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Solow meets Marx: Economic growth and the emergence of social class

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June 15, 2011

Abstract

This paper reconciles neoclassical models of economic growth ("Solow") with the formation of social classes during economic transition ("Marx"). An environment with missing capital markets and no labor divisibility is shown to lead to a steady state with no aggregate inefficiencies, but a very polarized wealth distribution. When capital cannot be rented, people must choose between self-production, potentially including hiring workers, and wage employment. As the first path is more profitable for the rich than the poor, inequality increases.

The model is calibrated to illustrate polarization and increasing inequality in early modern Europe, starting from a continuous pre-industrial wealth distribution. During the early industrializing period, when labor markets operate and capital markets do not, inequality increases and a distinct working class emerges. Even if capital markets later improve, the polarization is persistent.

The mechanism also has relevance for modern developing countries, where capital market access is limited. If a substantial amount of capital is needed in order to earn the market return, the poor have few incentives to save.

JEL codes: O11, E21, O43, G32

Keywords: Inequality, polarization, social class, economic growth, capital market frictions

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1 Introduction

How to explain increasing inequality and polarization in the early modern period? This paper reconciles models of economic growth with an important empirical fact: over the last five hundred years, inequality has tended to go up with modernization, a process starting before the Industrial Revolution.

Two observations motivate the theory. First, the development of full-scale capital markets is a relatively recent phenomenon, yet it is frequently assumed even in long-run growth models. Second, workers and capitalists have often been distinct social groups with low between-group mobility; correspondingly, the economic environment faced by each of the groups has been different.

When discussing heterogeneity in modern macroeconomic ("Solow") growth models, authors usually have in mind either a continuous distribution of wealth across people, or a discrete, technologically given, set of occupations.[1] On the other hand, contemporary discussions of inequality in nineteenth-century industrial economies (for example by Marx) were mainly about social class, or different occupation and income groups. The model in this paper attempts to reconcile these two approaches; to incorporate into a modern growth model the emergence of social class during the early modern period.

1.1 Empirics: Increasing inequalities

The inverse U-shape of inequality over time was first pointed out by Kuznets (1955), and is illustrated in Figure 1 with data from Milanovic *et al.* (2011). For the three countries with several inequality data points, inequality first goes up, then down.

Most studies argue that inequality in early modern Europe was increasing, at least after 1500. Van Zanden (1995) argues that economic growth and inequality growth largely went together in Europe between 1500 and 1800. Hoffman *et al.* (2002) agree, and further differentiate by looking at the price of various items in the consumption basket. They identify a general inequality-

[1] An example of the former is Caselli & Ventura (2000); the latter, Mookherjee & Ray (2003) and Galor & Zeira (1993).

increasing trend in England, France and Holland between 1500 and 1650, as well as later inequality increases in all countries.

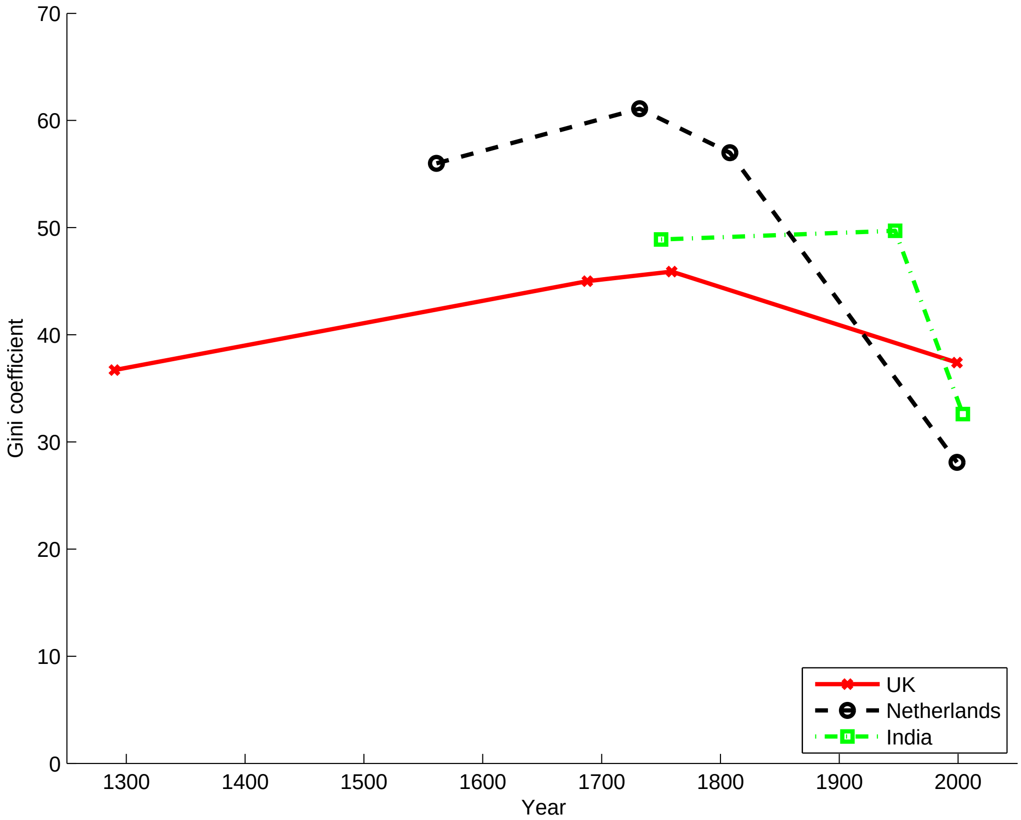

Figure 1: The Kuznets curve — data from Milanovic *et al.* (2011)

Moving to later periods, Figure 2 shows changes in the income distribution in 33 different regions after 1820, as reported by Bourguignon & Morrisson (2002). The study relies extensively on interpolation between countries and time periods; consequently, a lot of the region distributions are unchanged for several time periods. The upper panel shows changes of the income share of the highest 20%. It is evident that before 1910, all recorded changes are toward higher shares for the rich, except for the United Kingdom and France. After 1910 there are developments in both directions. The lower panel of Figure 2 shows the income share of the poorest 20%. There is also here a clear trend towards higher inequality; again only UK and France show

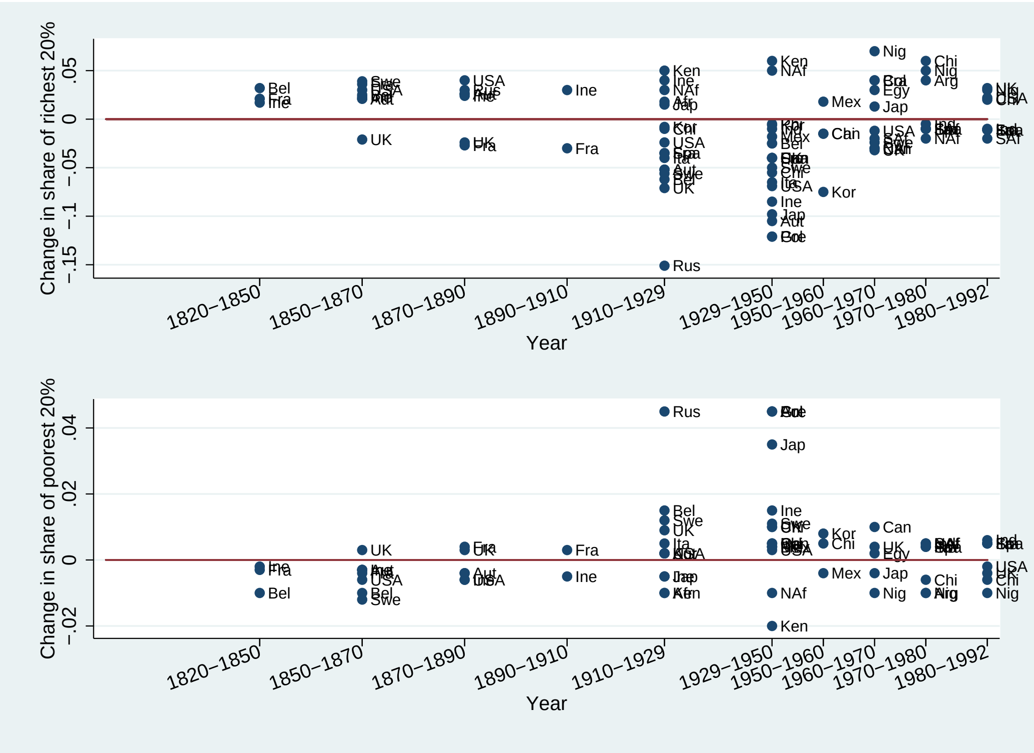

Figure 2: Worldwide inequality — changes in income shares of rich and poor. Data from Bourguignon & Morrisson (2002)

a trend toward less inequality in the early phase.[2]

1.2 Inequality and social class

The focus of this paper will be the period of increasing inequality. Several theoretical works have approached the question of how inequality can increase with modernization.

Kuznets' original theoretic contribution considered between-sector inequality. In a dual economy with an agricultural and an industrial sector, the industrial sector will tend to be more productive and grow faster than the traditional agricultural sector. Hence, as people move from the low-income to the high-income sector, inequality will first go up and then down as the size of the traditional sector approaches zero. The two-sector framework is used in a wide range of models in both growth and development economics, with the major early contribution being Lewis (1954).

However, this simplified dual economy structure is only part of the picture and hides other important differences, notably inequalities between rich and poor people in the same sector; for example, inequality between workers and capitalists in early industrial cities. Such "class differences" are characterized by the interdependence of social groups; in this example, the working class is defined explicitly by its relationship with capitalists.

Marx & Engels (1888) famously describe all history up to their period as "a history of class struggles". They focus mainly on the industrializing period, describing how the middle class is gradually out-competed by large, capitalist enterprises, leading to a society that is more polarized between capitalists and the proletariat. Within-sector inequality models also abound in more recent literature, in many cases with a two-group inequality structure motivated by analytical tractability. Galor & Zeira (1993) extend the polarization argument to human capital. In their paper, dynasties have the option of acquiring (a fixed level of) education. They study how group size depends on initial conditions, given a combination of convex investment returns and

[2] Only points with recorded changes shown in the figure. Changes are in percentage points. Country legend is given in the Appendix, section A.1.

constraints on borrowing and lending. Other important contributions in the same flavor are Banerjee & Newman (1993), Ghatak & Jiang (2002) and Erosa (2001). A common feature in many such models is that inequality stems from minimum investments — and the poorest do not have the resources to realize this minimum. In that sense, the investment technology has the "big push" feature of Murphy *et al.* (1989), but at the individual level rather than at that of society. An extensive review of the literature on growth and inequality is given by Bourguignon (2005), discussing sectoral shifts, differences and nonlinearities in savings rate and market access.

Common to many models of growth and inequality is a limited scope for forward-looking decisions by agents. Several papers have agents living for two periods, or only caring for immediate descendants (that is, next period utility). While this is a useful simplification in many contexts, it makes it harder to discern to what extent individual decisions that lead to increasing inequality are taken under a shroud of ignorance; agents willfully decreasing their (or their descendants') utility in the far future simply because it does not feature in their utility function. On the other hand, models of economic growth and saving at a national level, starting with Ramsey (1928), usually feature a discussion of accumulation with an infinite horizon.

1.3 Economic growth and capital market constraints

Classical economists were concerned about the functional distribution of income: how the value of production is divided between capitalists, land-owners and workers. In most modern models of economic growth, featuring perfect markets and the Cobb-Douglas production function, this issue is not discussed, as the income shares to capital and labor are by construction constant over time.[3] The worker earns the marginal product of labor, which is a fixed share of total production.

In Marx' economics (see, for example, Marx 1894, chap. 48), the worker is described as getting only minimum wage; this is part of the exploitation of

[3]Classical economists, including Ricardo and Marx, commonly separate the factors of production into three: land, capital and labor. Land will not be discussed here.

the working class. A fair system, according to Marx, would give the laborers all the output, as all production ultimately derives from labor. Assumptions of perfect markets and workers earning their marginal contribution to production is in some sense intermediate between Marx' description (minimum wage) and prescription (all the output).

Growth models with explicitly modeled wealth and income distributions have agents contributing both money and capital, severing the link between the functional and the personal distribution of income. If everyone have access to capital markets, and also contribute labor, they are all both "capitalists" and "laborers". To synthesize growth models with Marx' analysis, the population groups must be more clearly separated. If there are barriers to using the capital markets, there will be groups in the population who do not have capital income.

Over the last couple of years, several papers have emphasized the importance of capital market frictions for understanding how the aggregate economy works. Restuccia & Rogerson (2008) show how imperfect factor allocation can explain firm dynamics in the US, while Hsieh & Klenow (2009) and Buera & Shin (2010) extend the analysis to developing countries. With a more specific model, Song *et al.* (2011) argue that raising capital internally — financing investment out of savings — is an important feature of the private sector in China, and that differences in capital access explain why low-productivity state-run firms coexist with high-productivity private firms. Common for these papers is an illustration of how heterogeneity among establishments can help explain the growth paths of aggregate variables. Banerjee & Moll (2010) relate this discussion of misallocation to the more general inequality literature discussed above.

Keeping the usual neoclassical growth model setup, this paper will assume a departure from the free-flowing capital market models used in modern macroeconomics. The benefits given by two important institutions in modern economies will be assumed away: the banks, giving most of the population indirect access to capital markets, and well-defined frameworks for the rental of capital. Combined with labor effort being indivisible, this leads to the emergence of distinct social classes and high inequality in steady state.

The early Industrial Revolution was accompanied by a move from independent households to wage work. At the same time, capital markets as we know them today were not available for the larger part of the population. The theory behind this paper is that this asymmetry in the institutional arrangements for the two major components of production — capital and labor — was a strong driver of the increasing inequality. Hence, neoclassical studies of industrialization, such as Hansen & Prescott (2002), should be augmented by models with capital market limitations.

2 Model

The model builds on the neoclassical framework of production and saving, variously characterized as the "Solow" or "Ramsey-Cass-Koopmans" model.[4]

The main feature distinguishing the model in this paper from most other heterogeneous-agent models is an assumption that capital can only be successfully used by its owner. In a society of limited trust and institutional enforcement, how can the lender know if the borrower will use the capital in a sustainable way? The representation of the moral hazard problem is taken all the way in the sense that the model does not allow capital rental at all. In addition, it is assumed that the capital owners cannot split their own time between working with their own capital and being regular wage workers in someone else's firm.[5] These assumptions can alternatively be rationalized

[4]If one insists on making a distinction between the two labels, the utility functions used in this model correspond more closely to the Ramsey-Cass-Koopmans model than the Solow model, as the Solow model features an exogenous savings rate s rather than a utility function. However, the use of neoclassical models to study growth across countries is to a large extent a legacy of Solow's 1956 paper. For this reason, the term "Solow model" is used throughout the paper. The dynamics of inequality in standard neoclassical models are thoroughly studied in Stiglitz (1969), Chatterjee (1994) and Caselli & Ventura (2000).

[5]If they could, it can be shown that the previous restriction would not lead to a departure from the capital-rental model. There is a small tradition of indivisible labor models in neoclassical models. Hansen (1985) explain unemployment is by labor being indivisible — there is no intensive labor market margin. He argues that the fact that most people are either employed fully or not is a reasonable justification for such a model, and gives some interesting, if somewhat over-stylized, results. Mookherjee & Ray (2003) discuss how indivisibilities in investment affects inequality, and show that when capital markets are

from a geographic point of view: given location-bound capital (land, farm buildings, resource claims), capital may not be easily transportable, and the labor markets (in cities or at larger farms) may not be situated in the same place as the agent's own capital.

As will be shown, the result of these restrictions is the emergence of three distinct occupations, which correspond to social classes. For **workers** it is more profitable to work at someone else's firm than using their own capital. **Independents** own their own firm, but do not own enough capital to make hiring workers profitable. **Capitalists** have a capital level such that income is maximized by hiring workers to supplement their own labor effort.[6]

2.1 Production and income

Agents are infinitely lived dynasties that maximize the discounted utility of present and future consumption.

$$\max_{\{c_{i,\tau}\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{i,\tau}) \tag{1}$$

The function $u(c)$ is concave, homogeneous and satisfies the Inada conditions. The discount rate β is less than 1.[7]

Production takes place in a continuum of constant-returns-to-scale firms with the Cobb-Douglas production function

imperfect, the degree of indivisibility matters for inequality.

[6]A similar three-occupation result is found in Banerjee & Newman (1993).

[7]The choice of modeling utility with an infinite horizon makes the model harder to solve. The modeling choice is motivated by the fact that one of the characteristics of the working class is low return to capital. Even within one's own lifetime, saving enough to escape this position may not be possible. However, there is a possibility of passing on to future generations a slightly improved position in society, leading to potential upward mobility. To incorporate such mechanisms, the unit of decision-making in the model presented here will be the dynasty; with the presently active agent caring for all future generations, decisions that sacrifice some consumption today for potential future improvements will be important.

$$y_{i,t} = (k_{i,t})^{\alpha} \ell_{i,t}^{1-\alpha} \tag{2}$$

Labor productivity is assumed to be constant and equal across agents; without loss of generality, it is set to 1. Exogenous technological growth can be added to the production function without changing the main results.[8]

The budget constraint for the individual is

$$c_{i,t} + a_{i,t+1} = m(a_{i,t}, s_{i,t}, w_t) \tag{3}$$

The function m defines the agent's income as a function of wealth a, occupation s (worker, independent or capitalist, to be defined below) and the going wage level w.

For the agent, there are three possible distinct labor market outcomes each period:

A **worker** gets the going market wage w. In addition to labor income, the wage worker's capital can be stored, with a return $\nu \leq 1$. This gives the income function

$$m(a_{i,t}, \text{worker}, w_t) = w_t + \nu a_{i,t} \tag{4}$$

Anyone who is not a wage worker uses his wealth for production; in other words, he owns a firm. There is only one type of capital, hence $k_{i,t} = a_{i,t}$. The firm owner's maximization problem is to hire the amount of labor ℓ that maximizes output minus wage costs:

$$\max_{\ell_{i,t}} a_{i,t}^{\alpha}(1+\ell)^{1-\alpha} - w\ell$$

[8]This is covered in detail in the Appendix.

From this, optimal labor demand can be shown to be

$$\ell_{i,t}^{*} = w_t^{-\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}a_{i,t} - 1$$

and a corresponding income (profit) function can be derived. However, as firm owners cannot rent their own labor to someone else, ℓ cannot be negative.

If $\ell^* < 0$, that is $w_t^{-\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}a_i - 1 < 0$, meaning wealth is below the threshold $a_c(w_t) = w_t^{\frac{1}{\alpha}}(1-\alpha)^{-\frac{1}{\alpha}}$ no labor is hired, and income is just the production function with the agent's own labor inserted.

This leads to the definition of the two remaining occupations:

An **independent** produces with his own wealth, but does not hire outside labor. This constrained type of agent (with $a < a_c$) is hence not participating in the labor market. When capital is used productively, it depreciates uniformly at a rate δ, giving the income function for independents

$$m(a_{i,t}, \text{independent}, w_t) = a_{i,t}^{\alpha} + (1-\delta)a_{i,t} \tag{5}$$

If labor is hired, returns to savings are linear, as labor can be hired to better match the scale of capital holdings.[9] The income function is

$$m(a_{i,t}, \text{capitalist}, w_t) = (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha w_t^{-\frac{1-\alpha}{\alpha}}a_{i,t} + w_t + (1-\delta)a_{i,t} \tag{6}$$

The lower bound on a is set to 0 (no borrowing allowed).

Lemma 1 *The occupation decision is monotonous in income. For any given wage w, there exists an income function $m^*(a_{i,t}, w_t) = \max_{s_{i,t}} m(a_{i,t}, s_{i,t}, w_t)$.*

This follows from there being no cost associated with switching occupations, and from the specifications in (4, 5, 6).

Figure 3 shows individual income, and how it is increasing with wealth. Below the point a_w agents will get the highest income from being a worker.

[9] It is assumed that several capitalists can pool their wealth together to hire "fractional" labor units.

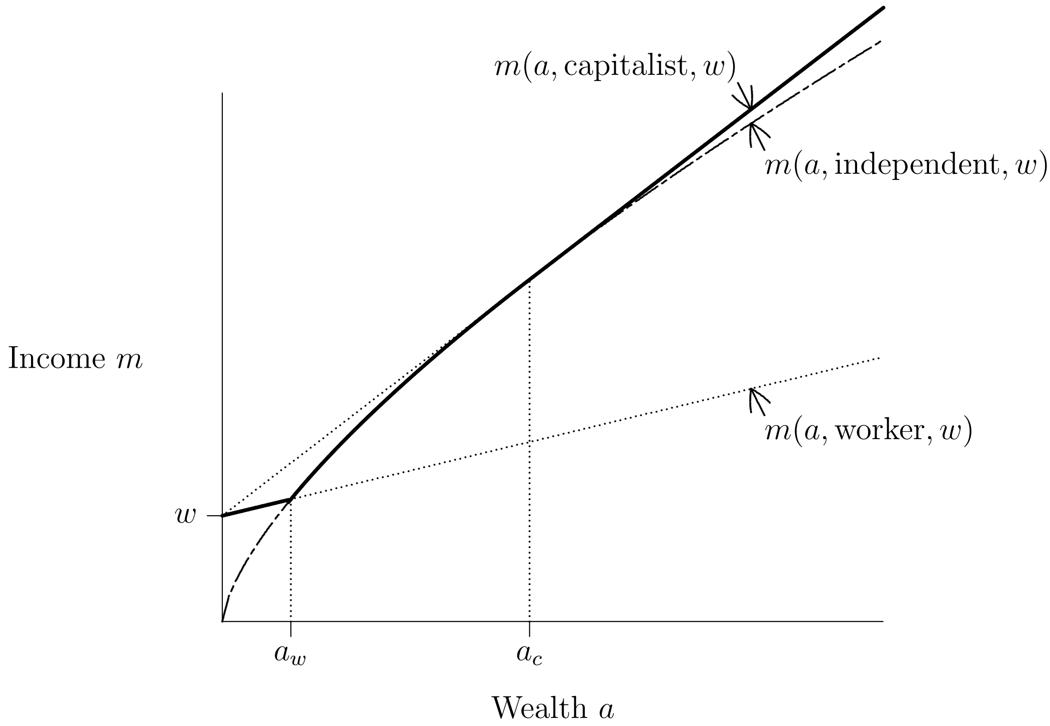

Figure 3: The relationship between wealth, occupation and income

Between a_w and a_c independence is preferred, while above a_c hiring labor (being a capitalist) is optimal. The bold line in the figure is the income function with choices incorporated; $m^*(a_{i,t}, w_t)$.

Figure 4 shows the slope of the income function, and illustrates how marginal returns to saving are low for workers (the storage return), high and falling for very poor independents and constant for capitalists. Much of the discussion in this paper relates to whether workers will have a strong enough incentive to accumulate wealth, given that saving over a long time period could mean a transition into a different social class.

2.2 Wage determination and labor market clearing

There are two markets that clear in each period: the market for the final good, and the market for labor. The price of the final good is normalized to 1 and the wage rate is determined in the labor market.

Labor market equilibrium, and the wage, depends on the mass of people in each occupation. Denoting workers, independents and capitalists as belonging to the sets $\mathcal{L}_W$, $\mathcal{L}_I$ and $\mathcal{L}_C$, the total wealth holdings of each group is denoted $A_i = \sum_{i\in\mathcal{L}_j} a_i$. The size of each group is denoted L_j, and with no population growth the identity $L_W + L_I + L_C = 1$ holds.

The supply of labor is given by $\sum_{\mathcal{L}_W} 1 = L_W$. The demand for labor is given by $\sum_{i\in\mathcal{L}_C} \ell_i^*$. Setting supply equal to demand yields the market clearing wage

$$w_t = (1-\alpha)\left(\frac{A_{C,t}}{L_{C,t} + L_{W,t}}\right)^{\alpha} \tag{7}$$

Note that the mass and wealth holdings of the independent group does not feature in the wage equation. The workers' stored capital, not part of production, also does not influence the market wage. Given sufficiently high inequality, there will always exist both workers and capitalists.[10]

The labor market clears every period. Hence, from an initial wealth distribution, one can calculate the occupational structure and market wage in a given period. Based on this income, and knowledge of the expected future wealth pattern, agents make savings and consumption decisions. Aggregating the savings decisions makes it possible to calculate next period's wealth distribution.

2.3 Consumption and saving decisions

While the current-period occupation of an agent is determined by individual wage and the market wealth, decisions over time also have to factor in expectations of the future. Rational expectations are formed over future wage paths.

Decisions on saving factor in the marginal return to wealth in future periods. The marginal return - the derivatives of the income functions (4)-(6) - are shown in Figure 4.

[10] With very low inequality, it could be the case that there is no wage such that the richest agent will prefer to be a capitalist while the poorest agent prefers to be a worker. This would, however, require rather strict bounds on the distribution. Moreover, as soon as inequality is large enough to accommodate this difference, "sufficient inequality" will

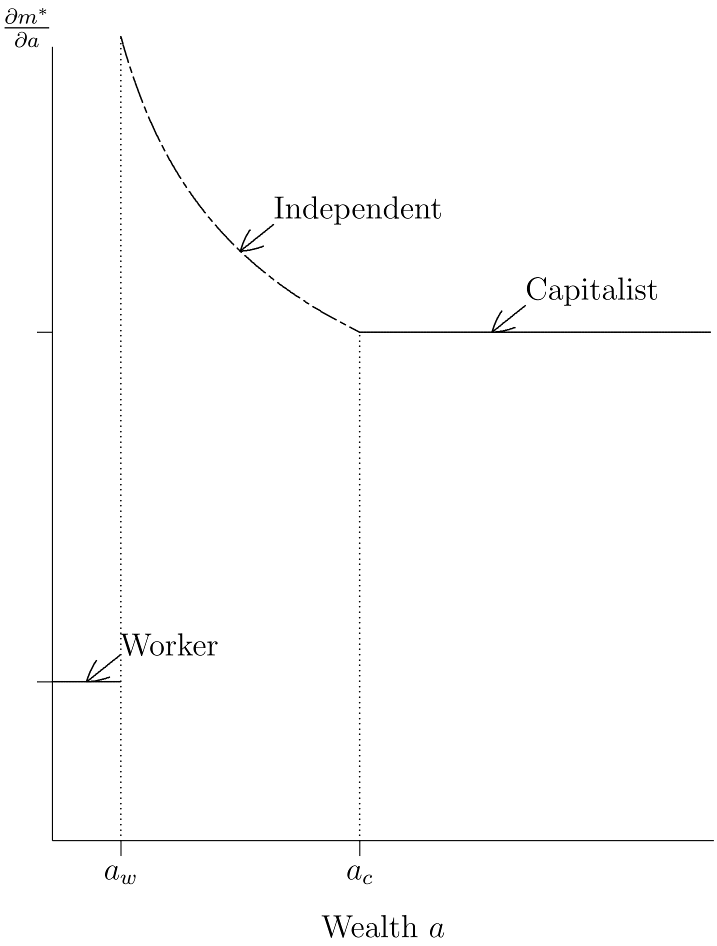

Figure 4: Marginal returns to saving

As a_w and a_c depend on the wage, and change over time, agents close to these thresholds may need to take several future periods into account. The setup deviates from the normal assumptions on dynamic optimization, because the derivative of the income function is not monotonous. For this reason, it is necessary to study the dynamics of wealth accumulation in several steps.

Agents only differ in their wealth holdings — there is no uncertainty or productivity differences — and have concave utility functions. We have the following intermediate result:

Lemma 2 *The sorting of agents by wealth does not change over time; the richest n% will remain the richest n% forever.*

As income functions are increasing in wealth, consumption smoothing motives will work in the same direction for all agents. Wealth paths "crossing" would correspond to individuals smoothing income in opposite directions, and are therefore not feasible.[11]

To see the dynamics of the transition, first consider the richest agent. As he will always be richest, he will always be a capitalist. As the income function for this agent is monotonous in wealth, we can use the Euler equation to calculate optimal intertemporal decisions. The Euler equation is found by maximizing the utility function (1) with respect to the budget constraint (3), and is given by

$$\beta \frac{\partial m^*(a_{i,t+1}, w_t)}{\partial a_{i,t}} \frac{u'(c_{i,t+1})}{u'(c_{i,t})} \leq 1, \qquad = 1 \text{ if } a_{i,t} > 0 \tag{8}$$

Over time, as long as the long-run wage level is stable, capitalists' wealth will reach a steady-state level. Increasing capital accumulation lowers the wage, while lower wage increases capital accumulation.

Second, consider the poorest agent, who will always remain a worker. In this case we can also use the Euler equation. As the return to capital for the

always prevail (by the discussion in the next sections).

[11] A formal proof is given in the Appendix, section A.2.5.

worker, ν, is lower than $\frac{1}{\beta}$, the poorest worker will always dissave and end up with zero wealth.

The remaining agents can be classified in three groups. Relatively rich agents start out as capitalists and find it optimal to remain so. Their wealth accumulation follows the same rules as the richest agent. Relatively poor agents follow the same rules as the poorest agents. Agents in the middle, however, have to take into account the non-monotonous marginal return profile shown in Figure 4. The trade-off between utility in the short and long run can then be found by the value function

$$\max_{a_{i,t+1}} u(m^*(a_{i,t}, w_t) - a_{i,t+1}) + \beta V(a_{i,t+1}, \mathbf{w}) \tag{9}$$

At each time t, the individual faces the problem (9) for a given future wage path $\mathbf{w}$, current period wealth $a_{i,t}$ and wage w_t.

In general, the exact wealth path of this middle group cannot be shown in closed form.[12] For example, a worker right below a_w might find it optimal to become an independent next period even though the local derivative is low. However, over time, it is evident that everyone in the middle group end up in either the "poor" or the "rich" group. Consumption smoothing motives means that a jump between worker and independent status is not optimal. Over time, wealth paths will point downward, leading into the working class, or upward, leading into the richer independent class. Finally, observe that the independents have higher marginal returns to saving than the capitalists. Hence, for a given level of capitalist aggregate capital A_C, with a corresponding wage, independents will want to save more than the capitalists. For this reason, a steady state cannot contain independents, as they would still want to save.

It follows that the economy will settle down in a steady state, as all members of the intermediate group is absorbed into either the "poor" or the "rich" group as time passes. In the steady state, everyone in the "poor" group will be workers and everyone in the "rich" group will be capitalists. As the

[12] A numerical solution to the transition problem is shown in the next section.

capitalists all face the same marginal return, and the returns to saving for this group goes down when aggregate savings go up, the return will settle down to a unique level in the long run — corresponding to a unique wage level. From Equation (6), this steady state wage level is given from technology and preference parameters. A wage marginally higher than this level would make capitalist want to dissave, as returns would be lower (see Equation (11) below), while a lower wage would induce higher saving.

With this analysis of the dynamics in place, we are ready to discuss the steady state.

2.4 Steady state

Proposition 1 *(Characterization of the steady state.) In the long-run steady state, agents will belong to one of two groups. Some will be workers with zero wealth, and some will be capitalists with wealth at or above a_c.*

As workers face a return rate lower than their subjective discount rate ($\nu < 1/\beta$), consumption and wealth holdings of agents that remain workers will decrease over time until wealth is at zero. If any initial workers become independents over time, they will face a return higher than their subjective discount rate, as marginal returns to saving are higher for independents than capitalists due to oversupply of labor within the small-scale independent firms. This means that independents will continue to save until reaching the wealth level $a_c = w_t^{\frac{1}{\alpha}}(1-\alpha)^{-\frac{1}{\alpha}}$.[13]

It follows from Proposition 1 that there is no misallocation of capital in steady state — workers hold no capital, so $A_W = 0$; there are no independents, so $A_I = 0$. Consequently, all wealth is held by the capitalists. As in a neoclassical model without frictions, their savings behavior is shaped by the aggregate return to wealth; in this case, $\partial m(a, \text{capitalist}, w)/\partial a$. Capitalists will save until the return to capital is equalized, which determines the unique steady-state level of aggregate capital, regardless of the number of capitalists. We see that the denominator of the wage equation (7) in steady state

[13] See Appendix, section A.2.3 for a formal exposition.

is 1, and combining this with constant consumption in the Euler equation (8) and the income function for capitalists (6) the expression for aggregate capital in steady state — all held by the capitalists — is found:

$$A_{C,LR} = \frac{\alpha^{\frac{1}{1-\alpha}}}{(\beta^{-1} - (1-\delta))^{\frac{1}{1-\alpha}}} \tag{10}$$

By the same argument, and using equation (7), there is also a unique wage level in steady state.

Proposition 2 *(Wage in steady state.) The wage in steady state is uniquely determined, and is given by*

$$w_{LR} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{(\beta^{-1} - (1-\delta))^{\frac{\alpha}{1-\alpha}}} \tag{11}$$

regardless of the number of workers in the steady-state population.

Another way to derive the unique long-run wage is to consider the capitalists as a single representative agent. This agent will save until market returns to aggregate capital equals his own discount rate, regardless of the mass of this representative agent, and as capitalist wealth returns are linear, the distribution across the capitalists does not matter for the aggregate allocation. It follows from concavity of utility (Equation 1) and linearity of capitalist returns (Equation 6, illustrated in Figure 4) that at some point the marginal utility of saving will equal the marginal utility from consumption. Because the labor market smooths saving for capitalists, they act *as if* there was full capital market access. As shown by Chatterjee (1994), when returns are linear, the within-group distribution of wealth is not required to calculate aggregate savings behavior. The wealth return for capitalists — the return rate in Equation (6) — will have a unique value in the long run, and the long-run equilibrium wage can be straightforwardly calculated.[14]

[14]The calculation follows from Equations (1), (8) and (6); see Appendix, section A.2 for full exposition of the calculations of w_{LR} and $A_{C,LR}$.

It is worth noting that with more forward-looking agents (higher β), everyone has higher steady-state gross income, even the zero-wealth workers. Higher β gives higher steady-state capital, which increases production. From the Cobb-Douglas production function, labor share of income is constant, meaning that increased production gives better conditions for the workers as well.

As there is no misallocation of capital in steady state, the aggregate level of capital is no different than it would be in a model without frictions. Capitalists can achieve the optimal wealth level by hiring the appropriate amount of labor. Only under-utilized wealth among workers and independents would keep the economy from reaching full utilization. As this non-capitalist wealth is zero in steady state, the economy has full utilization. To see this, compare (10) to the steady-state capital arising from a representative-agent model with perfect markets and Cobb-Douglas production; the two will be equal. Similarly, as there are no independent agents in the long run, the labor force allocations are also similar.

Proposition 3 *(Production in steady state.) In steady state, aggregate economic activity (aggregate production, aggregate capital and the wage level) is as if there were no indivisibilities and full capital market access.*

The result extends to the case of constant exogenous technological growth.[15]

2.5 Inequality in steady state

Even though the aggregate characteristics of the steady state economy is unaffected by the capital market frictions and labor indivisibility, the underlying wealth and income distributions are not.

First, there is substantial inequality between workers and capitalists. As stated in Proposition 1, agents group into two classes, with the workers holding zero wealth. Because the marginal return of independents is higher than that of capitalists, there will be no independents in steady state. $a_c = w^{\frac{1}{\alpha}}(1-\alpha)^{-\frac{1}{\alpha}}$ denotes the "threshold wealth" to be capitalist, and all

[15]The model adjusted for constant technological growth is shown in the Appendix.

capitalists will be above this level. Hence, by comparing the capitalist income at a_c (from Equation (6)) to the income of the zero-wealth worker (Equation (4)), we have a measure of the distance between the worker and capitalist groups in steady state.[16]

Proposition 4 *(Between-class inequality.) In the long-run steady state, the ratio of the consumption (net income) of the poorest capitalist to that of the richest worker will be*

$$\Omega = \frac{1}{1-\alpha} - \frac{\delta}{(1-\alpha)} \left(\beta^{-1} - (1-\delta)\right)^{\alpha} \tag{12}$$

The first term in Equation (12) is total production divided by the part of production going to labor. Capitalists receive both capital and labor income from production, and at the margin a_c they have just enough capital to employ themselves.

The second part of the equation derives from the income the capitalist loses to depreciation. This depends on the steady-state level of capital; with more forward-looking agents (higher β), the steady-state level of capital is higher, to the betterment of both capitalists and workers. However, for the workers, this "comes for free"; the increased depreciation cost is covered by the capital owners. This means that high β, through increased aggregate capital stock and depreciation losses, reduces the steady-state between-group inequality.[17]

Because of the constant-returns-to-scale assumption, the wage level and hence the conditions of the working class is uniquely determined in steady state. However, the number of workers that live under these conditions, the number of capitalists, and the average wealth of the capitalists, depend on the initial conditions.

[16] For discontinuous initial distributions, this is a minimum value; it could be the case that there is no population at the position ending up in a_c; in this case, the poorest capitalist would be slightly *richer* than a_c and Ω must be seen as a lower bound on between-class inequality.

[17] See Appendix, section A.2.4 for calculation of (12).

Proposition 5 *(History-dependence of the steady state distribution). In steady state, any strictly positive number of workers and capitalists are supported; the distribution of people between these classes depends on the initial wealth distribution. Moreover, the steady-state distribution of wealth among capitalists depends on the initial distribution.*

The first part of this proposition can be explained by an example: consider a distribution with everyone concentrated at one of two points: ϕ agents have a wealth of 0, and $(1-\phi)$ have a wealth of $A_{C,LR}/(1-\phi)$. Then, for any ϕ strictly above 0 and strictly below 1, the steady state conditions will be satisfied and individual wealth will remain stable.

The second part follows from the wealth formation of capitalists. Even though the aggregate level is determined by interactions between agents (the adjustment of the wage level), the distribution is determined by initial conditions.

It has now been shown that the aggregate environment in steady state, as well as the polarization between workers and capitalists, follow directly from the assumptions of the model. To discuss the size of the various groups, the distribution of wealth among capitalists, and the time and shape of economic transition, a specific initial distribution must be specified. The next section will discuss such a transition, and place it in a historical context.

3 Increasing inequality, economic transition, and phases of development

In the model presented in the preceding section, the transition to steady state corresponds to a period of increasing inequality and the emergence of economically distinct social classes. Poorer people become poorer over time, while those with higher wealth consolidate into a capitalist group. In this section, the model will be embedded into a long-run theory of institutional development, representing a stage when labor markets have emerged, but well-function capital markets do not yet exist.

3.1 A factor market theory of economic development

As stated in the introduction, while capital market access is near-universal in the developed world today, it was not so in the past. North & Weingast (1989) date the emergence of "modern" capital markets to late seventeenth century Great Britain. However, capital market access for everyone did not come immediately. In the early phases, the mechanisms that fostered trust and large-scale exchange mostly applied to the well-off.

If capital markets did not fully develop before the industrializing period, why was not the population completely economically polarized from the beginning? A key mechanism of the model presented here is labor markets, and like capital markets, these have developed over time. In Western Europe, a society that was "modern" in a labor market sense had developed around 1500. For the case of England, North & Thomas (1971) and Brenner (1976) describe a move from the manorial system, where obligations took the shape of fees, land rents and various degrees of unfree labor, to more formalized economic relations. In addition, Outhwaite (1986) and North (1991) describe the growth of cities, and with it, both specialization and wage work.[18]

These facts suggest the following stylized institutional framework of economic development. The process of increasing inequality in the early modern period can be explained by three stages, with labor markets improving before capital markets do.

Stage 1 is the pre-market stage, with no markets for rental of factors of production. Missing labor markets can be explained by big distances between farms, missing institutions, serfdom, or land abundance. In this environment, everyone operate in autarky, earning income by Equation (5). The initial distribution depends on geographical, social and economic mechanisms outside the model.

In **Stage 2**, labor can be freely rented — that is, employed. Capital can be bought and sold but there is no capital rental — no financial markets exist. This can be thought of as the early framework of the Industrial Revolution,

[18]The gradual evolution of labor markets is also evident in studies from modern developing countries. Fafchamps & Shilpi (2003) show that in Nepal, villages far from urban centers have a much lower rate of wage work and correspondingly more self-employment.

or as today's reality for poor people in developing countries. The agents face the environment of the model in Section 2 of this paper. This is a period of increasing class differences, a "transition to inequality"; the upward-sloping part of the Kuznets curve.

In **Stage 3**, the restrictions on capital rental are removed. This gives credit market access for all, and the environment is well described by the standard neoclassical growth model without frictions. Note, however, that if the steady state conditions of Stage 2 are satisfied, the distribution will also be a steady state outcome in Stage 3, meaning that inequality does not necessarily decrease.

The three-stage process is also illustrated in Table 1. The framework shares some characteristics with the discussion of exchange regimes in Townsend (1983). In Townsend's model of growth and financial development, the regime transitions are driven by population increase; a higher population means that agents are on average closer, facilitating more efficient exchange.

This evolution of institutions is not meant to explain all aspects of inequality development over time. There is clearly merit to Kuznets' story of the dual economy, and other mechanisms are at work as well. The contribution of this discussion is the emergence of inequality in interdependent relationships; both workers and capitalists becoming more defined as groups precisely by interacting with each other.

		Capital	Labor
Stage 1:	Pre-modern	Bought and sold, no rental	No rental
Stage 2:	Early modern	Bought and sold, no rental	Rented
Stage 3:	Late modern / Industrial	Bought and sold, rented	Rented

Table 1: A three-stage theory of economic development: Factor markets

3.2 Calculating the transition

Taking a continuous initial income distribution as given and corresponding to a pre-industrial equilibrium with no labor markets or mobility, the next sections will examine the paths of growth and polarization for a period with no capital markets.

Where pre-industrial data exists, the model will be fitted to Great Britain. The model could equally well be fitted to data for other major European countries, but British data is the most readily available to use for calibration.

The transition path is found by solving the value function (9) for all agents, given wage paths, and then iterate until the wage paths converge.[19] An overview of the parameters used is given in Table 2. Each time period in the model will be set to equal 10 years.

Technology (Cobb-Douglas production function)		
Capital share α	1/3	
Depreciation rate δ	0.1	
Storage return ν	0.8	
Preferences (CRRA)		
Discount rate β	0.979	Matching GDP growth 1500-1800
Utility curvature η	10	
Initial conditions (Pareto wealth distribution)		
Upper bound	6.09	"Golden rule" wealth in autarky
Lower bound	0.15	England 1290 (Campbell)
Mean	0.87	England 1290 (Campbell)

Table 2: Parameter values (see text for explanations)

Timing. Stage 2 is set to start in 1500, the commonly accepted beginning of the "early modern period" in Europe. The transition to Stage 3, when capital markets start to operate, is assumed to start in 1850. The 1850s saw a series of new corporate legislation in the United Kingdom. For example, in 1856, the British Parliament extended the availability of limited liability to all registered corporations (Rosenberg & Birdzell, 1986, p. 198).

Technology. The capital share parameter, $\alpha = 1/3$, is commonly used in the macroeconomic literature. The depreciation rate δ is 0.1, corresponding to little over 1 per cent annually. This is lower than that used in modern studies. Some evidence suggests that the depreciation in the pre-industrial period was lower than today. The share of land was higher, and technological progress slower.[20] The storage return ν is set to 0.8; around two per cent of

[19]A full description of this, including issues arising from the shape of the income functions, is given in the Appendix.

[20]This is discussed by Voigtlander & Voth (2006, p. 339) who also use a low depreciation

non-used capital disappears each year.

Preferences. The discount rate β is calibrated to match aggregate growth of capital from 1500 to 1900 — with a hypothetical steady state for pre-industrial Britain being reached in 1900. According to Maddison (2010), GDP per capita in Britain increased by roughly a factor of six in this period. In the model, there are two mechanisms for growth: reallocation and capital accumulation. In the pre-market period (Stage 1), misallocation gives a lower GDP than wealth alone should imply. The misallocation effect is found by calculating production for the initial distribution under autarky. This is explained in detail in the Appendix. The remaining growth is allocated to capital accumulation.

The reciprocal of the intertemporal elasticity of substitution, η, is set to 10. There are two motivations for this low level of intertemporal substitution elasticity. First, at mean incomes much lower than those in contemporary Western economies it is likely that intertemporal elasticities of substitution are lower, even if we assume the elasticity to be constant locally. Second, there are no frictions to capital reallocation in the model, and some of the "sluggishness" of adjustments could therefore be picked up by this parameter instead — for lower values of η the calibrated transition lasts for a very short period.

Initial conditions. The initial wealth distribution at the end of the period of autarky is described by a truncated Pareto distribution. The upper bound is given by the autarky income function (5); rational agents will not hold wealth higher than $a^* = \left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}$, where the marginal return to saving is zero. The two remaining parameters are set to fit two characteristics of a medieval income distribution, as described by Campbell (2008).[21] The mean of the initial distribution is set to 1/7 of the upper bound, corresponding to the ratio between the mean of the entire population and the mean of the

rate for this period, and comment that to fit their model, their depreciation rate should perhaps have been even lower than the .02 they use (≈ 0.22 if ten-year periods).

[21]Campbell describes the income and wealth situation in 1290. However, there are no other sources of income inequality in Britain later than this and before 1500. As the initial distribution should describe the "pre-modern" period, the 1290 data will be used. The land distribution is used as a proxy for the wealth distribution.

richest group in Campbell's data. In addition, the 90/10 wealth ratio of the initial distribution matches the 90/10 ratio in the data.

With these parameters, the wealth space is discretized into a grid of around 30 000 points. The steady state is assumed to be reached at a finite point far into the future; transition paths can then be calculated back from this point.[22]

The following sections present the results from the calibrated transition — from the initial condition, where wealth levels are calibrated to medieval England and there is no labor market — through opening labor markets, increased specialization and wealth adjustment to the steady state.

3.3 Transition with labor markets: Capital accumulation and class formation

Figures 5 and 6 show the evolution of the economy during transitions from Stage 1, where everyone is in autarky, to stage 3, with full capital and labor markets.

At t=3 (year 1500 in the figures), the institutional arrangement is changed from Stage 1 to Stage 2, that is, labor markets are introduced and the population splits into three occupations. At t=38 (year 1850 in the figures), Stage 3 is introduced, and full capital markets are also available. The following paragraphs go through the dynamics of the transitions.

Aggregate production

Aggregate production is increasing steadily over the entire period. The overall trend is due to capital accumulation; steady-state capital (and hence output) is six times higher than initial capital, and the Solow-type catch up goes on over the entire period. In addition to the capital accumulation, there are some aggregate productivity effects from reallocation. The biggest reallocation jump is seen at the transition from Stage 1 to Stage 2. At this

[22] For details on parameter choice and solution method, see the Appendix, sections A.3-A.4. The no-crossing specification in Lemma 2 greatly reduces the number of wealth paths that need to be analyzed.

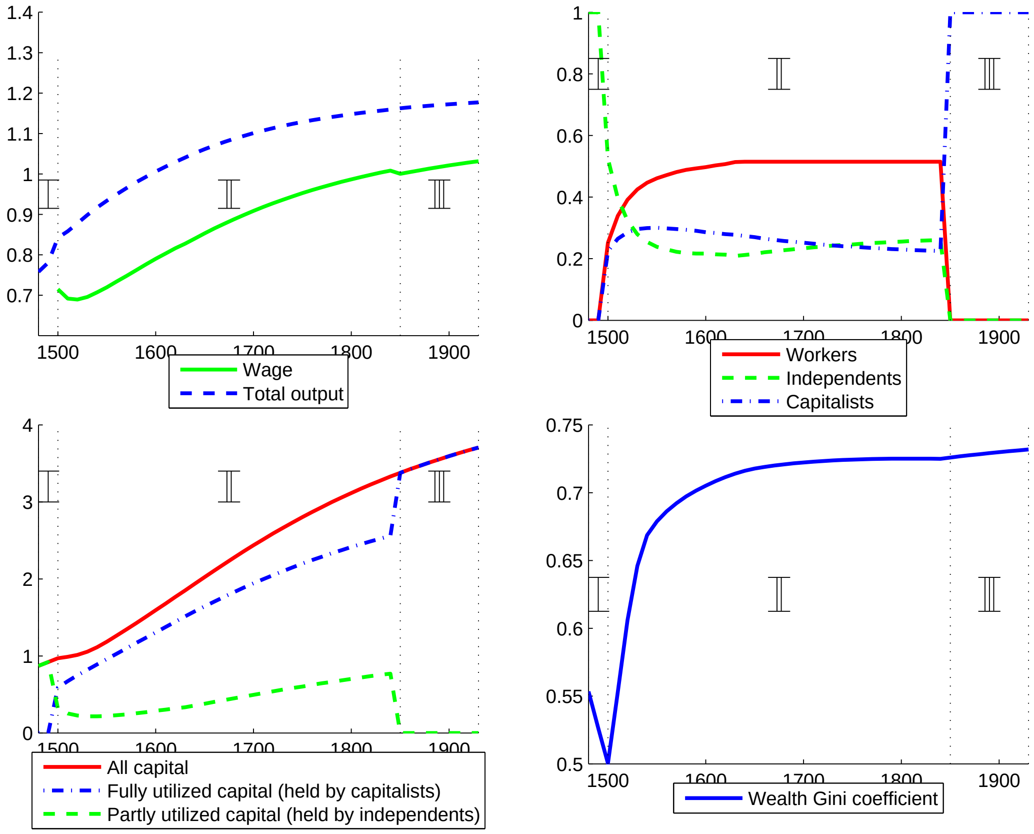

Figure 5: Model simulation, aggregates

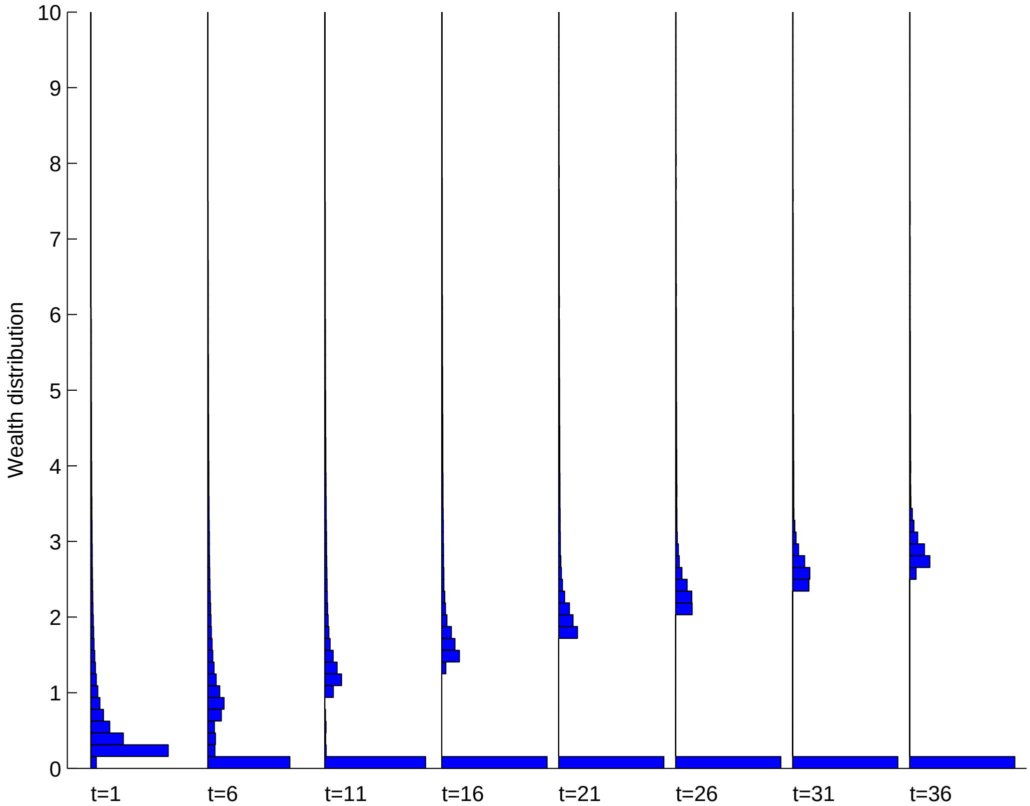

Figure 6: Model simulation, wealth distribution at selected time periods (upper limit truncated)

point in time, the high-wealth individuals go from autarky, where diminishing returns take a large cut of production, to constant returns to scale, because labor can be hired. The classification of utilized capital is shown in the lower left panel. Capital accumulation increases labor productivity, and hence the wage; the capital-accumulating independents stay independent for a long time, as hiring labor becomes more expensive at roughly the same rate as capital is accumulated.

There is no major change in aggregate production or capital accumulation at the introduction of complete capital markets (Stage 3). While all capital is now fully utilized, as low-capital independents can rent capital from high-capital capitalists, most independents are just below the capitalist threshold and the quantitative effect is not very large. The workers, who have zero wealth, have almost no incentive to save, even though that would give them the market return rate.

Social classes

The upper right panel of Figure 5 shows the population in each of the three social classes. Following the introduction of labor markets in Stage 2, the poorest individuals immediately become workers — this is more profitable than using what little capital is available to produce. Over time, more people become workers as wealth profiles adjust to the new institutions.

The number of independents and capitalists are affected by two opposing trends. First, as discussed in Section 2, all richer independents will accumulate capital and be capitalists in the long run. After roughly twenty periods all independents belong to this group. Second, the wage is increasing because of aggregate capital accumulation. The increasing wage moves the threshold for being a capitalist up — for a capitalist close to the zero-hiring ($\ell^* = 0$) border, an increasing wage means going from capitalist to independent. In the medium run, for these parameter values, the second effect dominates.

At the transition to Stage 3, all agents are, by the definitions in this paper, capitalists. If capital can be rented, even very poor people can combine wage work with a stake in a bigger investment, as in the standard neoclassical model. The effect of this can also be seen in the wage curve. The rich group of independents can now also contribute a small amount of labor in the market; this increase in labor supply temporarily lowers the wage.

Within-group inequality

Figure 6 shows wealth histograms for the entire population at selected time periods. The first histogram reflects the initial conditions; the others show the results from the calibrated model.[23] From the beginning of Stage 2, the distribution is split; there is a wealth threshold that separates the agents who will become workers in the long run from those who become capitalists in the long run. While the histogram is still continuous in period 6, the split is visible and most poorer agents are at zero wealth. Over the first fifteen periods, the long-run worker population is fully established; as shown at t=16, the group has zero wealth.

The rest of the population follow the standard neoclassical capital accumulation path; dispersion goes slightly down, and the shape of the original distribution is preserved. Hence, while overall inequality goes up, the groups become more coherent and within-group inequality decreases for all social classes. As is shown in the lower right panel of Figure 5, the wealth Gini coefficient goes up by a lot in the first periods of the transition, and stabilizes later. The periods of Stage 1, with no labor markets, illustrate the rapid convergence that would have taken place if labor markets were never introduced; wealth inequality falls rapidly when poor people have to save, but starts to increase as soon as the possibility to increase consumption by wage work is introduced.

The benchmark calibration shows that for reasonable parameters, the model replicates a transition path with increasing polarization, a sharp jump in inequality and a homogeneous, large working class. As explained in section 2, while the steady state wage does not depend on the initial wealth distribution, the number of people in the working class is history dependent.

[23] The numerical solution of the model has a larger and denser wealth space than shown in the histograms. As is evident from the figure, the richer groups are nearly empty. See Appendix for details on the numerical solution of the model.

For this reason, it is of interest to examine how different initial distributions affect the transition paths and the long-run outcomes.

3.4 Long-run distribution effects

To evaluate the long-run effects of initial inequality, we can simulate the transition for different parameters on the initial wealth distribution. Keeping the upper bound and mean constant, the shape of the distribution is altered to give different initial 90/10 wealth ratios. The results of the simulations are given in Table 3. The 90/10 ratio of 13.044 is the one used in the previous section.

Initial 90/10 wealth ratio	Initial wealth Gini	Workers at end of Stage 2	Wealth Gini at end of Stage 2	Wealth Gini at end of simulation
5.000	0.40	28%	0.49	0.50
8.000	0.48	41%	0.63	0.64
10.000	0.52	46%	0.67	0.68
(ref.) 13.044	0.55	50%	0.71	0.71
20.000	0.60	55%	0.74	0.75
30.000	0.64	58%	0.77	0.77

Table 3: Effects of initial inequality

The initial inequality is persistent. Wealth inequality increases in all cases, but the end inequality is much higher for the high-initial-inequality societies. In addition, the existence of many initially poor agents give larger working classes, and less people share the ownership of the steady-state wealth.

Overall capital density in steady state follows directly from the model parameters. The between-class inequality comes from the rich agents “crowding out” more moderate-wealth agents from the steady state wealth distribution. If, initially, some agents have high capital levels, more of the capital will be used in firms participating in the labor market, bidding up wages. This will lead to some intermediate agents choosing the “worker path” instead of the “capitalist path”, giving a higher fraction of workers in the long run.

4 Discussion: Development, polarization and poverty

The model presented here is discussed in a context of historically increasing inequality — a process seen in most parts of the world. While developed countries are now “class-less” in many senses of the word (for example, nearly everyone has access to capital markets), many developing countries remain deeply polarized. Rutherford & Arora (2009) argue that there is a huge, unmet demand for saving services by poor people in developing countries. They give examples of savings products where administrative fees would correspond to an annualized interest rate of minus 30% (with the depositor still bearing all risk for “bank failure” of the informal deposit collector) — which are envied by residents of neighboring areas who have no access to semiformal saving at all. Similar stories — of how financial instruments have to be improvised for 40% of the people alive today — are told in Collins *et al.* (2009). This illustrates that the model also has relevance in today’s developing countries. The next sections discuss some applications.

4.1 Is poverty a rational choice?

A key element of the model results is “rational poverty” — by the standard definitions, it is optimal for the agents in the lower end of the wealth spectrum to run down their wealth and become workers in the long run, even if saving and becoming a capitalist is feasible by reducing current consumption. Of course, in real world applications, many other factors are responsible for poverty, and this paper does not address any of those. If uncertainty was introduced to the model, for example through stochastic labor productivity, some mobility could ensue, and there would be savings also among poor people; such precautionary savings are reported by Collins *et al.* (2009).

However, the model does capture an important point: if returns for poor people are lower, they will save less. Is this a poverty trap? On the individual level, perhaps; if one poor person was initially given a large transfer, he would have ended up as a capitalist instead. But the *existence of the working class* can not be done away with. In this model, for any equilibrium, there will be *some* people for whom it is profitable to be workers rather than independents. These people will have zero wealth in the long run.

If, in modern society, there are some jobs that do not require human capital at all, we can imagine an inequality-inducing process like the one portrayed in this paper. Those using human capital in their daily work have a higher incentive to accumulate more. Those who do not have daily use of human capital choose not to accumulate — not because they have different preferences, but simply because they do not earn the same return to it. In this example the initial wealth would be human capital endowed by the previous generation. If this line of thought holds, it means that low-paid low-skill jobs are not likely to disappear with economic growth; while markets for physical capital and monetary wealth are now relatively well-developed in many parts of the world, markets for human capital (where a person holds his low-skill job while renting his human capital to a person in a high-skill job) do not seem likely to emerge.

Because the capitalists are also supplying labor effort, increases in the marginal product of labor (which are results of capital accumulation) cannot help in closing the gap between rich and poor.[24] Even with a human capital interpretation of model wealth, where a is interpreted more broadly as both human and non-human capital, there is a component of raw labor (ℓ) of which the rich get the same reward for any wage increase as the poor do.

4.2 Social policy

Redistribution schemes and poverty alleviation

In the model presented here, wages in the long run follow from capital density, and capital density in the long run follows from preferences. Hence, there is a unique equilibrium in the model in terms of the conditions of the poor. The number of poor, however, is not uniquely determined, and in that sense one

[24]The minimum between-class inequality was given in Equation (12). The only way to have wage increases lowering inequality would be to have several types of labor with imperfect substitutability (such as high- and low-skill labor).

could envision transfers that would elevate some workers to capitalist status permanently. In addition, any transfer will improve welfare in the short run even if they have no effect in the long run. It follows that in order to work, poverty alleviating policies need to be in force continuously, for example in the form of tax-and-transfer schemes.

As an alternative to tax-and-transfer schemes, one could consider other policies such as regulating the wage level. Assuming that this could be done and enforced properly, which is not a trivial matter, this would largely work in the same way as taxes; wage costs would go up, leading to reduced capital accumulation.

Unemployment

The main body of this paper talks about the "working poor". The working class earn a wage, but due to the institutional framework they do not get capital income in equilibrium. Many polarized countries also face high unemployment. For example, South Africa had an unemployment rate of 23 % in 2008 (UN HDI database).

There is no explicit subsistence income in the model. However, if the equilibrium wage is at a very low level — corresponding to a low equilibrium capital density — one could imagine that employers would get productivity gains by increasing wages, for example through workers eating better. This higher wage would then cause a "job lottery" and unemployment would rise. This is similar to the mechanisms in Harris & Todaro (1970), and would in this model be explained through low savings propensities (low β). These could again be caused by high systemic uncertainty — risk of expropriation, war or disease — increasing probability of a future state where current saving does not matter. Such systemic uncertainty would effectively work like a lower discount rate, with a reduced focus on the future.

Lowering systemic risk would lead to increased capital accumulation, increasing the wage rate. A higher wage rate, closer to subsistence income, would mean lower unemployment rates. It would, however, not translate into a less stratified society.

The third class: neither capitalists nor workers

In the model, "workers" work full-time for wages and have no capital income, and "capitalists" own firms and get income from them. It should not be difficult for the reader to conjure images of members of these groups, be it in eighteenth-century England or twentieth-century Uganda. But who are the "independents"?

The independent group in the middle of the wealth distribution can be "intermediate" in the sense of human capital, physical capital or land, or a combination of all these factors. For this reason, the definition of capital in the model has been intentionally vague. Starting with a predominantly agricultural economy, land and farm capital (animals, seeds etc.) are the most important elements in production. Independents would hence be self-sufficient farmers; a group with great heterogeneity. Improving labor markets would change the identity of poorer independents, as poor farmers seek employment on big farms or in cities. The group would also shrink at the top, as large farms could expand their production by hiring labor; the richer independents become capitalists. Hence, the decline in subsistence agriculture is consistent with the model.

The notion of "independent" in this model also has a counterpart in modern developed countries. While markets for physical capital are fairly well developed (the recent financial crisis notwithstanding), there are still no markets for renting human capital. Hence, many professional workers — doctors, lawyers, artists, others — organize their work independently, through self-employment or small partnerships. They can be contrasted from those holding large wealth, and hence owning big companies, and those who work in these big companies.

Prospects for redistribution and social reform are often analyzed as a one-dimensional policy choice; in the setting of this paper, it would be workers against capitalists.[25] In some cases, however, the interests of the independents may not be aligned with either group. For example, policies favoring large establishments and the formally employed might find the rich and poor

[25] See, for example, Galor & Moav (2006).

forming an alliance. A three-class population could be a useful addition to further studies in the political economy of labor market institutions.

4.3 The timing of institutional change

The factor market theory of economic development outlined in Section 3 contains two major revolutions: introductions of the labor markets first, and then introduction of capital markets. The timing of these have important implications for the evolution of inequality. While in Europe both can be seen as the results of internal processes, the rise of Europe and later colonialism imposed different economic regimes as economic shocks in other parts of the world. Hence, the state of the economy at the point of market introduction probably differed, as did the time between institutional reforms.

One example is the European colonization of Africa. The imposition of “modern”, capitalist economies is likely to have fueled a great increase in inequality, even if labor was not forced or otherwise unfree, as detailed by the mechanisms in this model. Moreover, capital market access for the broader population did not follow; consequently, poor people had no incentive to save. This story is complementary to the “extractive institutions” discussed in, for example, Acemoglu *et al.* (2001). With the mechanisms outlined here, the colonial institutions would not be bad *per se*. However, prolonged periods of labor markets but no capital markets would increase polarization, potentially leading to conflict or other bad institutional equilibria.

5 Concluding remarks

The model has accounted for one reason why inequality increased over the centuries leading up to and including the Industrial Revolution. Indivisibilities in labor, combined with missing capital markets, lead to a large group of the population not holding wealth in the long run, instead choosing wage work for themselves and their descendants. While the standard mechanisms of the neoclassical growth model operate for the richer part of the wealth distribution, they do not apply to the poor.

The results are different from other studies in several dimensions. First, in contrast to two-sector models of industrialization, the poor and rich groups produce together, and are defined by their interaction with each other. Second, compared to previous class difference studies like Banerjee & Moll (2010) and Banerjee & Newman (1993), there are no increasing returns in the production technology itself, giving straightforward aggregation properties and easy comparison to the neoclassical growth model. Third, comparing the results to the literature on misallocation (Buera & Shin, 2010), the trade-off between efficiency and inequality effects of misallocation are highlighted. In the institutional setting of labor markets and no capital markets, misallocation disappears in the long run, but only because the constrained agents stop holding capital.

Improved capital market access for the poor is an often-proposed policy; see, for example, Collins *et al.* (2009). However, the success of such a reform depends on the existing income distribution. If integrated labor markets have existed for a long time, class differences in the population can limit the effect of such reforms.

Another policy implication of the model is that welfare effects of transfers to the poor may not be persistent if the class gap is large. Without capital markets, we have a neoclassical ("Solow") economy in the aggregate, but class differences with fundamentally different environments for rich and poor ("Marx") are persistent.

A Appendix

A.1 Notes to figures

A.1.1 Note to Figure 1

Of the twenty-eight Gini coefficients given by Milanovic *et al.* (2011, p. 263), only three countries have more than one pre-industrial observation. The series labeled "UK" is denoted "England and Wales" for the pre-industrial observations. The series labeled "Netherlands" is denoted "Holland" for 1561 and 1732. The series labeled "India" is denoted "Moghul India" in 1750 and "British India" in 1947.

A.1.2 Legend for Figure 2

The country groups labeled in Figure 2 are defined by Bourguignon & Morrisson (2002). In the figure, the following abbreviations are used:

`Lat`: "37 Latin American countries", `Asi`: "45 Asian countries", `Afr`: "46 African countries", `Arg`: "Argentina-Chile", `Can`: "Australia-Canada-New-Zealand", `Aut`: "Austria-Czechoslovakia-Hungary", `Pak`: "Bangladesh-Pakistan", `Bel`: "Benelux-Switzerland-Micro-european states", `Bra`: "Brazil", `Gre`: "Bulgaria-Greece-Rumania-Yugoslavia", `Chi`: "China", `Col`: "Colombia-Peru-Venezuela", `Ken`: "Cote d'Ivoire-Ghana-Kenya", `Egy`: "Egypt", `Fra`: "France", `Ger`: "Germany ", `Ind`: "India", `Ine`: "Indonesia", `Ita`: "Italy", `Jap`: "Japan", `Kor`: "Korea-Taiwan", `Mex`: "Mexico", `Nig`: "Nigeria", `NAf`: "North-Africa", `Phi`: "Philippines-Thailand", `Pol`: "Poland", `Spa`: "Portugal-Spain", `Rus`: "Russia", `Swe`: "Scandinavian countries", `SAf`: "South-Africa", `Tur`: "Turkey", `UK`: "United Kingdom-Ireland", `USA`: "United States"

A.2 Model calculations and propositions

A.2.1 Calculating the long run wage and capital level

Denoting long-run variables by and LR subscript, we get

$$\beta((1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha(w_{LR})^{-\frac{1-\alpha}{\alpha}}+(1-\delta))\frac{u'(c_{LR})}{u'(c_{LR})}=1$$
$$(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha(w_{LR})^{-\frac{1-\alpha}{\alpha}}+=\beta^{-1}-(1-\delta)$$

$$(w_{LR})^{\frac{1-\alpha}{\alpha}}=\frac{(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha}{\beta^{-1}-(1-\delta)}$$
$$w_{LR}=\frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{(\beta^{-1}-(1-\delta))^{\frac{\alpha}{1-\alpha}}}$$

This is denoted as Equation (11) in the main text. Combined with Equation (7), we get

$$(1-\alpha)\left(\frac{A_{C,}}{L_{C,t}+L_{W,t}}\right)^{\alpha}=\frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{(\beta^{-1}-(1-\delta))^{\frac{\alpha}{1-\alpha}}}$$
$$\left(\frac{A_{C,t}}{L_{C,t}+L_{W,t}}\right)^{\alpha}=\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{(\beta^{-1}-(1-\delta))^{\frac{\alpha}{1-\alpha}}}$$

$$A_{C,t}=\frac{\alpha^{\frac{1}{1-\alpha}}}{(\beta^{-1}-(1-\delta))^{\frac{1}{1-\alpha}}}(L_{C,t}+L_{W,t}) \tag{13}$$

From Proposition 1 it is known that $L_C+L_W=1$ in steady state, giving Equation (10).

A.2.2 Handling technological growth

Technological growth is constant, deterministic and characterized by $Z_t=Z_0(1+g)^t$, where g is the period-to-period growth rate. Without loss of generality, $Z_0=1$.

Define technology-deflated consumption $\hat{c}_t\equiv\frac{c_t}{Z_t}=c_t(1+g)^{-t}$, and correspondingly deflated values of a, w, k.

The utility function is still given by (1). Using the definition of $\hat{c}$ from above, the maximand is equivalent to

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} u((1+g)^{\tau} \hat{c}_{\tau}) \tag{14}$$

$$\tag{15}$$

Denoting as ψ the degree of homogeneity of the per-period utility function $u(\cdot)$, one gets

$$\beta^{-t} \sum_{\tau=t}^{\infty} \left(\beta^{\tau} (1+g)^{\tau\psi} u(\hat{c}_{\tau})\right)$$
$$\beta^{-t} \sum_{\tau=t}^{\infty} \left(\beta^{\tau} \left((1+g)^{\psi}\right)^{\tau} u(\hat{c}_{\tau})\right)$$
$$\beta^{-t} \sum_{\tau=t}^{\infty} \left(\left(\beta(1+g)^{\psi}\right)^{\tau} u(\hat{c}_{\tau})\right)$$

It follows that if $\hat{\beta} \equiv \beta(1+g)^{\psi}$, the equation analogous to (1) is

$$\max_{\{c_{i,\tau}\}_{\tau=t}^{\infty}} E_t \sum_{\tau=t}^{\infty} \hat{\beta}^{\tau-t} u(\hat{c}_{i,\tau}) \tag{1'}$$

and the dynamics work similarly. It also follows that the upper bound on β is $(1+g)^{-\psi}$, which is greater than 1 for $\psi < 0, g > 0$. (This corresponds to a upper bound of $\hat{\beta}$ of 1).

For the CRRA utility specification $u(c) = \frac{c^{1-\eta}}{1-\eta}$ used in Section 3, the homogeneity parameter is $\psi = 1 - \eta$.

The dynamics of the model are then fulfilled by replacing β by $\hat{\beta}$ in all equations. The intra-temporal equations are found by inserting for technology-adjusted levels:

$$y_{i,t} = (k_{i,t})^{\alpha}(Z_t l_{i,t})^{1-\alpha} \tag{2'}$$

$$\hat{y}_{i,t} = (\hat{k}_{i,t})^{\alpha}(l_{i,t})^{1-\alpha} \tag{2''}$$

For the intertemporal budget constraint, there is an additional technology adjustment (note that $Z_{t+1} = (1+g)Z_t$):

$$\frac{c_{i,t}}{Z_t} + \frac{a_{i,t+1}}{Z_t} = m(a_{i,t}, s_{i,t}, w_t) \tag{3'}$$

$$\hat{c}_{i,t} + (1+g)\hat{a}_{i,t+1} = m(\hat{a}_{i,t}, s_{i,t}, \hat{w}_t) \tag{3''}$$

The income functions all have the property that $m(\hat{a}, s, \hat{w}) = \frac{m(a,s,w)}{Z}$; the occupational choice is not distorted.

$$m(a_{i,t}, \text{worker}, w_t) = w_t + \nu a_{i,t} \tag{4'}$$

$$m(\hat{a}_{i,t}, \text{worker}, w_t) = \hat{w}_t + \nu \hat{a}_{i,t} \tag{4''}$$

$$m(a_{i,t}, \text{independent}, w_t) = a_{i,t}^{\alpha} Z_t^{1-\alpha} + (1-\delta)a_{i,t} \tag{5'}$$

$$m(\hat{a}_{i,t}, \text{independent}, \hat{w}_t) = \hat{a}_{i,t}^{\alpha} + (1-\delta)\hat{a}_{i,t} \tag{5''}$$

$$m(a_{i,t}, \text{capitalist}, w_t) = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha Z_t^{\frac{1-\alpha}{\alpha}} w_t^{-\frac{1-\alpha}{\alpha}} a_{i,t} + w + (1-\delta)a_{i,t} \tag{6'}$$

$$m(\hat{a}_{i,t}, \text{capitalist}, \hat{w}_t) = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha \hat{w}_t^{-\frac{1-\alpha}{\alpha}} \hat{a}_{i,t} + \hat{w}_t + (1-\delta)\hat{a}_{i,t} \tag{6''}$$

The within-period-equilibrium wage:

$$w_t = (1-\alpha) Z_t^{1-\alpha} \left(\frac{A_{C,t}}{L_{C,t} + L_{W,t}} \right)^{\alpha} \tag{7'}$$

$$\hat{w}_t = (1-\alpha) \left(\frac{\hat{A}_{C,t}}{L_{C,t} + L_{W,t}} \right)^{\alpha} \tag{7''}$$

The technology-adjusted expression for intertemporal decisions is

$$\beta \frac{\partial m(\hat{a}_{i,t}, \hat{w}_t)}{\partial \hat{a}_{i,t}} (1+g) \frac{u'(c_{i,t+1}/Z_t)}{u'(c_{i,t}/Z_t)} \leq 1$$
$$\beta \frac{\partial m(\hat{a}_{i,t}, \hat{w}_t)}{\partial \hat{a}_{i,t}} (1+g) \frac{u'(\hat{c}_{i,t+1}(Z_{t+1}/Z_t))}{u'(\hat{c}_{i,t})} \leq 1$$
$$\beta \frac{\partial m(\hat{a}_{i,t}, \hat{w}_t)}{\partial \hat{a}_{i,t}} (1+g) \frac{u'(\hat{c}_{i,t+1}(1+g))}{u'(\hat{c}_{i,t})} \leq 1$$

As the utility function is homogeneous of degree ψ, the derivatives are homogeneous of degree $\psi - 1$, giving

$$\beta \frac{\partial m(\hat{a}_{i,t}, \hat{w}_t)}{\partial \hat{a}_{i,t}} (1+g)(1+g)^{\psi-1} \frac{u'(\hat{c}_{i,t+1})}{u'(\hat{c}_i, t)} \leq 1$$
$$\beta \frac{\partial m(\hat{a}_{i,t}, \hat{w}_t)}{\partial \hat{a}_{i,t}} (1+g)^{\psi} \frac{u'(c_{i,t+1})}{u'(c_i, t)} \leq 1$$

Again defining $\hat{\beta}$ as $\beta(1+g)^{\psi}$, optimal intertemporal decisions in equilibrium are

$$\hat{\beta} \frac{\partial m(\hat{a}_{i,t}, \hat{w}_t)}{\partial \hat{a}_{i,t}} \frac{u'(\hat{c}_{i,t+1})}{u'(\hat{c}_i, t)} \leq 1 \qquad , = 1 \text{ if } \hat{a}_{i,t} > 0 \tag{8'}$$

Long run wage

$$\hat{w}_{LR} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{(\hat{\beta}^{-1} - (1-\delta))^{\frac{\alpha}{1-\alpha}}} \tag{11'}$$

Minimum inequality

$$\hat{\Omega} = \frac{1}{1-\alpha} - \frac{\delta}{(1-\alpha)} \left(\hat{\beta}^{-1} - (1-\delta)\right)^{\alpha} \tag{12'}$$

As the model can be formulated in a parallel way with technological growth, it follows that all the result for constant technology hold, with the variables replaced by their technology-adjusted counterpart and the discount rate β replaced by $\hat{\beta}$.

A.2.3 Proof of Proposition 1

Savings decisions follow from Equation (8). Consider the steady state.

A worker with strictly positive wealth faces the condition

$$\frac{u'(c_{i,t+1})}{u'(c_{i,t})} = \frac{1}{\beta\nu}$$

Hence, for $\nu < \frac{1}{\beta}$, the right hand side will be greater than 1, meaning that consumption should decrease over time. For workers with wealth of zero, (8) holds as an inequality with constant consumption.

For an agent with wealth above a_c, the steady-state wage is defined from (8), giving stable consumption for everyone in this group.

For independents, $\frac{\partial m(a_{i,t},w_t)}{\partial a_{i,t}}$ is decreasing in $a_{i,t}$ (from (5)). Hence, $\beta\frac{\partial m(a_{i,t},w_t)}{\partial a_{i,t}}$ will be larger than 1 for $a < a_c$, leading to increased saving over time until $a \geq a_c$.

A.2.4 Calculation of minimum inequality

From Proposition 1, in steady state, the poorest capitalist has a wealth of minimum a_c, while workers have a wealth of 0. It follows that the incomes are

$$
\begin{aligned}
m_C &= m(a_c, w_{LR}) - a_c \\
&= (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha w_{LR}^{-\frac{1-\alpha}{\alpha}} \cdot w_{LR}^{\frac{1}{\alpha}} (1-\alpha)^{-\frac{1}{\alpha}} + w_{LR} - \delta w_{LR}^{\frac{1}{\alpha}} (1-\alpha)^{-\frac{1}{\alpha}} \\
&= \frac{\alpha}{1-\alpha} w_{LR} + w_{LR} - \delta w_{LR}^{\frac{1}{\alpha}} (1-\alpha)^{-\frac{1}{\alpha}} \\
&= \frac{w_{LR}}{1-\alpha} - \delta \left(\frac{w_{LR}}{1-\alpha} \right)^{\frac{1}{\alpha}}
\end{aligned}
$$

$$
\begin{aligned}
m_W &= m(0, w_{LR}) - 0 \\
&= w_{LR}
\end{aligned}
$$

The consumption (net income) ratio is then

$$
\begin{aligned}
\frac{m_C}{m_W} &= \frac{\frac{w_{LR}}{1-\alpha} - \delta \left(\frac{w_{LR}}{1-\alpha} \right)^{\frac{1}{\alpha}}}{w_{LR}} \\
&= \frac{1}{1-\alpha} - \delta \left(\frac{1}{1-\alpha} \right)^{\frac{1}{\alpha}} w_{LR}^{\frac{1-\alpha}{\alpha}}
\end{aligned}
$$

Insert for the steady-state wage from (11) to get the expression in Proposition 4.

A.2.5 Proof of Lemma 2

Consider, for a given wage path $\mathbf{w}$, two agents with wealth a_H and a_L at time t, with $a_H > a_L$. Assume the corresponding utility-maximizing choices for the two agents (the $t+1$ wealth level) to be a'_H and a'_L. For these to be optima, it must be the case that

$$
\begin{aligned}
u(m(a_L, w_t) - a'_L) + \beta V(a'_L, \mathbf{w}) &\geq u(m(a_L, w_t) - a'_H) + \beta V(a'_H, \mathbf{w}) \\
u(m(a_H, w_t) - a'_H) + \beta V(a'_H, \mathbf{w}) &\geq u(m(a_H, w_t) - a'_L) + \beta V(a'_L, \mathbf{w})
\end{aligned}
$$

Reordering gives

$$u(m(a_L,w_t)-a'_H)-u(m(a_L,w_t)-a'_L) \leq \beta V(a'_L,\mathbf{w})-\beta V(a'_H,\mathbf{w})$$
$$u(m(a_H,w_t)-a'_H)-u(m(a_H,w_t)-a'_L) \geq \beta V(a'_L,\mathbf{w})-\beta V(a'_H,\mathbf{w})$$

Combining gives

$$u(m(a_L,w_t)-a'_H)-u(m(a_L,w_t)-a'_L) \leq u(m(a_H,w_t)-a'_H)-u(m(a_H,w_t)-a'_L)$$

This inequality compares the utility difference between the two choices for the two agents. From the optimality assumptions, the utility gain in picking a'_H over a'_L for Low must be lower (in absolute terms) than for High. Due to the concavity of u, the utility difference between choices is greater the lower u is, meaning that the inequality will only hold if both sides are non-positive, implying that $u(m(a_L,w_t)-a'_H) \leq u(m(a_L,w_t)-a'_L)$ giving $a'_H \geq a'_L$. This proof holds without any assumptions on V.

A.3 Calibration

A.3.1 Initial distribution

The initial distribution is assumed to be a truncated Pareto distribution of the form

$$f(x)=\frac{qb^q x^{-q-1}}{1-\left(\frac{b}{c}\right)^q} \qquad \text{if } b<x<c \text{, 0 otherwise} \tag{16}$$

where b is the lower bound, c is the upper bound and q is a shape parameter.

The **upper bound** of wealth follows from the model; in autarky, steady-state wealth is maximized at $a^*=\left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}$. The upper bound c is set to this level.

The remaining two parameters are set from Campbell (2008), Table 17, p. 940, taking "land" as a proxy for wealth.

The **mean wealth in the initial period** is set to 1/7 of the upper bound, in accordance with the difference between the mean of the richest group and the mean of the population.

The **dispersion in the initial period** is set so that the 90/10 ratio of the initial wealth distribution equals the 90/10 wealth ratio of Campbell's social table.

To sum up, the parameter c is set to match the upper bound, then the (unique) combination (q, b) are chosen to match the mean and 90/10 ratio. The calibration of (q, b) is done numerically. Uniqueness of the result is verified by visually exploring a plot of calculations for a large range of (q, b) values.

A.3.2 Discount rate

The discount rate is calibrated to match the growth rate of aggregate GDP from 1500 to 1900 (hypothesizing a steady state of the pre-modern economy in 1900 if other institutions and innovations had not come along). As mentioned in the text, the model has two mechanisms for growth: reallocation and capital accumulation. In the pre-market period (Stage 1), misallocation gives a lower GDP than wealth alone should imply. The misallocation effect is found by calculating production for the initial distribution under autarky. Letting $f(x)$ be the distribution function discussed in the previous paragraph, initial income is

$$y_0 = \int_b^c f\left(x^\alpha - \delta x\right) dx \tag{17}$$

Let y_e denote end production under complete markets; then end capital is found by

$$y_e = k_e^\alpha - \delta k_e \tag{18}$$

Setting $y_e/y_0 = 6$ — the total GDP growth over the period, as given by Maddison (2010) — and finding y_0 by Equation (17), determines the corresponding k_e, the aggregate steady-state capital.

From the steady state capital in 1900, the parameter β is found from Equation (10).

A.4 Solution method

Discrete state space. Discretize the state space for individual wealth a, with a large number of discrete points (here $2^{15}+1 = 32769$ points are used). Denote this grid $\mathcal{A}$. An initial wealth distribution is defined over this space, as described in the Calibration section.

Defining the long run. Impose that in the long run the wealth of each group should be constant. This follows from the discussion on steady states above. Following Auerbach and Kotlikoff (1987) as referred in Heer & Maussner (2009), I set this "long run" as starting at $t = T$, with T being a sufficiently large number; here $T = 500$ is used (and it is verified that changes in T do not affect the result).

Initial and end conditions. The wage in the initial period follows from the initial distribution. The wage in the end period follows from Equation (11). Guess a wage path $\bar{w} \equiv \{w_t\}_{t=0}^{T}$ between these points.

Iterative solution. For a given guess $\bar{w}$:

- Calculate the long run utility $\sum_{t=T}^{\infty} \beta^t u(m(a, w_{LR}) - a)$ for all $a \in \mathcal{A}$
- For each period $t < T$, calculate optimal a_{t+1} at all points in the grid: $\max_{a_{t+1}} u(m(a_t, w_t) - a_{t+1}) + \beta V(a_{t+1}, \mathbf{w})$ [26]

[26] As the derivative of the income function m(a,w) is not monotonous, conventional optimizing solutions cannot be used. However, the result that optimal wealth paths cannot cross speeds up the solution. Using a wealth space of $2^N + 1$ points, I first calculate the optimal a_{t+1} for the wealthiest agent (denote this a_{t+1}^H), evaluating all $2^N + 1$ possible values. Then, for the poorest agent, only the values equal to or less than a_{t+1}^H need be evaluated. Next, the middle agent is evaluated, again with upper and lower bounds given by the two previous calculations. In this fashion the calculation is divided into ever smaller decision spaces by dividing the grid in a similar fashion N times until the optimal decision at any point is found.

- When all optimal a_{t+1} at all points in the grid have been found, simulate the path of all agents in the initial distribution from $t = 1$ to $t = T$ and keep track of the size of each group and capital stock$(L_W, L_I, L_C, A_W, A_I, A_C)$ in each period

- Calculate the wage in each period using Equation (7) and use this to update the wage guess $\bar{w}$

Iterate on this until the initial and updated wage path guess are sufficiently close. The convergence criterion used is that the largest single difference between the wage guess and the updated wage should be less than 0.0001.

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