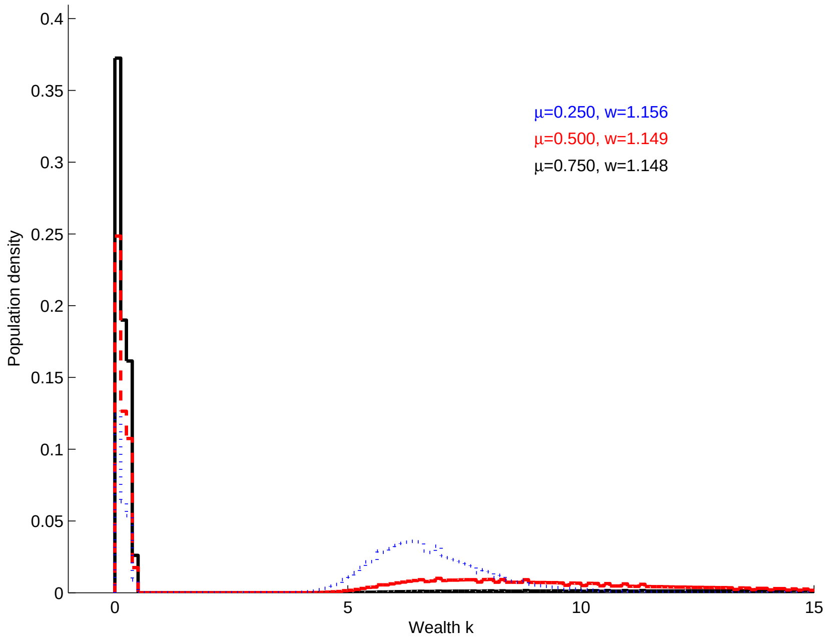

Figure 3: Three steady-state wealth distributions ($\Theta = \Theta_0$)

many capitalists, each of them is, on average, poorer. Moreover, when the average person in the rich group is poorer, more of the rich will have too little capital to actually employ people; they are independents, and their labor is under-utilized, giving higher demand for the labor that is offered in the market.

Conversely, when there are many poor, the rich are richer and total precautionary savings are lower. Even though there is large supply of workers, and even though the wages are low, the convexities in income are too large to make it beneficial for this group to save enough to utilize their capital productively.

2.7 Steady state with mobility

Now consider a higher level of idiosyncratic risk, $\epsilon = 0.4$, and define the "high-risk" parameter set $\Theta_1 = \{\theta_0, 0.4\}$. In this case, the combined effects of precautionary savings and income jumps from shocks ensure full mobility in steady state. The wealth distribution is shown in Figure 4. While there are still convexities in the savings functions, these are not large enough to cause a full population split. Poor agents will, given a long enough series of positive shocks, become rich.

Proposition 2 *When the level of idiosyncratic risk is sufficiently high, $\epsilon > \tilde{\epsilon}$, there is no history dependence and one unique steady state wealth distribution and wage.*

With high risk, the economy is similar to that in models without missing capital markets, like the canonical Aiyagari (1994) model, and has a unique steady state wealth distribution. For any other distribution of wealth, with a corresponding different wage, general equilibrium mechanisms will bring the economy back to steady state. The distribution of the steady state with $\Theta = \{\theta_0, \epsilon = 0.4\}$ is shown in Figure 4. The wealth distributions for the "poor" and "rich" are now connected. This steady state has the unique wage $w = 1.210$, with around 21% of the population being workers. Transferring a group of people from the "rich" to the "poor" would lead to the returns to saving going up, everyone saving more, and the distribution returning to the same steady state.

2.8 Mobility and income volatility

The separation of the two groups is held up by convexities in the savings function. Figure 5 plots the decision rules for the parameter set Θ_0 (giving history dependence) with half the population being workers ($\mu = 0.5$). Both the decision rule for high-productivity agents and that for low-productivity agents are convex for low levels of wealth, and both cross the 45-degree line ($k_t = k_{t+1}$) twice. This means that for the low-productivity decision rule, agents above the threshold level (around 2.2) will never become poorer than this. Similarly, agents with wealth below the threshold level where the high-productivity decision rule crosses the 45-degree line (around 1.9) will never save above this level. The area between the threshold levels will empty over time, as agents either move up into the "rich" group or down into the "poor" group; this is why two distinct groups emerge.

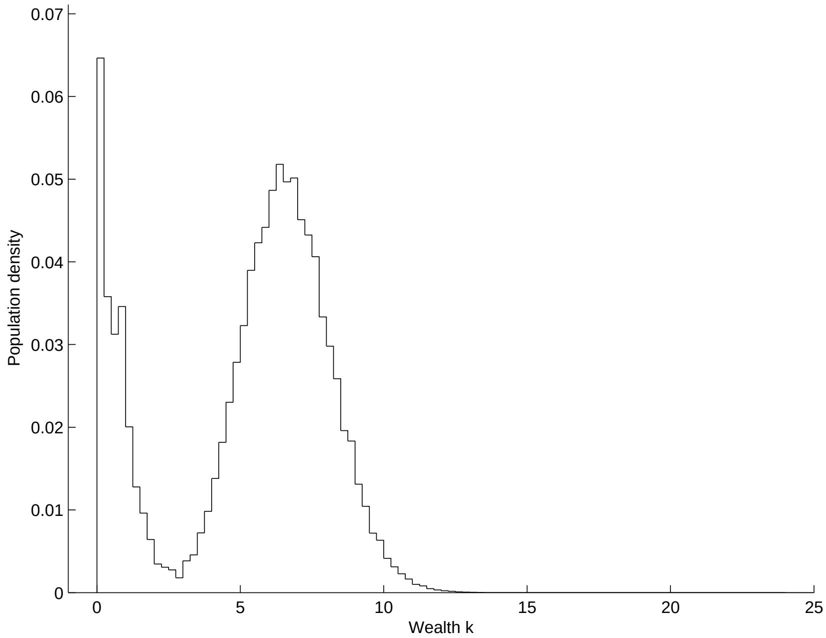

Figure 4: Steady state wealth distribution for $\Theta = \{\Theta_0, 0.4\}$

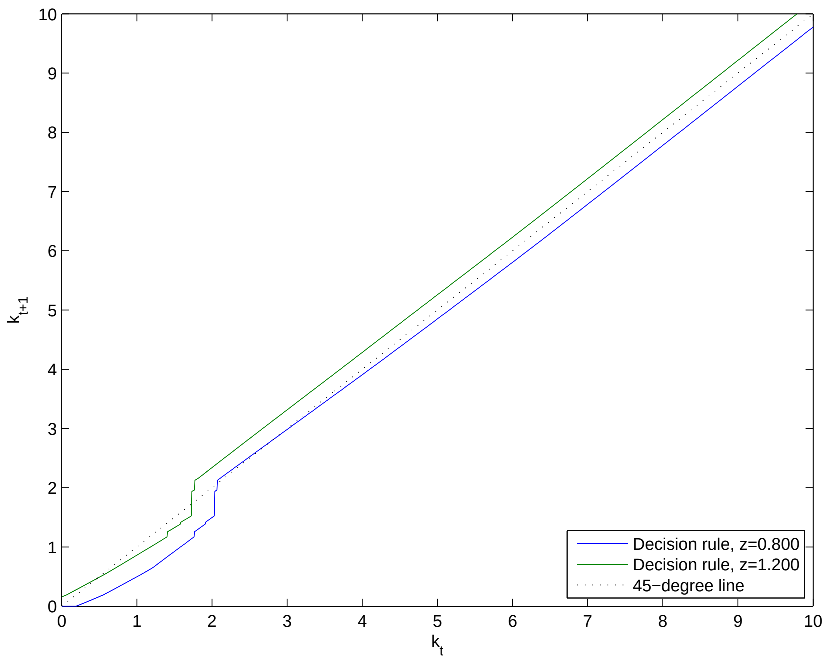

Figure 5: Decision rule in steady state (zoomed in) ($\Theta = \Theta_0, \mu = 0.5, w = 1.492$)

A *lower* wage decreases the threshold to move from being a low-return worker to a high-return independent; at some low wage level, the savings function will be above the 45-degree line for all high-productivity agents. With full mobility and a very low wage, all agents will save, capital accumulation will be very high, leading to high labor demand implying a high wage. Hence, in this case, a wage low enough to induce full mobility cannot be a feature of the steady state. Similarly, if the wage becomes very *high*, being a worker will be attractive enough that all capitalists would prefer this state and dis-save, and not support this high wage.

Figure 6 shows the steady state wages for values of the idiosyncratic risk parameter ϵ between 0.1 and 0.4. From the calculations, it is clear that the critical level $\tilde{\epsilon}$ is between 0.25 and 0.30. Below $\tilde{\epsilon}$, there is a range of feasible wages, as illustrated by three different between-group distributions. When the working class is large, risk has small effects on the wage level; see the line for $\mu = 0.75$, which is nearly flat. Because there are few capitalists, each capitalist is rich, and precautionary savings constitute a small part of the capitalists' wealth. Conversely, when the working class is small, capitalists are on average poorer and respond more, as a group, to changes in risk.[6]

Above $\tilde{\epsilon}$, we see that higher risk is universally associated with higher wages. While the low-risk economies support a range of between-group distribution, the high-risk economies do not. For this reason, it is possible for wage to go *down* when risk increases from a level below $\tilde{\epsilon}$ to a level above $\tilde{\epsilon}$; for example, the wage for $\mu = 0.25, \epsilon = 0.25$ is below the wage for $\epsilon = 0.30$.

It is also evident from Figure 6 that the wage increases quite fast with risk above $\tilde{\epsilon}$. For comparison, consider a model with the same parameters but without capital market frictions; this is the canonical Aiyagari model,

[6] Two features of the steady states are not evident from Figure 6. First, the upper bound on feasible wages is decreasing in ϵ below $\tilde{\epsilon}$. This is because the lower bound on μ — the smallest supported working class — is also decreasing in ϵ; with higher risk, distributions with very small working classes are not stable. Because these types of economies, with very few workers, are not economically very relevant, they are not discussed further here.

Second, for some levels of ϵ, "autarky" distributions are also supported; distributions so compressed that the richest agent does not find it profitable to employ the poorest wages. In this case, the wage is not defined.

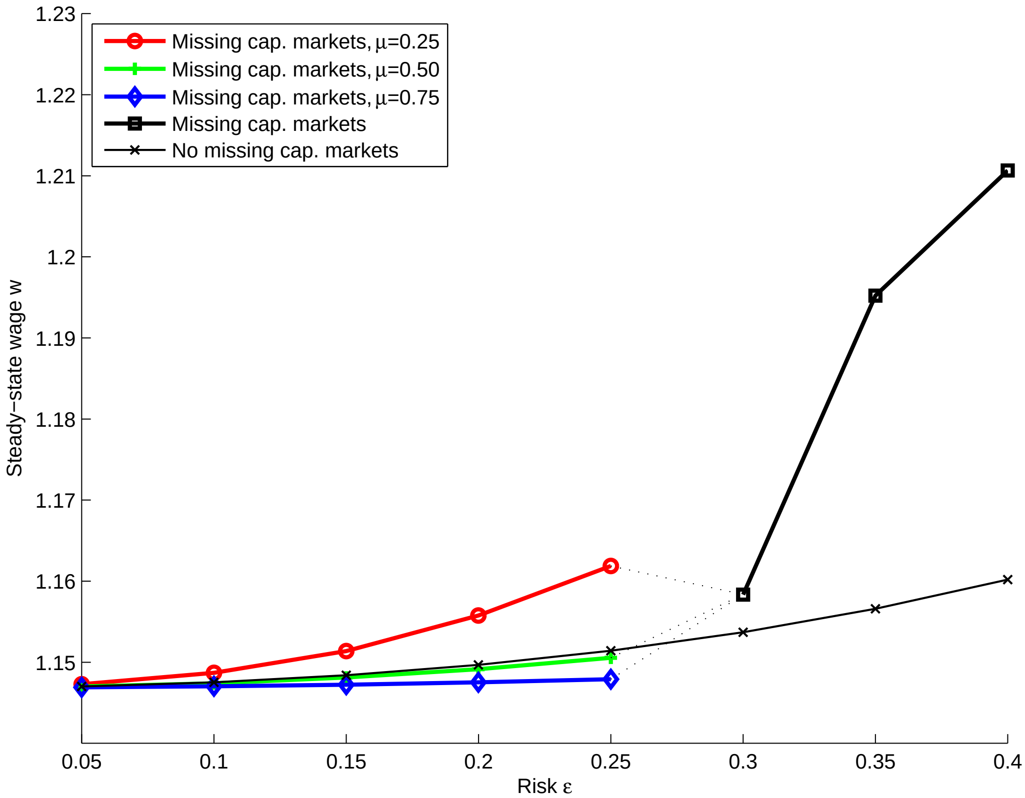

Figure 6: The steady-state wage for different levels of idiosyncratic risk ϵ

where the wage-risk relationship is much less steep.[7] The frictions of the model makes the wage rise fast for several reasons. First, because the return to saving drops fast when below a certain wealth level, there are more reasons to buffer-stock save; not only do agents want to avoid the zero-wealth state, the richer agents also want to avoid the poor state. Moreover, with higher risk, because of the increased income volatility, more of the richer agents are at a given time independents rather than capitalists, having just received a bad shock. This drives down labor demand, as a smaller share of capital is used to hire workers.

Proposition 3 *The steady-state wage is higher when risk is higher, except for some comparisons across $\tilde{\epsilon}$. Moreover, above $\tilde{\epsilon}$, the wage increases faster with risk than in the model with no missing capital markets.*

Having established how different steady states compare in terms of income opportunitites, we now turn to the question of welfare. How does the welfare of the different groups of income recipients differ across steady states?

3 Welfare analysis: Do any groups gain from more risk?

If the level of idiosyncratic risk in the economy is high, so that $\epsilon > \tilde{\epsilon}$, there will be full mobility; the poor (or their descendants) will be rich some time in the future. With the knowledge that higher risk gives more mobility, it is natural to ask the question: are any income groups *better off* when risk is high than when risk is low?

3.1 Gains and losses from changes in risk

When comparing the welfare for different parts of the wealth distribution, we can distinguish several effects that differ across agents. Consider an increase in idiosyncratic risk ϵ.

[7] The detailed setup is given in the Appendix, section A.1. This setup is also used in the lower right panels of Figures 8 and 9.

First, the increase in idiosyncratic risk leads to higher demand for self-insurance: agents save more. Saving means sacrificing consumption, and hence the net utility effect is negative. Moreover, self-insurance is not perfect. After a very long strike of bad luck, consumption will be low. This consumption volatility, or risk of consumption volatility, also affects the welfare of the population. The effect is strongest for the poor, who do not have as large buffer stocks and hence have to take current productivity more into account when choosing consumption levels.

Second, there are general equilibrium effects: the wage will change. Increased buffer-stock saving among capitalists increases the capital stock, thereby increasing the marginal product of labor. In isolation, this increases the welfare of workers. The welfare of capitalists goes down, as the return to capital is now lower.[8] The strength of these effects depends on the composition of the capitalists class. If there are few capitalists, each will have high wealth, and the need for extra precautionary savings is modest. If there are many capitalists, each will hold lower levels of wealth, and hence the increase in the capital stock will be higher.

Third, there are possible effects of changing mobility. If we compare the welfare distribution for levels of risk above and below the threshold $\tilde{\epsilon}$, the high-risk steady state will have mobility between groups and a unique distribution. This can be a positive welfare effect for some people in the poor group, as a strike of good luck now will lead to a permanent improvement in class status, as opposed to the "glass ceiling" with low risk and low mobility. There is a corresponding effect for those in the rich group, who now risk falling down into the working class if receiving a long series of bad shocks. In addition, mobility leads to a re-shaping of the relative sizes of the social classes as the history-dependent steady state of the low-risk economy is replaced by the unique steady state of the high-risk economy. If the number of capitalists is higher in the high-risk steady state, the average wealth of capitalists will be lower and hence the precautionary savings of capitalists will be higher, increasing the general equilibrium effect mentioned above. If,

[8]Risk and the functional distribution of income (wages vs. interest rates) is a central topic of the welfare analysis of models with precautionary risks in Davila *et al.* (2005).

on the other hand, there is a large group of capitalists in the low-risk steady state, the increasing risk could have an opposite effect: a slight decrease in the capital stock.

Figure 7 shows four sample consumption paths for four different steady states with different risk levels. The lines represents agents starting at the 20th, 40th, 60th and 80th wealth percentile, respectively, and then facing random shocks to productivity. In the two upper panels, where $\epsilon < \tilde{\epsilon}$, there is no mobility between groups. The agents represented by the lower two lines will never cross into the richer half of the population. While the two upper lines will cross, given enough time, the level of risk is sufficiently low that it does not happen during the first 100 periods. Note how consumption is much more volatile for the poor; the rich, with high capital stock, have no trouble smoothing consumption when risk is as low as $\epsilon = 0.10$. In the second panel, there is slightly more consumption variation for both rich and poor, but we see that the poor spend more resources to avoid the "rock bottom" consumption of around 0.9 consumption units; the minimum consumption level does not occur as frequently as in the first panel.

In the lower two panels risk is high enough for there to be mobility between groups. In the panel for $\epsilon = 0.30$, the agent at the fourtieth percentile makes a transition from the poor to the rich group. The agent that starts at the twentieth percentile, however, never has enough luck to escape being a worker, and the high risk means very high consumption volatility; this is further exaggerated when $\epsilon = 0.40$, though the borders between classes are even further blurred in that case.

3.2 Methodology

We can ask two distinct welfare questions. First, are any income quantiles better off in high-risk steady states than in low-risk steady states? Call the answer to this question "comparative statics": compare the welfare distribution in a steady state with one risk parameter to the welfare distribution in a steady state with a different risk parameter.

Second, if risk changes in a given society, do any groups gain? To answer

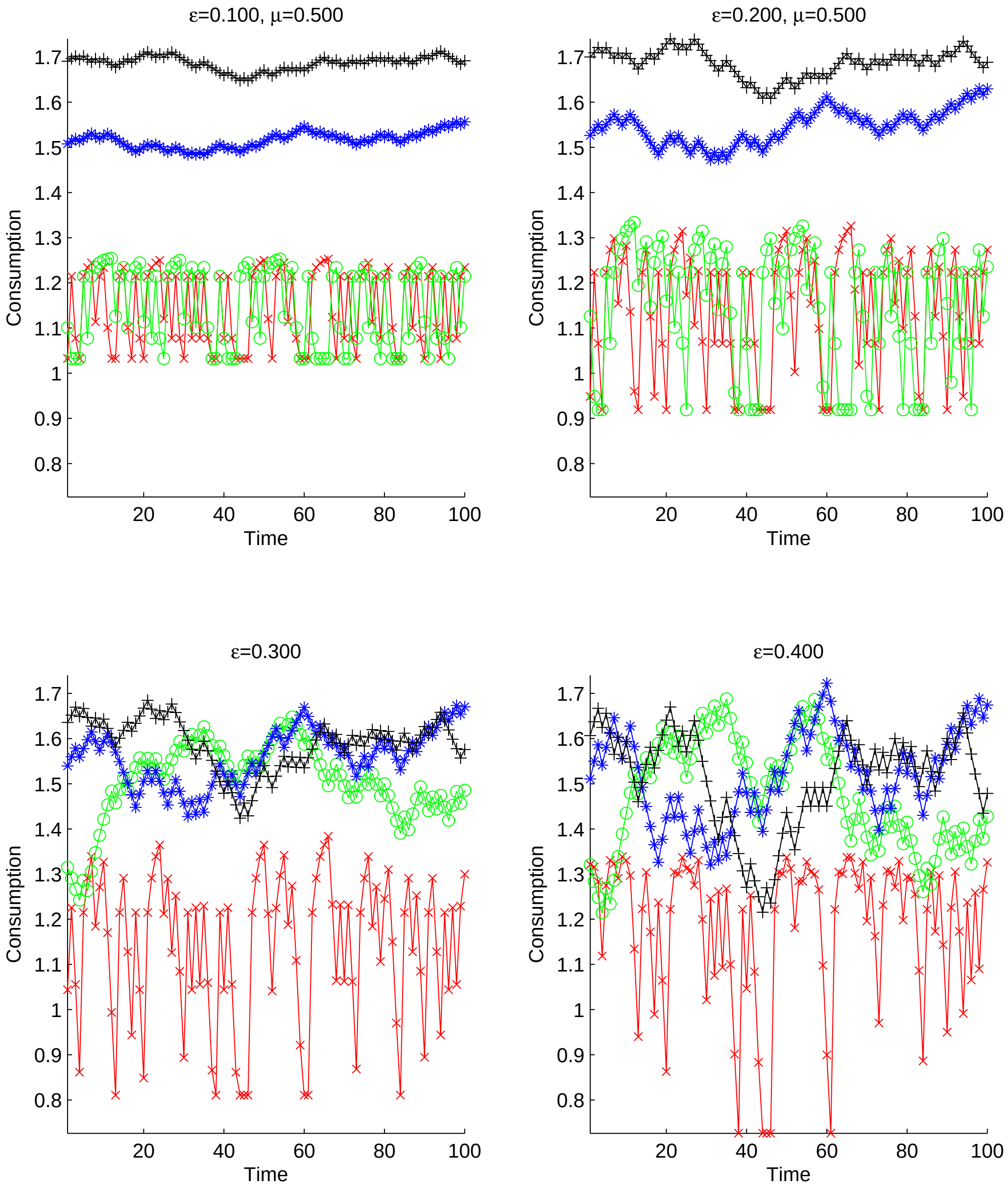

Figure 7: Sample consumption paths in four steady states with different risk parameters

this, use “dynamic welfare analysis”; calculate the transition between two steady states and see if welfare for some groups improve with the transition.

The comparison across societies — comparative statics — compares the welfare distributions in two steady states directly. The dynamic welfare analysis compares the welfare distribution at an initial steady state to the utility distribution *at the start of the transition* to a different steady state. In economic terms, this is equivalent to sudden and unexpected change of parameters from $\{\theta_0, \epsilon_0\}$ to $\{\theta_0, \epsilon_1\}$, and answering the question: do anyone gain from this shock?

Both approaches involve comparing two steady states. A steady state S is characterized by a parameter set $\Theta = \{\theta, \epsilon\}$ and, if there is history dependence, the share of people in the poor group μ. The parameter set θ_0, as defined in Table 1, will be used for all comparisons; only the risk parameter ϵ will differ. For the history dependent risk level three values of μ are used.

Utility will be compared using the value function: the utility function given optimal consumption and saving choices. For a given steady state with wage w, we can formulate the value function recursively from the utility function (2) as

$$V(k, z; w) = \max_{k'} u\left[m(k, z, w) - k'\right] + \beta E\left[V(a', z', w)|z\right] \tag{9}$$

Define the “welfare distribution” as the distribution of utility, defined by (9), across the population at a given point in time. This distribution is two-dimensional. When idiosyncratic risk is increased, the increasing dispersion in the z-dimension follows mechanically; the high-productivity and low-productivity agents move away from each other in utility terms. To focus the analysis on the dynamic general equilibrium effects, rather than the first-order effect of increased productivity dispersion, we only examine the utility distribution over capital. Define the average welfare for an agent with capital k as

$$\bar{V}(k) \equiv \frac{1}{2}\left(V(k, 1-\epsilon) + V(k, 1+\epsilon)\right) \tag{10}$$

While the productivity dispersion itself is abstracted from, all effects from the dispersion — including direct effects from more volatile consumption patterns — are taken into account.[9] $\bar{V}(k)$ is denominated in utils; to get a better impression of the numbers, the results will be presented in permanent consumption equivalents, where $\bar{c}(k)$ is defined implicitly from (1) as $V(k) = \sum_{t=0}^{\infty} \beta^t \frac{\bar{c}^{1-\eta}}{1-\eta}$. Because the per-period utility function is concave, a given consumption equivalent translates into more utils for the poor than the rich.

3.3 Comparative statics: Some groups are better off with risk

Figure 8 compares the welfare distributions for four different risk levels. All four plots show cumulative density functions; moving upward along the vertical axis, we examine the average welfare of agents at a given percentile of the wealth distribution, as shown in consumption equivalents on the horizontal axis.

The first panel has a large working class in the low-risk steady states; $\mu = 0.75$. We see that for the two lowest levels of idiosyncratic risk, $\epsilon = 0.10$ and $\epsilon = 0.20$, there is a jump in the welfare distribution at the seventy-fifth percentile; those in the poor population groups have much lower utility than those in the rich population group. Among the poor group, when comparing welfare for $\epsilon = 0.10$ and $\epsilon = 0.20$, higher risk gives lower welfare for all; the line for the low-risk steady state is to the left of the line for the higher-risk steady state. Even though the wage is higher when risk is higher, the volatility effect dominates.

[9] The averaging is straightforward when $\pi = 1/2$, because the wealth distributions for the high-productive agents and the low-productive agents will be similar. When $\pi \neq 1/2$, this is not the case. As all the examples here have $\pi = 1/2$, other ways of averaging will not be discussed here.

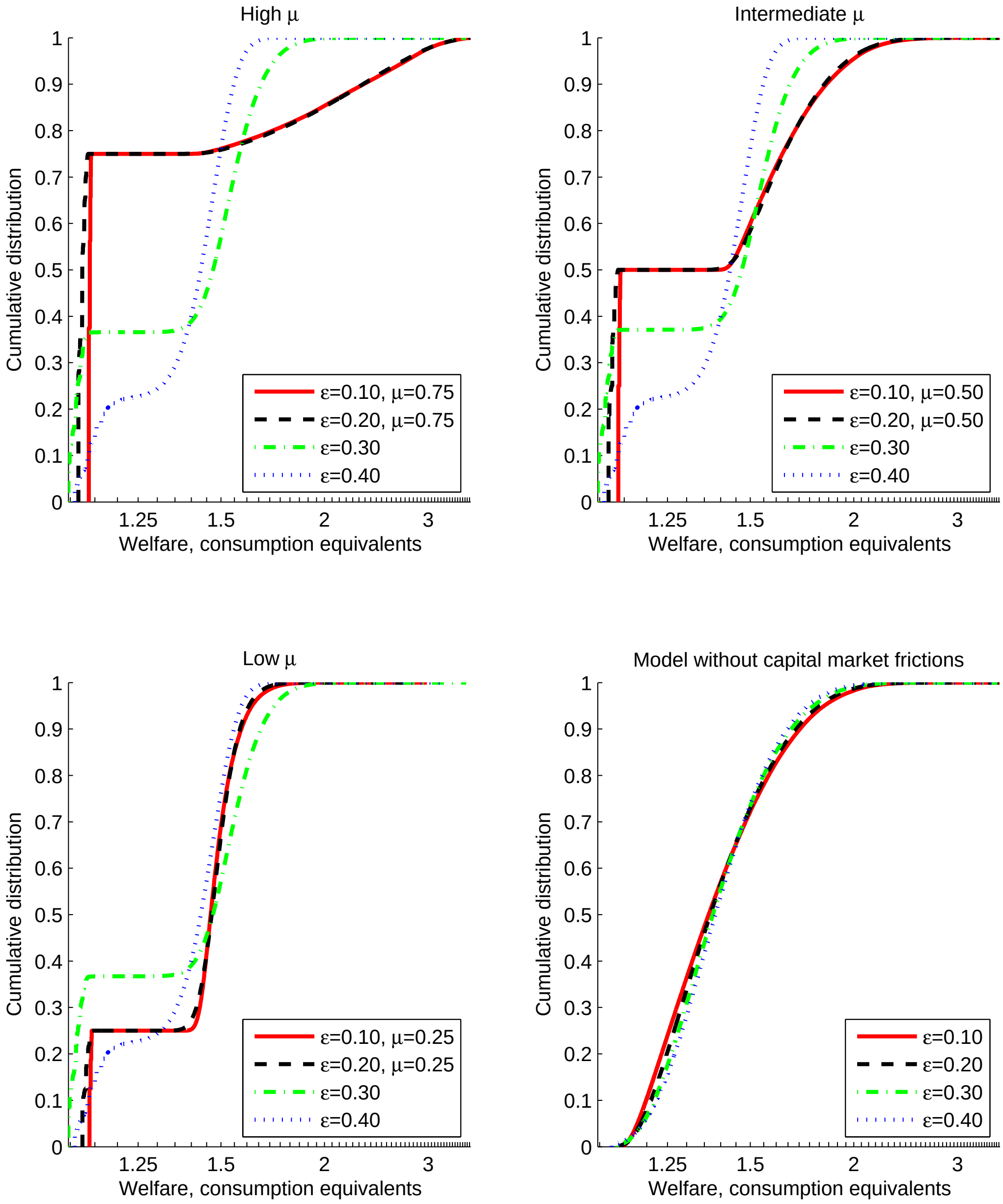

Figure 8: Comparative statics: Cumulative utility distributions under different risk parameters. Horizontal axis scale given in consumption equivalents

Staying in the upper left panel and moving to the richest twenty-five per cent of the population, the differences between the welfare distribution for the two lowest risk levels are small. For the poorest of the rich, there is a marginal advantage to be in the $\epsilon = 0.20$ steady state, while for the somewhat richer, $\epsilon = 0.10$ is better. As discussed in the previous section, the effects on the steady-state wage from changes in risk are very moderate when there is a large working class; this is the reason why the welfare distribution of the rich is roughly unchanged for changing risk below the threshold level $\tilde{\epsilon}$.

We then compare the two low-risk steady states to a steady state with full mobility; $\epsilon = 0.30 > \tilde{\epsilon}$. Though the cumulative distribution function still shows an almost flat shape, meaning the wealth distributions are almost completely disconnected, productivity jumps can now move agents across the gap between the groups. This means that there is now one unique steady state, where slightly more than a third of the population are workers. For the most part, these workers are worse off than in the workers in the low-risk economy.; the distribution function for the lower third of the population is to the left of the distribution functions for the low-risk steady states. While the wage has gone up, by a substantial amount, increased consumption volatility makes the poor worse off.

The population between the fourtieth and seventy-fifth percentile, however, are much better off in the $\epsilon = 0.30$ economy, as these groups are independents or capitalists in the higher-risk economy, self-insurance is not very expensive and increased risk does not translate into consumption volatility to the same extent as it does for the workers. The mobility induced by $\epsilon = 0.30$ makes the history dependence of the low-risk steady states disappear. While the low-risk economies had only 25% of the population in the richer group, it is now much higher. There are more capitalists who are, on average, poorer. Nearly all income quantiles containing capitalists in the low-risk steady state are worse off in the high-risk steady state.

The dotted line shows the welfare distribution for the highest risk level, $\epsilon = 0.40$. Comparing it to the $\epsilon = 0.30$ distribution, the poorer groups are better off while the richer groups are worse off. As there is full mobility in both cases, this is mainly caused by factor price effects; the large wage difference illustrated in Figure 6 translates into higher income for the poor and lower income for the rich. For the poor, the higher wage means higher income, and this dominates the volatility effect from increased risk. For the rich, the higher wage means lower return to capital.

The second panel has low-risk steady states with a smaller working class ($\mu = 0.50$); the wage is slightly higher than in the $\mu = 0.75$ case. The welfare distribution for the two lowest risk levels are qualitatively similar to the first panel, except that the cutoff is now at the fiftieth percentile. The two $\epsilon > \tilde{\epsilon}$ distributions are equal across all the three first panels, as there is no history dependence in this case.

In this case, it is also the case that intermediate-wealth agents are better off with higher risk; while the fortieth-percentile agent is a worker in the low-risk steady states, he is an independent producer when the level of idiosyncratic risk is at 0.30 or higher. Similarly, rich capitalists are universally worse off with more risk. Not only are there more capitalists, giving a lower level of capital per person and hence lower utility per person; precautionary savings among capitalists is also higher, meaning that the return to capital is lower.

The third panel shows an economy with a small working class; only 25% of the population are workers in the low-risk steady states. In this case, the capitalists are on average not so rich, and hence have high precautionary savings. Again, when comparing the two lowest risk levels, the poor lose from risk and the rich are largely indifferent. With mobility, we here see an opposite effect from the previous two examples: because there are so few workers in the low-risk steady state, increasing ϵ to a level above $\tilde{\epsilon}$ gives *more* workers. The high-risk steady states have fewer capitalists who are, on average, richer, hence the upper parts of the wealth distribution have higher utility. The wage is lower; the capital stock used in combination with hired labor is smaller.

The $\mu = 0.25$ economy depicted in the lower left panel is, with its small working class, a somewhat special case. As long as the size of the poor group in the low-risk steady states is smaller than the number of workers in the high-risk steady states, we have the general result

Proposition 4 *Comparing welfare across the wealth distribution, in most cases, the rich and the very poor are worse off in steady states with high risk. Workers with some wealth are better off in steady states with high risk.*

The effects are qualitatively similar to an economy where capital rental is allowed. The setup for such an economy, similar to Aiyagari (1994) and parameterized according to Table 1, is given in the Appendix, and the welfare comparisons are shown in the lower right panel of Figure 8. The results are analogous to the three panels with missing capital markets; in this case, the poorest 2% and the richest 30% are better off when risk is lower. In the model without frictions, however, the magnitude of the effects is much lower.

The steady state comparisons give interesting results when comparing societies with different risk environments, and how inequalities differ across societies. In many cases, however, it is more interesting to study changes in risk *within* a society. This is the topic of the next section.

3.4 Welfare and transition: Risk is bad

To take into account the welfare effects of transition from one steady state to the other, we first take one set of parameters as a starting point; here, it will be Θ_0 with the risk level $\epsilon = 0.2$. Then, we consider a sudden, unexpected introduction of a different level of idiosyncratic risk. When the environment changes, agents will adjust. While some of the adjustment is instantaneous, there will be a long transitionary period before the new steady state distribution is reached. By comparing the welfare distribution in the initial steady state to the welfare distribution *at the outset of the transition* — before the wealth distribution have had time to change — we can deduce the welfare effects from a change in idiosyncratic risk.

Figure 9 shows the results of such an exercise. As is evident from the figures, when the transition is taken into account, at no parts of the wealth distribution is there a welfare gain. The difference from the steady state comparisons lie in the transitionary period. In the higher-risk steady states, all agents hold more wealth for precautionary reasons. This also applies to the richer workers, who are indeed better off in the final steady state than

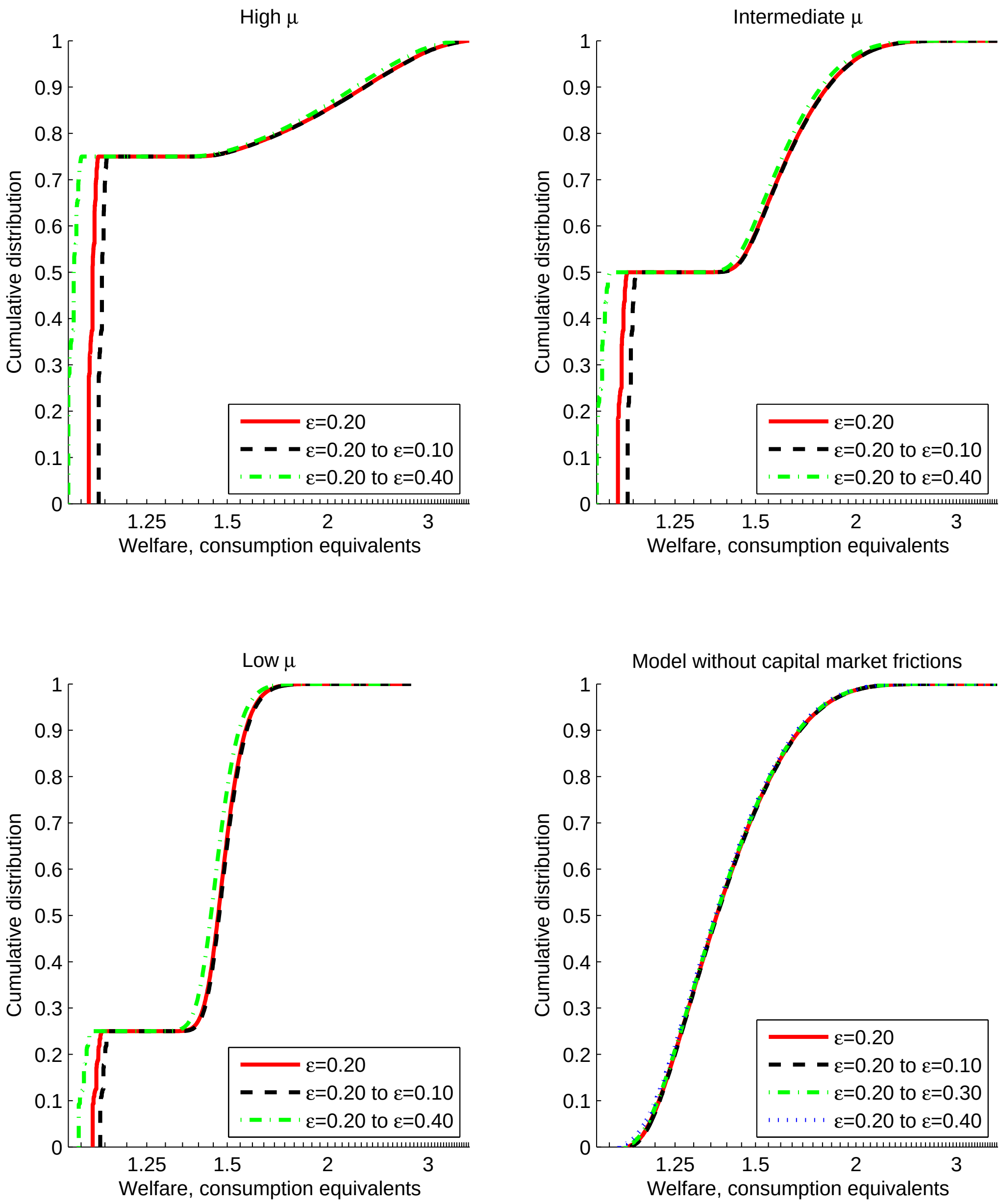

Figure 9: Utility gains and losses from changing risk, taking the transition into account

they were initially. However, while moving to the new steady state, all agents accumulate capital, meaning lower consumption during the transition. When this cost is accounted for, as it is in the calculations underlying Figure 9, this group also loses from the increased risk. Similarly, there is a welfare gain for all when the level of risk decreases, because wealth levels can also be decreased.

Proposition 5 *When moving from a steady state with low risk to a steady state with high risk, all agents lose from the change in the environment.*

A similar phenomenon is discussed in Mookherjee & Ray (2003), who discuss convex investment returns in a deterministic framework. In their model, also with perfect foresight, inequality develops over time even if everyone are equal in the initial state. In their case, the rich are rich not because their ancestors were lucky, but because all ancestors had a trade-off between consuming themselves and saving for their children — and the ancestors of the present-day rich chose low consumption. In the model presented in this paper, the middle class in the high-risk steady state are not poor because they, in an earlier period, chose to forgo consumption during the transition to the new steady state. *Ex ante*, the transition makes them worse off, but after the fact, they are better off than they were before the environment changed.

Even though there is a net loss for everybody from increasing the risk, the welfare effects are not evenly distributed. The workers take a large hit; while the wage goes up, increasing the buffer stock to better shield oneself is a costly affair. If the increased risk leads to full mobility ($\epsilon_1 > \tilde{\epsilon}$), this effect is even stronger, as the transitionary period entails a very large build-up of wealth as the richer workers transform themselves into capitalists. For the capitalists, on the other hand, the welfare effects from increasing risk are not very large.

4 Discussion

4.1 Polarization and risk

The model shows an inverse relationship between risk and polarization; higher idiosyncratic income risk gives less polarization. We see that taking the mechanism to the extreme verifies the result: no risk means no mobility, and infinite risk means that tomorrow is completely random. The model shows important mechanisms likely to be at work in developing economies. Periods of high risk can upset persistent inequalities, even if the economy returns to a low-risk situation. In economies with capital market frictions, inequality is likely to be history dependent unless the income process is very volatile.

In societies with no mobility across groups, cultural and economic characteristics could, over time, become more correlated with class identity. Fixing the membership of each social class would give time for divergence in any number of cultural traits. Moreover, the *aspirations* to get rich, not covered in the model of this paper, could further influence savings motivations; in steady states with no mobility across groups, workers only save to avoid desperate poverty, while in the higher-risk steady states, you also save to potentially become a rich entrepreneur.

4.2 The demand for social insurance

In polarized economies with a large working class, the rich are worse off in steady states with higher idiosyncratic risk. With high risk, the poor save more and very unequal distributions are not supported (for an example of the welfare effects, see the upper left panel of Figure 8). Could this explain some of the motivations for the introduction of social insurance systems in the nineteenth-century Western world?

The development of social security is frequently seen as a response to implicit or explicit threats of revolt by the poor; se Lindert (2004) for a general survey and Acemoglu & Robinson (2000) for a specific model. As emphasized by Moene & Wallerstein (2001), government social spending is not only a matter of transfers, it is also about *social insurance*; in this case, reducing labor market risk. Lindert describes "the eternal search for the worthy poor"; relief should be given to the unlucky and not the lazy. In the framework of the model presented here, we can see social insurance and poor relief as policies decreasing risk.

In a situation where risk was increasing, for example by accelerating technical progress, the rich would have an incentive to combat this increase by introducing social insurance. If the increase was avoided, the aggregate capital stock would not have to go up, the wage would stay lower, and the average wealth among the rich would stay high.[10]

Moreover, the model helps explain middle-class opposition to social reform. Acemoglu & Robinson (2000, p. 1190) discuss how the middle classes in nineteenth-century Britain and other countries opposed franchise extension for the poor. Lindert (2004, chapter 4) argues that the gradual extension of voting rights explains a decrease in poor relief and then an increase. The landed gentry who hired large amounts of labor favored insurance; the middle classes, who did not hire labor, were not at a great risk of poverty, but had to pay taxes, opposed it, while the poor benefited directly. Hence, when the franchise was extended to the middle classes in the mid-nineteenth century, support for social insurance actually went down. In the model of this paper, the better-off in steady states with higher risk are those with intermediate wealth, because the wages are higher. The very poor lose from more risk, because of increased income volatility; the very rich also lose, because of the increased labor cost.

4.3 Concluding remarks

This paper has shown that there is a trade-off between risk and polarization; given missing capital markets, for otherwise similar environments, low-risk societies can be polarized while high-risk societies are not. A large part of the population is better off in the high-risk steady state than the low-risk steady state; the loss from high risk is carried by the rich and the very poor.

[10] Another mechanism of saving for reasons over and above marginal utility that lead to different social structures is given in Doepke & Zilibotti (2008).

Still, if we consider a transition from a low-risk steady state to a high-risk steady state, everybody lose.

The model presented in this paper has different steady state characteristics depending on the parameters; history dependence is endogenous and does not always occur. The model predicts a "threshold level" of risk, above which mobility between social classes exists, and the history of the economy is no longer relevant for the wealth and welfare distribution in steady state.

A final lesson is that missing capital markets not only matter because of limited scope for upward mobility. As this paper shows, the utility costs of income volatility caused by inferior self-insurance possibilities can be substantial.

A Appendix

A.1 The model with capital markets

The model with capital markets is as given by Aiyagari (1994). Agents earn income from labor and capital. Capital depreciates at a rate δ. When agents can rent and borrow capital, the indivisibility of labor does not affect income opportunities. Consequently, aggregation of (2) is straightforward, and aggregate production is given by $Y_t = K_t^\alpha L_t^{1-\alpha}$, where L_t is 1 (the total population) and $K_t = \sum_i k_{i,t}$. Labor and capital are paid their marginal product. Each period, the market for final goods as well as the rental markets for capital and labor clears. The wage per efficiency unit of labor is $w_t = (1-\alpha)K_t^\alpha$ and the return rate on capital $r_t = \alpha K^{\alpha-1}$; it will be convenient to re-write the capital return rate $r_t = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} w_t^{-\frac{1-\alpha}{\alpha}}$ and keep w_t as the variable characterizing the aggregate economy. The income of any individual is then given by

$$m(k_{i,t}, z_{i,t}, w_t) = w_t z_{i,t} + \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} w^{-\frac{1-\alpha}{\alpha}} k_{i,t} + (1-\delta)k_{i,t} \qquad (11)$$

which is the same as "capitalist" income in (5).

With the parameter set Θ_0, as shown in Table 1 (ν has no role in this version of the model), there is a unique steady state with a single peaked wealth distribution and a wage $w = 1.150$.

A.2 Solution method: Steady states

The model is solved in the following way.

A.2.1 Setup

Construct a discrete grid for k over 4097 grid points, where $\log k$ is uniformly spaced. Set the upper bound to $k = 50$ and the lower bound to $k = 0$. The individual shock is discretized as explained in the main text.

Then, guess an equilibrium wage w_q and use Equation (5) to calculate income at all points in the grid. Denote this income function $m(k, e)$.

A.2.2 Value function iteration

Guess a value function V_0. Update the value function by iterating over $V_{j+1}(k, e) = \max_{a'} u(m(k, e) - k') + \beta E(V_j(k', e')|e)$ until V_j and V_{j+1} are sufficiently similar. This procedure also gives the decision rule $k' = \psi(k, e)$.

As the derivative of the income function is not monotonous, maximizing $u + \beta V$ is not trivial. To make sure that the optimization does not stop at local equilibria, the entire relevant income range must be examined. To speed up the procedure, take advantage of the fact that agents' wealth paths will not cross; given individual productivity, a an agent with wealth k_1 will not pick a value for a' larger than that picked by an agent with wealth $k_2 > k_1$.

Choose a wealth grid of the form $N = 2^{\tilde{N}} + 1$ (here, $\tilde{N} = 12$ and hence $N = 4097$).

- First, calculate the optimal choice for the richest and poorest wealth grid point, examining the entire wealth space. ("level 0"; $L = 0$)
- Then, calculate the optimal choice for the middle wealth grid point, using the decisions of the richest and poorest agents as bounds for the optimizations. ("level 1"; $L = 1$)
- Then, for $L = 2$, calculate the 2^{L-1} intermediate positions on the grid, using the grid points from $L = 1$ and $L = 0$ as bounds.
- For each increase in L, use the two relevant grid points from the previous level as bounds for the optimization. This makes the area to search over smaller with each iteration.
- The last level, $L = N$, evaluates half the grid points in relatively short time, as the bound for each point is quite narrow.

The number of grid points need not be of the form $2^{\tilde{N}} + 1$, however, it simplifies the code by making it easier to divide the grid space into relevant "levels". Level 0 could be further sped up by evaluating the richest agent before the poorest; to keep the code simple and take advantage of parallel computing, this was not done. Each level can be computed in parallel, as the inputs only depend on the level above.

To further refine the result, for each point k, now construct a piecewise interpolation of the value function between the chosen next-period wealth grid point k' and the grid points before and after. Then use golden section search on $u(m-k')$ plus β times the interpolated value function to get a more accurate value for k' that does not necessarily lie on the grid. This is the value for k' that will be used later. This also gives a more accurate calculation of V.

A.2.3 Analyze decision rules

Analyze $\psi(k,e)$ to check whether there are one or two distributions in equilibrium. For each level of e, starting from the lowest wealth grid point $i=1$ and increasing i, examine the sign of $\psi(k_i,e)-k_i$. The number of sign changes determines the number of distinct groups in steady state. In the setup used here, this number, denoted Ξ, will always be 1 or 2. Index the groups by ξ.

A.2.4 Simulate individual decisions

For each population (there are one or two), simulate the decision rules. Start at an initial distribution $\Phi_{0,\xi}$ over (k,e); which one does not matter for the result, as full mobility within each population is ensured in the previous step (it might of course matter for computation time). For each grid point in $\Phi_{j,\xi}$, observe the optimal saving decision k'. If this is between two grid points, allocate the population mass between these two points according to the distance from k' to each of these points (this resembles linear interpolation in the simulation). This gives next period population distribution $\Phi_{j+1,\xi}$.

Iterate until $\Phi_{j,\xi}$ and $\Phi_{j+1,\xi}$ are sufficiently similar.

If the simulation yields a large population mass at the upper bound of k, this means that the wage is too low. In the solution framework, this is corrected for by inflating the return wage (mimicking a situation where the agents kept on saving beyond $k = k_{max}$ to a very high capital stock level).

A.2.5 Calculate aggregates

- If $\Xi = 1$: Calculate the aggregate capital and labor for each occupation definition, and get the new wage w_{q+1} from (6).
- If $\Xi = 2$, weight the two groups (the two parts of the distribution) by μ and $1 - \mu$. Then calculate the new wage w_{q+1}.
- If $\Xi = 2$, and we are interested in the range of feasible steady states rather than steady states for specific μ: Calculate aggregate capital and labor in each group ξ. Define population weights θ and $(1 - \theta)$, for the two groups. Using the wage equation (6), see if any value for θ gives $w_{q+1} = w_q$ — by (6), there will be at most one solution. Once a solution is found, record this as one steady state with the wage $\hat{w}$. Then search in each direction — away from $\hat{w}$ upwards and downwards. When w_{q+1} is no longer equal to w_q, we have reached the bounds of the range of feasible steady states.

A.2.6 Wage search

Calculate $\text{diff}_q = w_q - w_{q+1}$. Use bisection search to try new values of w and repeat the whole procedure until w_q and w_{q+1} are sufficiently close.

A.3 Solution method: Transitions

To calculate the transition, first calculate the initial steady state S_0 and final steady state S_1 by the method above. Usually these will differ in the risk parameter, but any difference should in principle be possible.

Then define a "long time" T that is sufficiently high that the entire transition from S_0 to S_1 can take place between period 1 and period T (Auerbach-Kotlikoff method). From T onwards, the environment is fixed at the S_1 steady state; hence the value function in period T, V_T, is known.

A.3.1 Iteration

Guess a wage path $\left(\{w_t\}_{t=1}^T\right)_q$.

At $T-1$, using the S_1 environment, calculate the value function for all k and e as $V_{T-1}(k_{T-1}, e_{T-1}) = \max_{k_T} u(m(k_{T-1}, e_{T-1}, w_{T-1}) - k_T) + \beta E(V_T(k_T, e_T, w_T)|e_{T-1})$ and iterate backwards to period 1. Then, simulate from the initial distribution (from the population distribution in S_0) using the decision rules obtained in the value function iteration. This gives a wealth and occupation distribution that is then used to calculate a new wage series. A weighted average of these produce the wage path $\left(\{w_t\}_{t=1}^T\right)_{q+1}$, which is then used in the next iteration.

Iterations proceed until the wage paths are sufficiently close.

A.4 Steady state details

Legend:

- ϵ: Idiosyncratic income risk
- μ: Size distribution of groups in low-risk steady states (exogenous)
- (A)=Ayiagari model (no friction) — μ is not relevant. All agents can be defined as "capitalists"
- w: Steady state wage (endogenous)
- L_i: Size of each population class (endogenous)
- A_i: Aggregate capital of each population class (endogenous)

ϵ	μ	w	L_W	L_I	L_C	A_W	A_I	A_C	Hist. dependence
0.05	(A)	1.147	0.000	0.000	1.000	0.000	0.000	5.102	No
0.10	(A)	1.148	0.000	0.000	1.000	0.000	0.000	5.100	No
0.15	(A)	1.148	0.000	0.000	1.000	0.000	0.000	5.112	No
0.20	(A)	1.150	0.000	0.000	1.000	0.000	0.000	5.129	No
0.25	(A)	1.151	0.000	0.000	1.000	0.000	0.000	5.152	No
0.30	(A)	1.154	0.000	0.000	1.000	0.000	0.000	5.183	No
0.35	(A)	1.157	0.000	0.000	1.000	0.000	0.000	5.222	No
0.40	(A)	1.160	0.000	0.000	1.000	0.000	0.000	5.271	No
0.05	0.25	1.147	0.250	0.032	0.718	0.000	0.158	4.934	Yes
0.10	0.25	1.149	0.250	0.067	0.683	0.008	0.327	4.772	Yes
0.15	0.25	1.151	0.250	0.113	0.637	0.018	0.540	4.569	Yes
0.20	0.25	1.156	0.250	0.165	0.585	0.032	0.774	4.353	Yes
0.25	0.25	1.162	0.250	0.217	0.533	0.049	1.005	4.161	Yes
0.30	—	1.158	0.367	0.126	0.507	0.103	0.575	4.516	No
0.35	—	1.195	0.206	0.441	0.353	0.081	1.993	3.222	No
0.40	—	1.211	0.205	0.495	0.299	0.111	2.183	3.022	No
0.05	0.50	1.147	0.500	0.006	0.494	0.000	0.030	5.059	Yes
0.10	0.50	1.147	0.500	0.012	0.488	0.016	0.056	5.040	Yes
0.15	0.50	1.148	0.500	0.018	0.482	0.035	0.088	5.014	Yes
0.20	0.50	1.149	0.500	0.026	0.474	0.063	0.122	4.988	Yes
0.25	0.50	1.151	0.500	0.036	0.464	0.097	0.166	4.956	Yes
0.30	—	1.158	0.371	0.125	0.504	0.104	0.572	4.489	No
0.35	—	1.195	0.206	0.441	0.353	0.081	1.993	3.222	No
0.40	—	1.211	0.205	0.495	0.299	0.111	2.183	3.022	No
0.05	0.75	1.147	0.750	0.000	0.250	0.000	0.002	5.089	Yes
0.10	0.75	1.147	0.750	0.001	0.249	0.024	0.005	5.087	Yes
0.15	0.75	1.147	0.750	0.002	0.248	0.053	0.009	5.087	Yes
0.20	0.75	1.148	0.750	0.003	0.247	0.094	0.012	5.086	Yes
0.25	0.75	1.148	0.750	0.003	0.247	0.145	0.016	5.087	Yes
0.30	—	1.158	0.365	0.127	0.508	0.102	0.576	4.528	No
0.35	—	1.195	0.206	0.441	0.353	0.081	1.993	3.222	No
0.40	—	1.211	0.205	0.495	0.299	0.111	2.183	3.022	No

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