China's Savings Multiplier*

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Abstract

China's growth is characterized by massive capital accumulaton, made possible by high and increasing domestic savings. In this paper we develop a model with the aim of explaining why savings rates have been high and increasing, and we investigate the general equilibrium effects on capital accumulation and growth. We show that increased savings and capital accumulation stimulates further savings and capital accumulation, through an intergenerational distribution effect and an old-age requirement effect. We introduce what we term the savings multiplier, and we discuss why and how the one-child policy has stimulated growth.

Keywords: Overlapping generations, Growth, Savings.

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1 Introduction

Real GDP in China has has grown by almost 10% per year since 1978 (Xu 2011). The high and sustained growth in China is characterized by massive capital accumulation. More than 40% of GDP has been invested over the last years. The high investments have been made possible by high and increasing savings, where presently more than 50% of GDP is saved. Unlike in the Asian tigers, domestic savings have exceeded domestic investments.

The high savings rate is the sum of high corporate savings and high household savings. High corporate savings can be explained by capital market imperfections, where profitable firms have financed their investments by retained profits (Song, Storesletten and Zilibotti 2011). A number of papers have investigated the maybe most puzzling fact, namely that households have increased their savings rate, despite being quite poor, having fast income growth, and receiving low returns on their savings. At present, houshold savings is the single largest component of total savings, and according to Yang (2012), the increase in the rate of household savings from 2000 to 2008 is also the most important contribution to the overall increase in the Chinese savings rate in the same period.

Several mechanisms have been proposed to explain the high and increasing household savings. Kraay (2000, p. 546) points out that that households "once covered by generous cradle-to-grave benefits through employment in state enterprises, are finding their futures increasingly uncertain", and most studies see the lack of a public welfare system as key to key to explain household savings patterns. As argued by Modigliani and Cao (2004), a main effect of the one-child policy decided in 1978 was to strengthen the needs to save for retirement. Blanchard and Giavazzi (2006, p. 7) similarly argues that "The high savings rate reflects a high level of individual risk, related to health costs, retirement and the financing of education", and show how the share of health spending that households pay themselves has increased from 16% in 1980 to 61% in 2001. Chamon and Prasad (2010) find that, among the households in their sample, expenditures on health and education grew from 2% of consumption expenditures in 1995 to 14% in 2005. They argue that the increased savings rates are (p. 93) "best explained by the rising private burden of expenditures on housing, education and health care". Song and Yang (2010) argue that a main reason for the increasing household savings rate is a change in the composition of income, where the income profile has flattened, so that the young workers earn a higher fraction of income than before, and, since the young have a high propensity to save, this increases aggregate savings. Wei and Zhang (2011) point to the rising number of boys relative to girls born (due to selective abortion), and find that parents of a boy save in order to increase the attractinveness of their son in the marriage market so as to increase the probability he finds a wife. This savings motive, in turn, spills over to other households, increasing savings further. Although these papers put different weight on different mechanisms, there seems to be some consensus in the literature that the high and increasing savings reflects precautionary motives, and that these are strenghthened by the sharp decrease in state enterprises, the missing welfare

Table 1: As elderly, what will you rely on? Do you worry ?

kids pool	1 girl	1 boy	2 girl	1 boy 1 girl	2 boys
rely on own savings	40.2	36.5	37.4	32.2	31.3
rely on children	44.9	50.0	54.7	61.8	63.7
"Yes I worry"	40.1	31.3	43.6	30.8	32.9

Table 2: Self reported reason for savings

age	≤ 44	45-54	≥ 55	all
children related	89.5	77.8	55.9	78.2
build house	21.6	18.4	11.2	18.2
retire	33.8	50.0	68.8	47.0
medical	11.4	18.4	35.0	18.9

system, the one-child policy, and the increased need to provide for own retirement and old-age care.

Such views can also be confirmed by household surveys. Table 1 and Table 2 shows the responses to a houshold survey carried out in 2001. In the first table parents with one ore two children are asked what funding they will rely on as elderly. They are aslo asked whether they worry.about becoming old. We see that parents with at least one boy worry least. We also see that parents with two boys rely on their children on a larger extent than those with only one girl. Table 2 shows frequencies of the **two** most important reasons for savings. The noteworthy pattern is that children related savings declines with age of head of household while medical and retirement reasons increases.with age of household head

Ma and Yi (2010) discuss effects of the rapid ageing process in China, pointing out that growth in labor supply will slow sharply, and as documented by e.g. Li et al. (2012) wage growth already exceeds GDP growth, and has done so since at least 1998. There thus seems to be a shift in the income distribution towards workers (and as argued by Song and Yang (2010), towards the young). Moreover, China has already reached a level of development where the share manufacturing employment out of total (paid) employment is decreasing, while that of service sectors is increasing. According to the ILO database, from 1993 to 2008 the share of workers in manufacturing decreased from 37% to 29%, whereas the employment share of e.g. health and social work increased from 2.8% to 4.7%.

In this paper we take as a starting point the studies investigating the high and increasing household savings in China. A main contribution of our paper is to investigate the general equilibrium macro implications for savings, capital accumulation, and structural change, from these micromotives for household savings. To do so we develop an OLG-model, extended to take into account that agents need to save for old-age care, and that this care is no longer exclusively provided by own childern or state enterprises, but must be purchased in the market. We show how high savings and capital accumulation gives rise to increased savings and further capital accumulation. In particular, savings react to increased future wages, pushing capital and future wages even further up. Thus, there is what we term a savings multiplier in the model. We believe this to be key to understand important characteristics of growth and structural transformation in China.

In addition to relating to the literature on savings and growth in China referred above, our paper also relates to the debate on "communist growth". According to Acemoglu and Robinson (2012), growth in China has important similarities with growth in the former Soviet Union, based on high savings and massive capital accumulation, but being unsustainable if institutions are not reformed to be more inclusive. In fact, investments rates in China and the former Soviet Union are at similar levels, both exceeding 40% of GDP. In the Soviet Union the mobilization of the high required savings rates was achieved partly with the suppression and collectivization of agriculture. Clearly this strategy to increase savings is not important in present day China. Our paper points out why the one-child policy, and the dismantling of state enterprises without replacing them with a welfare system, may achieve the same.

The rest of the paper is organized as follows. In Section 2 we extend a traditional overlapping generations growth model, to take into account that when old, agents need care. When agents cannot rely on the state or their family to provide such care, they must be prepared to arrange it on their own. This puts them on the demand side of the labor market as old. We show the static equilibrium of the model in Section 3. In Section 4 we study transitional capital accumulation and growth, and introduce what we term the savings multiplier. We discuss how and why the steady-state capital stock in our model differs from standard OLG-models, and show how, again compared to standard OLG-models, the effects on capital accumulation from one-child policies are magnified. Section 5 extends the model with the introduction of a welfare state, then discusses dynamic inefficiency in Subsection 5.2, before extending the model to study endogenous growth in Subsection 5.3. Section 6 concludes.

2 The Model

In this section, we develop our model of savings and growth based on an overlapping-generations (OLG) structure that takes into account that when old, agents are in different needs from when young. In particular, motivated by the one-child policy and also the modernization of Chinese society, parents can rely less on their children to provide old-age care and less on state firms to act as a substitute for a welfare state. Thus, differently from the standard OLG framework pioneered by Diamond (1965) – henceforth called the *canonical one-good model* – we separate between the production of goods and the production of care. One set of firms produces a *generic good*, used for investments and for consumption of both young and old agents. The second set of firms provides specific services to old agents, which we interpret as *old-age care*. The production of care is more labor intensive than the production of the generic good, and labor is perfectly mobile between the two sectors. The crucial departure from the canonical one-good model is that young agents foresee their future needs for old-age care, and therefore adjust their saving behavior on the basis of the expected cost of services purchased in the second period of life. As we will see this introduces novel effects for the general equilibrium dynamics of savings and growth.

2.1 Households

We consider an overlapping-generations environment where each agent lives two periods $(t, t+1)$. Total population, denoted N_t, consists of N_t^y young and N_t^o old agents, and grows at the exogenous net rate $n > -1$;

$$N_t = N_t^y + N_t^o, \quad N_t^y = N_t^o \cdot (1+n), \quad N_{t+1} = N_t \cdot (1+n). \tag{1}$$

Households purchase two types of goods over their lifecycle: a generic consumption good and old-age care services. The generic good is consumed in both periods of life. Old-age care services, instead, are exclusively purchased by old agents. The canonical one-good model is therefore a special case of our model in which we exclude old-age care services from the analysis. The utility of an agent born at the beginning of period t takes the additive form

$$U_t \equiv u(c_t) + \beta \cdot v\left(d_{t+1}, h_{t+1} - \bar{h}\right), \tag{2}$$

where c_t and d_{t+1} represent consumption levels of the generic good in the first and second period of life, respectively, h_{t+1} is the amount of old-age care consumed when old, $\bar{h} \geqslant 0$ is the *minimum requirement* – i.e., the minimum amount of old-age care required by old agents – and $\beta \in (0,1)$ is the private discount factor between young and old age. A constraint of the consumer problem is that the minimum requirement is at least weakly satisfied,

$$h_{t+1} - \bar{h} \geqslant 0. \tag{3}$$

As is standard, we first study existence and uniqueness of interior equilibria where old-age care obeys (3). We then verify ex-post the conditions under which $h_{t+1} > \bar{h}$ holds.[1] The case where $\bar{h} = 0$, so that there is no minimum old-age care requirement, is of special interest. As we will see, this case transparently isolates what we term the intergenerational distribution effect in our model. For this reason, in Section 4 where we study the dynamics of the model, we first put emphasis on this case. We then turn to the more general case of $\bar{h} \geqslant 0$, in which what we term the old-age requirement effect is also present.

We assume that only young agents work, supplying inelastically one unit of homogeneous labor. The only source of income in the second period of life is interest on previous savings.

[1] In fact, in our main model which is the neoclassical case with constant returns to scale in generic-good production, there always exists a stable long-run equilibrium in which the allocation of labor between generic-good and health-care production exhibits stable shares consistent with the interior solution $h_{t+1} > \bar{h}$. We discuss cases where this may not be the case in Section 5.3 where we extend the model to allow for linear returns to capital at the aggregate level. Then, under certain conditions, the accumulation process may drive the economy towards long-run equilibria where labor is pushed away from the health-care sector so that the constraint $h_{t+1} - \bar{h} \geqslant 0$ becomes binding.

Personal lifetime income is entirely consumed at the end of the second period. Taking the consumption good as the numeraire in each period, the budget constraints read

$$c_t = w_t - s_t, \tag{4}$$

$$s_t R_{t+1} = d_{t+1} + p_{t+1} h_{t+1}, \tag{5}$$

where w_t is the wage rate, s_t is savings, R_{t+1} is the (gross) rate of return to saving, and p_{t+1} is the price of old-age care. Savings consist of physical capital, which as in the one good OLG model is homogeneous with the generic consumption good. Assuming full depreciation within one period, market clearing requires that aggregate capital at the beginning of period $t+1$ equals aggregate savings of the young agents in the previous period, $K_{t+1} = N_t^y s_t$.

In order to make our new mechanisms as transparent as possible, we consider a specific, yet flexible form of preferences:

$$u(c_t) \equiv \log c_t, \tag{6}$$

$$v\left(d_{t+1}, h_{t+1} - \bar{h}\right) \equiv \log\left[\gamma \cdot (d_{t+1})^{\frac{\sigma-1}{\sigma}} + (1-\gamma) \cdot \left(h_{t+1} - \bar{h}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

where $\gamma \in [0, 1]$ is a weighting parameter and $\sigma > 0$ is the elasticity of substitution between consumption goods and care services in the second period of life: d_{t+1} and h_{t+1} are strict complements if $\sigma < 1$, strict substitutes if $\sigma > 1$. In the limiting case $\sigma \to 1$, the term in square brackets reduces to the Cobb-Douglas form $(d_{t+1})^{\gamma}(h_{t+1})^{1-\gamma}$.

Assumptions (6)-(7) imply two fundamental properties. First, we can treat the canonical one-good model as a special case: letting $\gamma = 1$ (and $\bar{h} = 0$), old-age care services disappear from private utility and, hence, are not produced in equilibrium. Second, the utility functions (6)-(7) exhibit a unit elasticity of intertemporal substitution. This property allows us to describe the effects of old-age care on saving rates in the clearest way. Setting $\gamma = 1$, we obtain the logarithmic version of the canonical model, in which the saving rate is constant over time because consumption propensities are independent of the interest rate.[2] Hence, in the general case $0 < \gamma < 1$, any departure from constant saving rates in the model is exclusively due to the inclusion of old-age care services.

2.2 Production Sectors

Old-age care is labor intensive. In our framework this implies that the factor price of interest to old agents is not only the interest rate, but also the wage rate. This contrasts with standard one-good OLG models. There, old agents are on the supply side of the capital market, and the only relevant factor price when old is the (real) interest rate. In the present model, old agents are still on the supply side of the capital market, but since when old they need care, they are in addition on the demand side of the labor market. Therefore, unlike standard one-good

[2] More precisely, the savings rate of the young is constant with logarithmic preferences. When production is Cobb-Douglas, the income share of the young is constant, and thus also the aggregate savings rate is constant.

OLG models, the wage rate is not irrelevant for old agents. To clarify this, and to capture in a simple way that care is more labor intensive than the production of the generic consumption and investment good, we assume that care is produced with labor as the only factor of production, .

We denote by ℓ_t the fraction of workers employed in the generic sector, and by $1-\ell_t$ the fraction employed in the care sector. Perfect labor mobility and perfectly competitive conditions in the labor market ensure wage equalization in equilibrium. In the old-age care sector, there is a simple constant returns to scale production technology:

$$H_t \equiv \eta \cdot (1-\ell_t)\, N_t^y, \tag{8}$$

where H_t is the aggregate output of care services, and $\eta > 0$ is a constant labor productivity parameter.

In the generic good sector, we consider a specification displaying constant returns to scale at the firm level. A continuum of firms, indexed by $j \in [0, J]$, exploits the same Cobb-Douglas technology

$$x_t^j \equiv (k_t^j)^\alpha (a_t \ell_t^j N_t^y)^{1-\alpha} \text{ for each } j \in [0, J]\,, \tag{9}$$

where x_t^j is the output of the generic good produced by the j-th firm, k_t^j and $\ell_t^j N_t^y$ are the amounts of physical capital and labor employed at the firm level, $\alpha \in (0,1)$ is an elasticity parameter, and a_t is labor productivity in the generic-good sector.

2.3 Labor Productivity

Specification (9) assumes that the generic good technology displays constant returns to scale *at the firm level*, so that income shares are determined according to standard zero-profit conditions. In the main model of our paper we also make the standard neoclassical assumption of constant returns to scale at the *aggregate level*. We here impose that a_t equals an exogenous constant $B^{\frac{1}{1-\alpha}}$ in each period: the generic production sector exhibits strictly diminishing marginal returns to capital also at the aggregate level, and aggregate sectoral output $X_t \equiv Jx_t^j$ is given by

$$X_t = B \cdot (K_t)^\alpha \left(\ell_t N_t^y\right)^{1-\alpha} \tag{10}$$

where $K_t \equiv Jk_t^j$ is aggregate capital and $\ell_t \equiv J\ell_t^j$ is aggregate labor employed in the generic sector. This is the setup of the canonical model since the seminal work of Diamond (1965).

In Subsection 5.3 we extend the model to allow for endogenous growth in its simplest fashion. Following Romer (1989), we include learning-by-doing whereby the productivity of workers employed in the generic sector increases with the amount of capital that each of these workers uses. In this case the labor productivity is governed by the spillover function $a_t = A^{\frac{1}{1-\alpha}} \cdot K_t / \left(\ell_t N_t^y\right)$, where A is an exogenous constant. Since a_t is taken as given at the firm level, income shares are still determined by the usual zero-profit conditions, but aggregate sectoral output is proportional to aggregate capital:

$$X_t = AK_t. \tag{11}$$

We next describe the equilibrium conditions that hold independently of the assumed technology for the generic good, then put our main emphasis on the neoclassical case, before returning to the extension of the model to endogenous growth in Subsection 5.3.

3 Static Equilibrium

This section discusses the static equilibrium conditions holding in each period for a given stock of capital per worker. We first study the profit-maximizing conditions for firms, the utility-maximizing conditions for households, the labor market equilibrium, and the goods market equilibrium. We then study the joint (static) equilibrium of all the markets, the implications for the aggregate savings rate, and finally the implied mapping to capital accumulation.[3]

3.1 Firms

In the service sector for old-age care, the technology (8) implies that the wage equals the market price of services times the labor productivity,

$$w_t = p_t \eta. \tag{12}$$

Market clearing requires that total output of old-age care services matches aggregate demand by old agents, $H_t = N_t^o h_t$. The existence of a minimum requirement, $h_t \geqslant \bar{h}$, requires that total production H_t exceeds $N_t^o \bar{h}$, which implies a constraint on sectoral employment shares: using the production function (8), we obtain

$$\ell_t \leqslant \frac{\eta(1+n) - \bar{h}}{\eta(1+n)} \equiv \ell^{\max}, \tag{13}$$

where $\ell^{\max}$ is the maximum level of employment in the generic sector that is compatible with a level of old-age care output equal to the minimum requirement.[4] In the remainder of the analysis, we will work under the parameter restriction

$$\bar{h} \leqslant \eta(1+n), \tag{14}$$

which implies $\ell^{\max} \geqslant 0$. By construction, when the minimum requirement is $\bar{h} = 0$, we have $\ell^{\max} = 1$.

In the generic good sector each firm maximizes own profits $x_t^j - R_t k_t^j - w_t \ell_t^j N_t^y$ subject to technology (9). Denoting capital per young agent as $\kappa_t \equiv K_t / N_t^y$, the zero-profit conditions in the sector can be aggregated across firms and written as

$$w_t = a_t^{1-\alpha}(1-\alpha)(\kappa_t/\ell_t)^{\alpha}, \tag{15}$$

$$R_t = a_t^{1-\alpha}\alpha(\ell_t/\kappa_t)^{1-\alpha}. \tag{16}$$

[3] Unless otherwise specified, all equations in this section are valid in the neoclassical case as well as in the AK-case. Thus, to avoid repetitions when we extend the model to endogenous growth in Subsection 5.3, in the present section we continue to use a_t for the labor productivity, without specifying if growth is neoclassical or endogenous (when not necessary).

[4] Formally, the level of health-care output equal to the minimum requirement is $H_t^{\min} \equiv \eta \cdot (1 - \ell^{\max}) N_t^y = N_t^o \bar{h}$.

3.2 Consumers

Each agent maximizes (2) subject to the budget constraints (4)-(5). Denoting the derivative of the u-function with respect to c_t by u_{c_t}, and so on, the solution to this problem yields two familiar first order conditions; the Keynes-Ramsey rule, $u_{c_t} = \beta R_{t+1} v_{d_{t+1}}$, and an efficiency condition establishing the equality between the price of care services and the marginal rate of substitution with second-period generic goods consumption, $v_{h_{t+1}}/v_{d_{t+1}} = p_{t+1}$. Under preferences (6)-(7), we show in the Appendix that these conditions result in the following relationships.

The consumption and savings of young agents is given by

$$c_t = \frac{1}{1+\beta} \cdot \left(w_t - \frac{p_{t+1}}{R_{t+1}} \bar{h} \right) \text{ and } s_t = \frac{1}{1+\beta} \cdot \left(\beta w_t + \frac{p_{t+1}}{R_{t+1}} \bar{h} \right). \tag{17}$$

Note that when $\bar{h} = 0$, these expressions are equivalent to those in the simplest version of the canonical OLG model, where young agents save a constant fraction of their wage income, which is then used to provide old age consumption.[5] When $\bar{h} > 0$, individual decisions on c_t and s_t are no longer fixed proportions of young age income. Young age consumption is lower, and savings higher, the larger is $\bar{h}$. More interesting, the strength of the effect is related to the *future* relative factor price, since $p_{t+1}/R_{t+1} = \eta w_{t+1}/R_{t+1}$. A high future wage w_{t+1}, and low returns on savings R_{t+1}, implies that much must be saved today in order to purchase the minimum amount of care tomorrow. We term this the *old-age requirement effect*. The old-age requirement effect implies that future relative factor prices affect present savings.[6]

Turning next to generic consumption in the second period of life, each old agent purchases

$$d_t = (1+n) \left[\ell_t - (1-\alpha) \right] \cdot a_t^{1-\alpha} \left(\kappa_t / \ell_t \right)^{\alpha}, \tag{18}$$

which is the residual (per-old) output of the generic sector after consumption and savings of young agents have been subtracted. Result (18) implies that second-period consumption is positive only if $\ell_t > 1 - \alpha$, which, as we will see, always turns out to be the case in equilibrium.

Finally, the relative demand for old-age care links the old agents' expenditure shares over the two goods to their relative price:

$$\frac{p_t \cdot \left(h_t - \bar{h} \right)}{d_t} = \left(\frac{1-\gamma}{\gamma} \right)^{\sigma} \cdot p_t^{1-\sigma}. \tag{19}$$

[5] As we will return to, however, this does not imply that the dynamics are equivalent to the canonical OLG model. As we will see, these are quite different also in the case where $\bar{h} = 0$ due to our intergenerational income distributon effect.

[6] In particular, the feature that the future wage is relevant for individual consumption and savings decisions is in contrast to one-good versions of the OLG model, where the only future factor price relevant is the return to savings. Moreover, note that in general this feature is the result of old-age care in the model, and does not require $\bar{h} > 0$. For instance, with an intertemporal elasticity of substitution that falls short of one, a higher future wage would imply higher young age savings also in the case where $\bar{h} = 0$.

Finally, to preview some intuition, note that since the future wage affects young age savings, it is already clear at this stage that the general equilibrium dynamics will be quite different from one-good OLG models. For instance, higher future wages implies higher savings and thus higher future capital stock, in turn increasing future wages even more.

Expression (19) shows that the expenditure share of old agents on net health care, $h_t - \bar{h}$, increases (decreases) with the price when the two goods are complements (substitutes). The reason is that a ceteris paribus increase in p_t always reduces the 'physical consumption ratio' between net care and generic consumption, $\left(h_t - \bar{h}\right)/d_t$, but in the usual fashion the final effect on the 'expenditure ratio' $p_t\left(h_t - \bar{h}\right)/d_t$ depends on the elasticity of the relative demand for net care. Under complementarity, the demand is relatively rigid: if p_t increases, the price effect dominates the quantity effect and the expenditure share of net care increases. Under substitutability, instead, net old-age care demand is relatively elastic and the quantity effect dominates: an increase in p_t decreases the expenditure share of care. These substitution effects will imply that variations in the price of care have an impact on the labor allocation between the two production sectors.[7]

3.3 Labor Market

The labor demand schedules of both production sectors determine a unique equilibrium in the labor market. Combining (12) with (15), we obtain

$$p_t = (1/\eta)(1-\alpha)\, a_t^{1-\alpha}(\kappa_t/\ell_t)^{\alpha} \equiv \Phi(\ell_t, \kappa_t, a_t)\,. \tag{20}$$

Condition (20) establishes that, in equilibrium, the wage rate must be equalized between the two production sectors. In particular, (20) defines p_t as the level of the price of care ensuring equal wages between the two sectors for given levels of sectoral employment, capital per worker, and productivity.

The labor market equilibrium differs between the neoclassical case in our main model, denoted by $i = 1$, and the extension to the AK case in Subsection 5.3, denoted by $i = 2$. By substituting the relevant value of labor productivity a_t in each of the two cases, we obtain an expression for the labor market equilibrium in each case $i = (1, 2)$:

$$\Phi(\ell_t, \kappa_t, a_t) = \left\{ \begin{array}{ll} \Phi(\ell_t, \kappa_t; 1) = (B/\eta)(1-\alpha)(\kappa_t/\ell_t)^{\alpha} & \text{(Neoclassical)} \\ \Phi(\ell_t, \kappa_t; 2) = (A/\eta)(1-\alpha)(\kappa_t/\ell_t) & \text{(Linear AK)} \end{array} \right\}. \tag{21}$$

In each case $i = (1, 2)$, function $p_t = \Phi(\ell_t, \kappa_t; i)$ is strictly decreasing in ℓ_t; for a given capital per young κ_t, higher employment in the generic sector decreases the marginal productivity of labor, implying a lower wage, and thus a lower price of care.

3.4 Goods Markets

In the Appendix we show that solving the demand relationship (19) for the price of care, and substituting $p_t h_t/d_t$ with the market-clearing and zero-profit conditions holding for the producing

[7] As usual substitution effects only disappear with Cobb-Douglas preferences: when $\sigma = 1$, the expenditure shares of generic goods and old-age care are independent of the relative price, and are exclusively determined by the relevant preference parameter γ.

firms, we obtain

$$p_t = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\sigma-1}} \cdot \left[\frac{(1-\alpha)\left(\ell^{\max}-\ell_t\right)}{\ell_t-(1-\alpha)}\right]^{\frac{1}{1-\sigma}} \equiv \Psi\left(\ell_t\right). \tag{22}$$

This expression defines p_t as the price of care that ensures equilibrium in the goods market.[8] The most important insight of (22) is that the function $p_t = \Psi(\ell_t)$ is strictly decreasing when $\sigma < 1$, and strictly increasing when $\sigma > 1$. When $\sigma < 1$ the price of care is positively related to the employment share in the care sector $1 - \ell_t$. The reason is that a ceteris paribus increase in p_t increases the expenditure share old consumers devote to care services relative to generic consumption and, consequently, attracts labor in the care sector. When $\sigma > 1$, in contrast, a higher price of care means a lower expenditure share of care, and thus less labor in the care sector and more labor in the generic sector.[9]

3.5 Employment and Capital Co-Movements

Consider now the joint equilibrium of the markets for labor and for goods. The two relevant conditions, (21) and (22), imply that the price of health care and the employment shares of the two sectors in each period t depend on the level of capital per worker κ_t. Formally, in each case $i = (1,2)$ the employment share of the generic sector for a given level of κ_t, denoted by $\ell_t = \ell(\kappa_t)$, is the fixed point

$$\ell(\kappa_t) \equiv \arg\text{solve}_{\{\ell_t \in (1-\alpha, \ell^{\max})\}} \left[\Phi(\ell_t, \kappa_t; i) = \Psi(\ell_t)\right], \qquad i = (1,2). \tag{23}$$

Our assumptions guarantee the existence and uniqueness of this fixed point – a result that is shown in the Appendix and that can be verified in graphical terms in Figure 3. On the one hand, the function $\Phi(\ell_t, \kappa_t; i)$ is strictly decreasing in ℓ_t and exhibits positive vertical intercepts at the boundaries of the relevant interval $\ell_t \in (1-\alpha, \ell^{\max})$. On the other hand, the function $\Psi(\ell_t)$ is decreasing (increasing) under complementarity (substitutability) with limits

$$\begin{array}{l} \lim_{\ell_t \to 1-\alpha} \Psi(\ell_t) = \left\{ \begin{array}{cccc} \infty & \text{if } \sigma < 1; & 0 & \text{if } \sigma > 1 \end{array} \right\}, \\ \lim_{\ell_t \to \ell^{\max}} \Psi(\ell_t) = \left\{ \begin{array}{cccc} 0 & \text{if } \sigma < 1; & \infty & \text{if } \sigma > 1 \end{array} \right\}. \end{array}$$

These properties[10] ensure the existence and uniqueness of the fixed point $\Psi(\ell_t) = \Phi(\ell_t, \kappa_t; i)$, and that it is contained in the relevant interval $\ell \in (1-\alpha, \ell^{\max})$. The fixed point (23) simultaneously determines employment shares and the price of care, which is measured along the vertical axis of Figure 3. Substituting $\ell(\kappa_t)$ in $\Psi(\ell_t)$ we obtain the equilibrium price of care for given capital per worker,

$$p(\kappa_t) \equiv \Psi(\ell(\kappa_t)). \tag{24}$$

[8] Note that the term in square brackets only contains ℓ_t because, with Cobb-Douglas technologies, the sector allocation of labor alone determines the output ratio $X_t/p_t H_t$. If we deviate from Cobb-Douglas technologies, the term in square brackets would also contain capital employed in generic production: see the derivation of (22) in the Appendix.

[9] It should be noted that, in the special case of unit elasticity of substitution, $\sigma = 1$, expression (22) does not hold because price and quantity effects on the demand side balance each other. As a result, the equilibrium between demand and supply in the goods market is characterized by constant employment shares, with $\ell_t = \frac{(1-\alpha)(\gamma\ell^{\max}+1-\gamma)}{\gamma(1-\alpha)+1-\gamma}$ at each t.

[10] Along with the further concavity properties of both curves described in the Appendix.

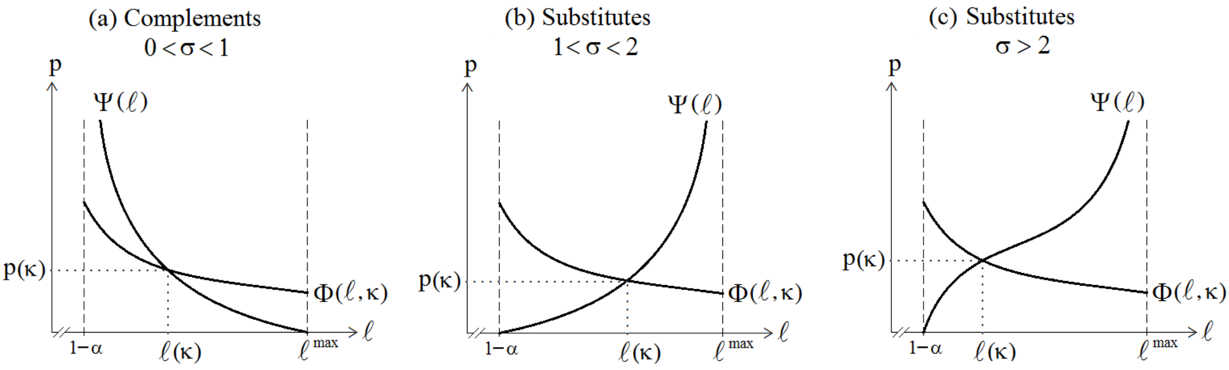

Figure 3: Determination of the equilibrium employment level ℓ_t for given κ_t according to condition (23) under complementarity ($\sigma < 1$) and substitutability ($\sigma > 1$). Qualitatively, the graphs do not change between the neoclassical case and the linear AK case. The case of strong substitution ($\sigma > 2$) implies concavity of $\Psi(\ell)$ for low values of ℓ but does not alter existence and uniqueness properties.

Even though we have not yet specified whether and how capital grows, result (24) clarifies how capital accumulation affects the price of care and employment shares:

Lemma 1 *An equilibrium trajectory with positive accumulation implies a rising price of care. Under complementarity the employment share in the generic sector is decreasing. Under substitutability the employment share in the generic sector is increasing;*

$$\kappa_t > \kappa_{t-1} \qquad \Longleftrightarrow \qquad p_t > p_{t-1} \text{ and } \{\ell_t < \ell_{t-1} \text{ if } \sigma < 1; \quad \ell_t > \ell_{t-1} \text{ if } \sigma > 1\}.$$

Lemma 1 is easily proved in graphical terms by means of a comparative-statics exercise. Because $\Phi(\ell, \kappa; i)$ is positively related to κ, a higher stock of capital per young implies an uprightward shift in the $\Phi(\ell, \kappa; i)$ curves in Figure 3. The new equilibrium price $p(\kappa)$ is higher in all cases but sectoral employment shares react differently depending on the value of the elasticity of substitution. The employment share of the generic sector $\ell(\kappa)$ increases (decreases) when $\sigma < 1$ ($\sigma > 1$). The intuition is that an increase in capital per young expands the production frontier of the generic good, and thereby increases the price of care. Under complementarity, old agents react to the price increase by raising the share of expenditure on net old-age care, which decreases the employment share in the generic sector $\ell(\kappa)$. Under substitutability, instead, old agents reduce the expenditure share on net care, and employment in the generic sector therefore grows. It is easily verified that the direction of these capital and employment co-movements is fully reversed when we consider an equilibrium trajectory with decumulation of capital per young – that is, when $\kappa_t < \kappa_{t-1}$.[11]

[11] Note that the results established in Lemma 1 hold in in both the neoclassical case 1 and the AK case 2 in our model: the co-movements of employment shares, price of health care and capital per worker are the same in both variants of the model.

3.6 Static Equilibrium Comparative Statics

For a given capital stock, the static equilibrium labor allocation depends on the parameters in the model. In particular, for later use we investigate how it depends on productivity B, on population growth n, and on the level of the minimum requirement $\bar{h}$. The properties of $\ell(\kappa_t) = \ell\left(\kappa_t; B, n, \bar{h}\right)$ are summarized in the following Proposition:[12]

Proposition 1 *In the static equilibrium with given* κ_t*,*

$$\frac{d\ell\left(\kappa_t; B, n, \bar{h}\right)}{dB} \equiv \ell'_B \; < 0 \quad \textit{if } \sigma < 1; \quad > 0 \quad \textit{if } \sigma > 1 \;\;, \tag{25}$$

$$\frac{d\ell\left(\kappa_t; B, n, \bar{h}\right)}{dn} \equiv \ell'_n > 0 \textit{ if } \bar{h} > 0 \quad (= 0 \textit{ if } \bar{h} = 0), \tag{26}$$

and

$$\frac{d\ell\left(\kappa_t; B, n, \bar{h}\right)}{d\bar{h}} \equiv \ell'_{\bar{h}} < 0 \textit{ if } \bar{h} > 0 \quad (= 0 \textit{ if } \bar{h} = 0). \tag{27}$$

Proof. See the Appendix. ■

A higher productivity B expands production possibilities of generic goods. When $\sigma < 1$, labor is pushed out of the generic sector, as consumers want to utilize the increased production possibilities to consume more services from the care sector. When $\sigma > 1$ in contrast, labor is drawn into the generic sector, since in this case old agents would like less care but more generic goods.

The effects of population growth and of minimum care requirement, instead, operate through the term $\ell^{\max} \equiv 1 - \frac{\bar{h}}{\eta(1+n)}$ that appears in (22). The intuition is that a higher rate of population growth n means that the share of young-age to old-age agents increases, shifting relative demand away from old-age care and towards generic goods. The labor share of generic goods then increases. A higher $\bar{h}$ has the opposite effect, since it increases the demand of old-age care.

3.7 Saving Rates and Accumulation

Before studying in detail the dynamics, it is instructive to describe the general relationships between saving rates, capital accumulation and sectoral employment shares. Considering the economy's aggregate income, the total labor share accruing to young agents is given by

$$\frac{w_t N_t^y}{X_t + p_t H_t} = \frac{a_t^{1-\alpha}(1-\alpha)(\kappa_t/\ell_t)^\alpha}{(\kappa_t)^\alpha (a_t \ell_t)^{1-\alpha} + w_t(1-\ell_t)} = \frac{1-\alpha}{1-\alpha\cdot(1-\ell_t)}, \tag{28}$$

where we have used the profit-maximizing conditions of both production sectors (see the Appendix). Equation (28) shows that, in static equilibrium, an increase in the generic sector employment share ℓ_t reduces the total income share of young agents. The intuition is that if labor moves from the care sector to generic production, the return to capital increases relative

[12] This proposition is also valid if the productivity term B from the neoclassical version of the model is replaced by the productivity term A in the AK version of the model.

to the wage rate. There is, thus, a shift in the income distribution away from the young towards the old. We term this effect the *intergenerational distribution effect*.

Since it is the young who save, the intergenerational distribution effect directly influences the economy's saving rate (and will, as we shall see, have important implications for capital accumulation). The savings rate, termed θ_t and defined as aggregate savings relative to the the total value of production, is found by using the saving function in (17) and expression (28), and then inserting for $\ell^{\max}$ from (13):

$$\theta_t \equiv \frac{N_t^y s_t}{X_t + p_t H_t} = \underbrace{\frac{\beta(1-\alpha)}{1+\beta}}_{\text{Canonical model}} \cdot \underbrace{\frac{1}{1-\alpha\cdot(1-\ell_t)}}_{\text{Intergenerational Distribution}} \cdot \underbrace{\left[1-\frac{(1-\alpha)\,\bar{h}}{\alpha(1+\beta)\,\eta(1+n)\cdot\ell_{t+1}}\right]^{-1}}_{\text{Old-age Requirement}}. \tag{29}$$

Expression (29) is a semi-reduced form showing that the savings rate is negatively related to both ℓ_t and ℓ_{t+1}. Again, to explain the intuition is it instructive to compare this result to the savings rate in the canonical OLG model with logarithmic preferences and Cobb-Douglas technology. There, the young save a fraction $\beta/(1+\beta)$ of their income, and the income share of the young is $1-\alpha$. The savings rate is therefore, in this case, given by the first of the three terms on the right hand side of (29), and it is time independent.

The present model implies that the savings rate is, in general, not constant over time. Moreover, it is always higher than in the canonical model for two reasons; the intergenerational distribution effect and the old-age requirement effect. First, as seen by the second term on the right hand side of (29), the presence of employment in the care sector implies higher labor demand, shifting the income distribution in favor of the young, and thus increasing savings. Second, as seen by the third term on the right hand side of (29), with $\bar{h} > 0$, as we have seen from (17), the young have an additional savings motive in that they need some minimum amount of old-age care, increasing the savings rate further.[13] The old-age requirement effect on savings is stronger the lower is ℓ_{t+1}, because lower future employment in the generic sector implies higher future wages, increasing the cost of purchasing the minimum requirement of care. The expected increase in the cost of health care in period $t+1$ prompts young agents to save more in period t and, therefore, to accumulate more capital.

The natural question concerns the general-equilibrium impact of both these mechanisms on economic growth. In this respect, the market-clearing condition equating investment to savings implies that capital per worker obeys the dynamic law

$$\kappa_{t+1} = \frac{1}{1+n}\cdot\left[a_t^{1-\alpha}\kappa_t^{\alpha}\ell_t^{1-\alpha} - c_t - \frac{d_t}{1+n}\right], \tag{30}$$

where the term in square brackets equals savings per worker. The next section discuss capital accumulation in the neoclassical variant of the model, while Subsection 5.3 extends the dynamics to the AK case.

[13] In the Appendix we show that restiction (14) and $\ell_{t+1} > 1-\alpha$ implies that $(1-\alpha)\,\bar{h} < \alpha(1+\beta)\,\eta(1+n)\cdot\ell_{t+1}$.

4 Neoclassical Growth

In the neoclassical case labor productivity in the generic sector equals $a_t = B^{\frac{1}{1-\alpha}}$ in each period. In this framework, when the economy reaches a long-run equilibrium where capital per worker is constant, generic production grows at the exogenous rate of population growth. Subsections 4.1-4.3 derive the stability properties of the long-run steady state and show that, given an initial stock below the steady-state level, capital per worker grows monotonously. We also show that under complementarity, these transitional dynamics are characterized by increasing savings rates. Under substitutability, on the other hand, savings rates decrease during the transition to steady-state. The intergenerational distribution effect and the old-age requirement effect both contribute to these results.

While under substitutability the steady state is always stable and unique, under complementarity the dynamics are more involved: since increased capital increases savings rates and thereby capital further, this opens for the possibility of (local) instability and multiple steady states. We show, however, that a departure from uniqueness and stability of the steady state can only occur under unreasonable high values of the elasticity of capital in generic production α.[14]

The case of complementarity is of particular interest when discussing growth in China, as it involves increasing savings rates and increasing (share of) employment in the care sector. Subsection 4.4 clarifies further how in this case the intergenerational distribution effect and the old-age requirement effect give rise to a *savings multiplier*, where savings and capital accumulation stimulates further savings and capital accumulation.[15] 4.5 performs comparative-statics exercises suggesting that one-child policies may boost capital accumulation via two channels – the negative impact on population growth and the increased need to purchase care services in the market rather than relying on own children to provide them. The final positive effect on long-run capital per worker is magnified by the savings multiplier.

4.1 Accumulation Law

The equilibrium path of capital is determined by the saving decisions of young agents. Using the utility-maximizing conditions of the household to substitute consumption levels in (30), we obtain a semi-reduced form of the accumulation law of capital per worker, which links κ_{t+1} to the previous stock κ_t and to the sectoral employment levels in the two periods (see the Appendix):

$$\kappa_{t+1} = \underbrace{\frac{B\beta\,(1-\alpha)}{(1+\beta)\,(1+n)}\kappa_t^{\alpha}}_{\text{Canonical model}} \cdot \underbrace{\ell_t^{-\alpha}}_{\text{Intergen. Distr.}} \cdot \underbrace{\left[1 - \frac{(1-\alpha)\,\bar{h}}{\alpha\,(1+\beta)\,\eta(1+n)\cdot\ell_{t+1}}\right]^{-1}}_{\text{Old-age requirement effect}} \tag{31}$$

This expression decomposes the accumulation law of capital in three parts. The first term on the right hand side of (31) is the dynamic law in the canonical one-good model: if we eliminate

[14] Nevertheless, for completeness we also solve the dynamics for this case in the Appendix.

[15] Naturally, the convergence of this multiplier process is guaranteed exactly when the steady state is unique and stable.

the care sector by setting $\ell_t = 1$ and $\bar{h} = 0$, capital per worker evolves according to this stable monotonic relationship, and the saving-output ratio is constant by virtue of constant income share of the young and logarithmic intertemporal preferences.

The second and third terms on the right hand side of (31) again directly follow from the intergenerational distribution effect and the old-age requirement effect. An increase in ℓ_t reduces κ_{t+1} because a lower current wage reduces young agents' income, and thereby, current savings. An increase in ℓ_{t+1} reduces κ_{t+1} because a lower future wage reduces the expected future cost of health care, and thereby, current savings.

To present the intuition in the most transparent way we first, in the next subsection, investigate the special case where $\bar{h} = 0$. This isolates the intergenerational distribution effect, and shows how this increases the steady state capital stock. In Subsection 4.3 we then expand the model to the case where $\bar{h} > 0$. This shows how the old-age requirement effect further increases the steady state capital stock.

4.2 Dynamics without Minimum Requirement

When there is no minimum health-care requirement for old agents, capital accumulation obeys a fairly simple dynamic law. In the main text, we assume that the elasticity of capital in generic production is not too high, that is:[16]

Assumption 1: $\alpha < \frac{3}{4}$.

This assumption is sufficient (but not necessary) for the steady state to be unique.[17] The next Proposition then establishes that the steady state is globally stable: under both complementarity and substitutability, the economy converges towards a long-run equilibrium in which capital per worker, the price of health care and employment shares are constant.

Proposition 2 *In the neoclassical case with $\bar{h} = 0$, capital per worker obeys*

$$\kappa_{t+1} = \frac{\beta\eta}{(1+n)(1+\beta)} \cdot p(\kappa_t), \tag{32}$$

where $p(\kappa_t)$ is the price of health care determined by (24). Under Assumption 1 the steady state $\kappa_{ss} = \frac{\beta\eta}{(1+n)(1+\beta)} \cdot p(\kappa_{ss})$ is unique and globally stable, implying

$$\lim_{t\to\infty} \kappa_t = \kappa_{ss}, \quad \lim_{t\to\infty} \ell_t = \ell(\kappa_{ss}), \quad \lim_{t\to\infty} p_t = p(\kappa_{ss}).$$

During the transition, given a positive initial stock $\kappa_0 < \kappa^{ss}$, both capital per worker and the price of health care increase, whereas employment in the generic sector declines (increases) and the saving rate increases (declines) under complementarity (substitutability):

$$\kappa_{t+1} > \kappa_t, \quad p_{t+1} > p_t, \quad \left\{ \begin{array}{l} \ell_{t+1} < \ell_t \text{ and } \theta_{t+1} > \theta_t \text{ if } \sigma < 1 \\ \ell_{t+1} > \ell_t \text{ and } \theta_{t+1} < \theta_t \text{ if } \sigma > 1 \end{array} \right\}.$$

[16] In the Appendix, we solve the model for the case in which Assumption 1 is not satisfied.

[17] Under substitutability the steady state is always unique and stable.

Proof. See the Appendix. ■

Proposition 2 suggests three remarks. First, the dynamic law for capital (32) shows that, when there is no minimum requirement, investment per-young is proportional to the price of care because savings only depend on current wages (w_t is proportional to p_t in each period). Second, the transitional dynamics of p_t and ℓ_t directly follow from Lemma 1: given that capital per worker grows monotonically, both the wage and the price of care increase over time. Employment shares, however, move in opposite directions depending on the value of σ, which determines whether old agents increase or decrease their expenditure share on old-age care in response to increasing prices. The third remark is that, under complementarity, the savings rate θ_t increases during the transition because rising care prices attract labor in the care sector and the income share of young agents then grows – i.e., the intergenerational distribution effect.

The steady-state implications of the intergenerational distribution effect is immediate by comparing the steady-state level of the capital stock, κ_{ss}, with that in the canonical version of the model, which we term $\kappa_{ss}^{\text{canonical}}$. Starting from (31), and imposing $\bar{h} = 0$ and $\kappa_{t+1} = \kappa_t = \kappa_{ss}$, we obtain

$$\kappa_{ss} = \frac{1}{\ell\left(\kappa_{ss}\right)^{\frac{\alpha}{1-\alpha}}} \left[\frac{B\beta\left(1-\alpha\right)}{\left(1+\beta\right)\left(1+n\right)}\right]^{\frac{1}{1-\alpha}} = \frac{1}{\ell\left(\kappa_{ss}\right)^{\frac{\alpha}{1-\alpha}}} \kappa_{ss}^{\text{canonical}}, \tag{33}$$

where the steady-state level of capital per worker in the canonical model (which is obtained by setting $\ell_t = 1$ in each period) is

$$\kappa_{ss}^{\text{canonical}} = \left[\frac{B\beta\left(1-\alpha\right)}{\left(1+\beta\right)\left(1+n\right)}\right]^{\frac{1}{1-\alpha}}. \tag{34}$$

It immediately follows from $\ell\left(\kappa_{ss}\right) < 1$ that $\kappa_{ss} > \kappa_{ss}^{\text{canonical}}$ always holds, i.e., capital per worker in our model is higher than in the canonical model independently of whether generic goods and old-age care are complements or substitutes. The need for care increases the demand for labor, pushing income distribution in favor of the young, and therefore increases savings. (The size of the gap between κ_{ss} and $\kappa_{ss}^{\text{canonical}}$ depends, obviously, on the elasticity of substitution as well as the other parameters of the model through the term $\ell\left(\kappa_{ss}\right)$, which we return to below).

4.3 Dynamics with Minimum Care Requirement

When the minimum old-age care requirement is strictly positive, $\bar{h} > 0$, the accumulation law (31) includes the dependency of current savings on future employment shares, i.e. the old-age requirement effect. Recalling result (23), equilibrium employment shares are a function of the capital stock per worker in each period. Substituting $\ell_t = \ell\left(\kappa_t\right)$ and $\ell_{t+1} = \ell\left(\kappa_{t+1}\right)$ into the accumulation law (31), we obtain

$$\kappa_{t+1} \cdot \left[1 - \frac{\left(1-\alpha\right)\bar{h}}{\alpha\left(1+\beta\right)\eta(1+n) \cdot \ell_{t+1}\left(\kappa_{t+1}\right)}\right]^{-1} = \frac{B\beta\left(1-\alpha\right)}{\left(1+\beta\right)\left(1+n\right)} \kappa_t^{\alpha} \cdot \left[\ell\left(\kappa_t\right)\right]^{-\alpha}. \tag{35}$$

This dynamic law determines the steady state(s) of the system and the associated stability properties. Under substitutability there is always a unique steady state. Under complementarity, i.e. $\sigma < 1$, we again in the main text assume that the elasticity of capital in generic production is not too high, now that is:[18]

Assumption 2: $\alpha < \frac{1-\alpha}{1-\sigma}$.

This assumption is sufficient (but not necessary) for the steady state to be unique. We then have:

Proposition 3 *Under Assumption 2 equation (35) exhibits a unique steady state* $\bar{\kappa}_{ss}$ *that is globally stable The transitional dynamics of* $p(\kappa_t)$ *and* $\ell(\kappa_t)$ *comply with Lemma 1.*

Proof. See the Appendix. ■

Proposition 3 establishes that, also with a minimum old-age care requirement, under complementarity the savings rate increase during a transition path where capital grows. Now, this is the combined result of the future-requirement and intergenerational distribution effects.

Imposing $\kappa_{t+1} = \kappa_t = \bar{\kappa}_{ss}$ in (35), the steady-state level of capital per worker must satisfy

$$\bar{\kappa}_{ss} = \left[\frac{1}{1 - \frac{(1-\alpha)\bar{h}}{\alpha(1+\beta)\eta(1+n)\ell(\bar{\kappa}_{ss})}}\right]^{\frac{1}{1-\alpha}} \cdot \frac{1}{\ell(\bar{\kappa}_{ss})^{\frac{\alpha}{1-\alpha}}} \cdot \kappa_{ss}^{\text{canonical}}, \tag{36}$$

where the term in square brackets is shown to be strictly positive (see Appendix), and strictly exceeds one when $\bar{h} > 0$. Comparing (36) to (33), we thus conclude that $\bar{\kappa}_{ss} > \kappa_{ss} > \kappa_{ss}^{\text{canonical}}$; the long-run level of capital per worker is higher when there is a positive minimum requirement of old-age care, which drives capital further above the level attained in the canonical model. The reason is the minimum-requirement effect, which prompts households to save more during the transition in response to the continuous increase of the price of old-age care.

4.4 The Savings Multiplier

We have now seen how steady state capital is affected by the intergenerational distribution and the minimum requirement effects. We now investigate how the same effects come into play when exogenous shocks affect the economy. This also sheds further light on transitional dynamics in the model, and allows us to clarify *the savings multiplier*. For this purpose, we in this subsection consider variations in the productivity level B, and we focus on the case of complementarity, $\sigma < 1$, which seems the most interesting scenario for discussing chinese household's saving behavior in the model.[19] The effects of exogenous shocks on income per capita may, as we will see in this subsection and the next, differ substantially from those predicted by the canonical model. For expositional clarity, we start out without the minimum requirement effect, before we extend the analysis to include this.

[18] Note that even in the limiting case where $\sigma \to 0$ this assumption is satisfied with the empirically plausible restriction $\alpha < \frac{1}{2}$. In the Appendix, we solve the model for the case in which Assumption 2 is not satisfied.

[19] However, all of the equations to follow are identical also in the case of $\sigma > 1$, the only difference being in the quatitative strength of the effects. As will be easily understood below, all savings multiliers which exceed one when $\sigma < 1$, falls short of one when $\sigma > 1$. Thus shocks that are magnified with complementarity, are instead dampened with substitutability.

Zero Requirement. In the canonical model, an exogenous increase in productivity increases the long-run level of (log) capital per worker in (34) by

$$\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{\mathrm{d}B} = \frac{1}{B\left(1-\alpha\right)}. \tag{37}$$

Now consider our model without minimum requirement. With $\bar{h} = 0$, the steady-state capital per worker is κ_{ss} defined in (33), and the impact of the productivity shock is determined by

$$\frac{\mathrm{d}\log\kappa_{ss}}{\mathrm{d}B} = \underbrace{\frac{1}{1-m_1\left(\kappa_{ss}\right)}}_{\text{Savings Multiplier}} \cdot \left(\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{\mathrm{d}B} + m_1\left(\kappa_{ss}\right)\cdot\frac{\ell_B'\left(\kappa_{ss}\right)}{\ell_\kappa'\left(\kappa_{ss}\right)\cdot\kappa_{ss}}\right), \tag{38}$$

where ℓ_κ' is the partial derivative $\mathrm{d}\ell\left(\kappa\right)/\mathrm{d}\kappa$, and ℓ_B' is the partial derivative $\mathrm{d}\ell\left(\kappa;B\right)/\mathrm{d}\kappa$ defined in Proposition 1, both evaluated in the steady state κ_{ss}. The crucial element in (38) is *the savings multiplier*, where $m_1\left(\kappa\right)$ is defined as the weighted elasticity

$$m_1\left(\kappa\right) \equiv -\frac{\alpha}{1-\alpha}\frac{\ell_\kappa'\left(\kappa\right)\cdot\kappa}{\ell\left(\kappa\right)}. \tag{39}$$

We show in the Appendix that $\sigma < 1$ under Assumption 1 implies $m_1 \in (0,1)$.[20] Hence, under complementarity, the savings multiplier in (38) is strictly higher than unity. Combining this result with $\ell_\kappa' < 0$ and $\ell_B' < 0$,[21] we conclude that the impact of a productivity shock on steady-state capital per worker is stronger than that predicted by the canonical model. There are two reasons for this, both related to the intergenerational distribution effect. The first reason, which is the result of the intergenerational distribution effect in the static part of the model, is represented by the term $m_1\frac{\ell_B'}{\ell_\kappa'\kappa} > 0$. The productivity increase pushes labor into care and out of generic production, increasing the wage further as compared to the canonical model. This means that the initial increase in the savings rate as a result of better productivity is higher than in the canonical model. The second reason, which is the result of the intergenerational distribution effect in the dynamic part of the model, is represented by the savings multiplier; the term $\frac{1}{1-m_1} > 1$. In our model, as the capital stock starts to grow, this further pushes labor out of generic production and into care, increasing the wage even further, thus magnifying the the initial increase in savings. The implication is that a higher productivity increases the capital stock and wages by more than in the canonical model. As we will see below, the savings multiplier is also key to explain why low population growth and one-child policies may have such a massive impact on capital accumulation.

Positive Requirement. To see how the old-age requirement $\bar{h} > 0$ modifies the savings multiplier, we define

$$m_2\left(\kappa\right) \equiv -\frac{\bar{h}}{\alpha\left(1+\beta\right)\eta\left(1+n\right)\ell\left(\kappa\right)-\left(1-\alpha\right)\bar{h}}\cdot\frac{\ell_\kappa'\left(\kappa\right)\cdot\kappa}{\ell\left(\kappa\right)}. \tag{40}$$

[20] While with $\sigma > 1$, $m_1 < 0$.

[21] Under complementarity, $\ell_\kappa' < 0$ follows from Figure 3 whereas $\ell_B' < 0$ follows from Proposition 1.

As above, we investigate the response of the steady-state capital stock to increased productivity, which from (36) is now given by

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{\mathrm{d}B} = \underbrace{\frac{1}{1-m_1(\bar{\kappa}_{ss})-m_2(\bar{\kappa}_{ss})}}_{\text{Savings multiplier}}\left[\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{\mathrm{d}B}+\frac{(m_1(\bar{\kappa}_{ss})+m_2(\bar{\kappa}_{ss}))\cdot\ell_B'(\bar{\kappa}_{ss})}{\ell_\kappa'(\bar{\kappa}_{ss})\,\bar{\kappa}_{ss}}\right], \quad (41)$$

Focusing again on the case of complementarity, $\sigma < 1$, we show in the Appendix that under Assumption 2 $m_1(\bar{\kappa}_{ss}) + m_2(\bar{\kappa}_{ss}) < 1$, that is, the savings multiplier in (41) is strictly higher than unity.[22]

Compared to the case with $\bar{h} = 0$ in (38), the effect of increased productivity on steady-state capital now involves two additional effects strengthening the impact of productivity on steady state capital. These are identified by the two appearances of the term m_2 in (41). First, in the static equilibrium of the model, the higher wage now also means higher cost of old-age minimum requirement of care, implying an additional increase in savings compared to in the case above. Second, the savings multiplier increases, strenghtening the feedback of capital on capital growth: the increase in the capital stock makes the wage rise over time, and this also increases the cost of the future minimum requirement, in turn increasing savings and the capital stock even more as compared to the case with $\bar{h} = 0$. Thus both the static and dynamic effects generated by the old-age requirement effect reinforce the steady-state response of capital to increased productivity.

Clearly an important channel of growth in China has been (transfer of labor from low productivity to) high productivity in the manufacturing sector. The analysis so far indicates that the effect of this may have been magnified by the savings multiplier. However, many would argue that one of the defining characteristics of Chinese policy, when compared to other countries, has been the one-child policy. Thus, we now investigate the effects on savings, capital accumulation and growth of such policies.

4.5 Population Growth and One-child Policies

The one-child policy has main effects on the chinese economy by lowering population growth, but also by changing society from one where parents could rely on their children for old-age care to one where parents receive less care from their offsprings. In particular, parents with one girl are strongly affected, since when married girls traditionally becomes part of the family of the husband. But parents of boys are also affected, because the one-child policy has implied rising sex ratios, increasing the probability that their boy may end up as unmarried. (In addition, naturally, general modernization may weaken the tradition of children to support their parents with old-age care.) In this subsection, thus, we study how the one-child policy may affect the economy through lower population growth and increased need to rely on alternative sources to own children for old-age care.

[22] While when $\sigma > 1$, the savings multiplier is strictly positive but less than unity.

As is well known from the canonical model, a lower growth rate of population increases the steady-state level of capital per worker, and from (34) we find get

$$\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{-\mathrm{d}n} = \frac{1}{(1+n)(1-\alpha)} > 0. \tag{42}$$

In contrast, from (36), and taking into account (26), we find that the effect in our extended model with minimum requirement, $\bar{h} > 0$, is given by

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{-\mathrm{d}n} = \frac{1}{1-m_1-m_2}\left[\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{-\mathrm{d}n} + \frac{\ell_n'}{(-\ell_\kappa')\cdot\bar{\kappa}_{ss}}(m_1+m_2) + \frac{\ell}{(1+n)\,\ell_\kappa'\bar{\kappa}_{ss}}m_2\right], \tag{43}$$

where to simplify notation we now write m_1 instead of $m_1(\bar{\kappa}_{ss})$, and so on.

There are, thus, five reasons the relative increase in capital with a lower growth of population is higher as compared to in the canonical model. The first two effects arise because of the intergenerational distribution effect and the old-age requirement effect, which through the savings multiplier ensures that capital growth has a stronger positive feedback as the capital stock increases. These two effects are represented by m_1 and m_2 in the savings multiplier. The third and fourth effects are represented by the presence of m_1 and m_2 in the expression $\frac{\ell_n'}{(-\ell_\kappa')\cdot\bar{\kappa}_{ss}}(m_1+m_2)$ in (43). They represent the effects on the labor share in the static model. With lower population growth there are fewer young relative to old agents at each point in time. This pull workers out of generic production and into care, increasing the wage. The increased wage increases the aggregate savings rate through both the intergenerational distribution effect and the old-age requirement effect. The fifth effect is represented by the term $\frac{\ell}{(1+n)\ell_\kappa'\bar{\kappa}_{ss}}m_2$ in (43). At each point in time, there is a higher fraction of old-age to young-age agents. Even for a fixed labor allocation this increases the wage. Through the old-age requirement effect, this increases the savings rate of the young in the static model even more, stimulating capital accumulation further. In total, this means that the effect of population growth in the present model may be substantially magnified compared to the effects in standard OLG models.

As we discussed above, another potential effect of one-child policies may be that, by lowering the number of young relative to the old, less care will be provided inside the family, and more care has to be purchased in the market. In the model, this can be represented by an increased amount of care that each agent must purchase as old, i.e. a higher $\bar{h}$. Obviously, this draws resources out of generic sector production and into the production of care. The effect on steady-state capital is found by (36) to be

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{\mathrm{d}\bar{h}} = \frac{\frac{\ell_{\bar{h}}'}{\ell_\kappa'\bar{\kappa}_{ss}}(m_1+m_2) + \frac{\ell}{\bar{h}\ell_\kappa'\bar{\kappa}_{ss}}m_2}{1-m_1-m_2}. \tag{44}$$

A higher minimum requirement of care has a direct effect on savings, represented by the term $\frac{\ell}{\bar{h}\ell_\kappa'\bar{\kappa}_{ss}}m_2$. In addition, the demand for labor increases, pushing the wage up. This shifts income distribution in favor of the young, and also makes care more expensive. For both reasons, savings increase, represented by the term $\frac{\ell_{\bar{h}}'}{\ell_\kappa'\bar{\kappa}_{ss}}(m_1+m_2)$. Thus, through three channels, a higher minimum requirement increases savings in the static model. Stimulated by the savings multiplier, steady-state capital increases by more than the immediate effect on savings and capital accumulation. As a consequence, the increased need for market based care may have a strong positive impact on capital accumulation in the generic sector.

5 Extensions

In this section we first extend the model to study social security in the form of a pay as you go system where the government provides old-age care. We then consider the possibility of dynamic inefficiency. We lastly extend the model to study endogenous growth dynamics.

5.1 Introducing the Welfare State

The savings motive in the economy comes partly from the need to pay for future old-age care. And, as we have seen, the accumulation of fixed capital increases the future cost of old-age care workers. Hence, accumulation may stimulate even further accumulation. The current debate about extending the welfare state in China has direct relevance within our model. In this section we consider the consequences of adopting a pay as you go scheme where the young pay a proportional tax τ_t on their income so as to finance free care g_t to the old living in the same period. A balanced budget then requires that $\tau_t w_t (1+n) = p_t g_t$, which from (12) is equivalent to

$$\tau_t \eta (1+n) = g_t$$

Thus a given tax rate, $\tau_t = \tau$, provides a given amount of care for each old, $g_t = g = \tau\eta(1+n)$. While (21) is unaffected by the tax, we show in the Appendix that (22) is now modified to

$$p_t = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\sigma-1}} \left[\frac{(\ell^{\max} - \ell_t)(1-\alpha)}{\ell_t - (1-\alpha)(1-\tau_t)}\right]^{\frac{1}{1-\sigma}} \equiv \Psi(\ell_t) \tag{45}$$

In the static part of the model, it can thus easily be verified that a higher tax rate reduces employment in the generic sector (and increases it in the production of care services). A further implication of this, in the static model, is that the decreased generic sector employment lowers the wage, and shifts income distribution further in the favor of the old and away from the poor.

In the Appendix we show capital accumulation is now given by

$$\kappa_{t+1} = \underbrace{(1-\tau_t)\frac{B\beta(1-\alpha)}{(1+\beta)(1+n)}\kappa_t^{\alpha}}_{\text{Canonical model}} \cdot \underbrace{\ell_t^{-\alpha}}_{\text{Intergen. Distr.}} \cdot \underbrace{\left[1 - \frac{(1-\alpha)\left(\bar{h} - \eta(1+n)\tau_{t+1}\right)}{\alpha(1+\beta)\eta(1+n)\cdot\ell_{t+1}}\right]^{-1}}_{\text{Old-age requirement effect}} \tag{46}$$

Compared to (31), we note that the accumulation is affected through decreased savings as income is redistributed from savers to non-savers. This is captured by the term $1-\tau_t$, and is the same as in the canonical model. In addition, we see that also the future tax rate enters directly through the old age requirement effect. A higher future tax rate used to provide old-age care, means that there is less need to save for the future. Thus capital accumulation, other things equal, decreases.

The total effect of taxes on the steady-state level of capital is found by imposing $\kappa_{t+1} = \kappa_t = \bar{\kappa}_{ss}$, $\tau_{t+1} = \tau_t = \tau$, and taking into account that generic sector employment is decreasing in the tax rate through the term . Taking into account that higher taxes decreases generic sector employment, we find from (46) that

$$\frac{\mathrm{d}\log\bar{\kappa}_{ss}}{\mathrm{d}\tau} = \frac{1}{1-m_1-m_3}\left(\frac{\mathrm{d}\log\kappa_{ss}^{\text{canonical}}}{\mathrm{d}\tau} - \frac{\ell'_\tau}{\ell'_\kappa\bar{\kappa}_{ss}}(m_1+m_3) - \frac{\eta(1+n)\ell}{\left(\bar{h}-\eta(1+n)\tau\right)\ell'_\kappa\bar{\kappa}_{ss}}m_3\right), \tag{47}$$

now with m_2 modified to

$$m_3 \equiv -\frac{\bar{h}-\eta(1+n)\tau}{\alpha\left(1+\beta\right)\eta\left(1+n\right)\ell\left(\kappa\right)-\left(1-\alpha\right)\left(\bar{h}-\eta(1+n)\tau\right)}\cdot\frac{\ell'_\kappa\left(\kappa\right)\cdot\kappa}{\ell\left(\kappa\right)} > 0.$$

The introduction of the welfare state reduces the steady-state capital stock. For four reasons, the reduction in the capital stock is larger than in the canonical model: there are both static and dynamic intergenerational distribution as well as old-age requirement effects.

We can also find

$$\frac{\mathrm{d}\log\kappa_{ss}}{\mathrm{d}\bar{h}} = \frac{\frac{\ell'_{\bar{h}}}{\ell'_\kappa\bar{\kappa}_{ss}}(m_1+m_3) + \frac{\ell}{\left(\bar{h}-\eta(1+n)\tau\right)\ell'_\kappa\bar{\kappa}_{ss}}m_3}{1-m_1(1+m_3)}. \tag{48}$$

Consider now a simultaneous increase in τ and $\bar{h}$ such that an increase in the minimum requirement is provided for free to the old and financed by increased taxes on the young, i.e. $\mathrm{d}\bar{h} = \eta(1+n)\mathrm{d}\tau$.

To Be Completed

5.2 Dynamic Inefficiency and Golden Rule

A pay as you go pension system lowers the steady state capital stock, and improves efficiency when the economy is dynamically inefficient in the first place. Dynamic inefficiency is characterized by a situation where the steady state capital stock is so high that it would be possible to consume part of it with no generation becoming worse off. This is the case if the capital stock is above the so called "golden rule" capital stock. The golden rule capital stock, κ^*, is found by letting a social planer optimize with respect to κ, ℓ, d, h, and c finding $(\kappa^*, \ell^*, d^*, h^*, c^*)$ such that the consumer in each generation gets equal and maximum utility. We can suppress time subscripts as all generations are considered equal fro the social planers point of view. Abstracting for simplicity from minimum health requirement in this subsection, this implies maximizing

$$U \equiv u\left(c\right) + \beta\cdot v\left(d,h\right)$$

under the the following constraints

$$\begin{aligned} x &= B\kappa^\alpha\ell^{1-\alpha} = c + \frac{d}{1+n} + \kappa\left(1+n\right) \\ 1 &= \ell + \frac{h}{\eta\left(1+n\right)} \end{aligned}$$

The first constraint states that production in one period should pay for consumption of the young, c and of the old d. In addition the capital labor ratio in each period should be preserved for the next period. The second constraint limits the labor use to the available labor force.

The solution for κ, can be found immediately. Optimality requires that, given the optimal choice of all other variables, there should be no gain from changing the value of a subset of variables. In particular, given (ℓ^*, d^*, h^*), there should be no scope for increasing c by altering κ.Hence in optimum

$$\frac{\partial x}{\partial \kappa} = (1+n) \Rightarrow \kappa^* = \ell^* \left(\frac{B\alpha}{1+n}\right)^{\frac{1}{1-\alpha}}$$

where $\ell^* < 1$ is the optimal generic sector employment fraction. The implications for the golden rule capital ratio of introducing the service sector is then immediate by comparing κ^* with that in the canonical version of the model, which we term $\kappa_{ss}^{*\text{canonical}}$. In the canonical model $\ell^* = 1$, hence

$$\kappa^* = \ell^* \left(\frac{B\alpha}{1+n}\right)^{\frac{1}{1-\alpha}} = \ell^* \kappa^{*\text{canonical}} < \kappa^{*\text{canonical}}$$

Therefore, as compared to the canonical model, our model with the service sector for old-age care for increases the potential relevance of dynamic inefficiency two reasons. First, from (33) we know that the care sector generates an income distribution effect that leads to a steady state capital stock higher in our model as compared to in the canonical model. Second, as the service sector reduces the labor available for the generic sector, this lowers the golden rule capital stock. Thus the actual capital stock in our model is higher than in the canonical model, while the golden rule capital stock is lower. Thus for parameters spaces where the canonical model is efficient, our could very well be inefficient, and moreover for parameter spaces where the canonical model is inefficient, our is even further away from efficiency.

5.3 Endogenous Growth

For simplicity, the analysis of the AK model excludes the minimum requirement. We show that, under complementarity (substitutability), accumulation is self-reinforcing (self-balancing) because capital growth induces positive (negative) feedback effects on saving rates and thereby subsequent accumulation. The consequences of these processes for long-run growth are discussed below.

Complementarity: Self-Reinforcing Accumulation and Traps

Setting $\bar{h} = 0$ and substituting the learning-by-doing specification of productivity $a_t = A^{\frac{1}{1-\alpha}} \cdot (\kappa_t/\ell_t)$ in (30), the linear AK model yields the accumulation law

$$\frac{\kappa_{t+1}}{\kappa_t} = \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} \cdot \frac{1}{\ell(\kappa_t)}. \tag{49}$$

Equation (49) holds independently of the elasticity of substitution between generic goods and health care. Considering complementarity, we may observe (aside from the very special case of permanent steady state discussed below) two types of growth paths:

(i) *Self-Reinforcing Accumulation.* Capital per worker and the price of health care grow forever. During the transition, the employment share of the generic-good sector declines and the saving rate grows. In the long run, the economy converges asymptotically to the equilibrium featuring

$$\lim_{t\to\infty}\frac{\kappa_{t+1}}{\kappa_t}=\frac{A\beta}{(1+\beta)(1+n)}>1 \text{ and } \lim_{t\to\infty}\ell(\kappa_t)=1-\alpha. \tag{50}$$

(ii) *Self-Reinforcing Decumulation.* Capital per worker, the price of health care and the saving rate decline over time while the employment share of the generic-good sector grows. In the long run, the economy converges asymptotically to the equilibrium featuring

$$\lim_{t\to\infty}\kappa_t=0 \text{ and } \lim_{t\to\infty}\ell(\kappa_t)=1. \tag{51}$$

Self-reinforcing accumulation results from the fact that, under complementarity, capital accumulation induces *positive feedback effects* on saving rate. An initial increase in capital per worker drives up the health-care price and reduces the employment share of the generic sector: the intergenerational distribution effect then implies a higher saving rate and thereby further capital accumulation. Symmetrically, when capital initially declines, we observe self-reinforcing decumulation because reduced health-care price expands employment in the generic sector, which further reduces capital via lower saving rates. Importantly, the same economy may undertake a permanent accumulation path or remain trapped in a permanent decumulation path depending on initial endowments. The next Proposition defines a critical level of capital per worker at time zero which acts as a threshold between accumulation and decumulation outcomes.

Proposition 4 *(AK model under complementarity) If $1-\alpha<\frac{(1+\beta)(1+n)}{A\beta}<1$, there exists a finite critical level $\tilde{\kappa}>0$ satisfying*

$$\ell(\tilde{\kappa})=\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)},$$

and acting as a separating threshold: if $\kappa_0>\tilde{\kappa}$ ($\kappa_0<\tilde{\kappa}$), the economy follows the self-reinforcing accumulation (decumulation) path forever. If $\frac{(1+\beta)(1+n)}{A\beta}<1-\alpha<1$, the economy follows the self-reinforcing accumulation path for any $\kappa_0>0$. If $1-\alpha<1<\frac{(1+\beta)(1+n)}{A\beta}$, the economy follows the self-reinforcing decumulation path for any $\kappa_0>0$.

Proposition 4 can be easily verified as follows. When $\sigma<1$, the equilibrium employment share $\ell(\kappa_t)$ is negatively related to κ_t. Therefore, if the initial stock is sufficiently high to satisfy $\kappa_0>\tilde{\kappa}$, we have[23]

$$\ell(\kappa_0)<\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)},$$

[23] A necessary condition for observing inequality $\ell(\kappa_0)<\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}$ at time zero is that parameters satisfy $\frac{A\beta}{(1+\beta)(1+n)}>1$, because $\ell(\kappa_0)/(1-\alpha)$ must be greater than unity in an interior equilibrium. This is why, in Proposition 4, permanent accumulation paths may arise only if $1>(1+\beta)(1+n)/A\beta$.

the economy exhibits positive capital growth in the first period, $\kappa_1 > \kappa_0$, the generic-sector employment share declines, $\ell(\kappa_1) < \ell(\kappa_0)$, and the growth rate of capital in the subsequent period is even higher, $\kappa_2/\kappa_1 > \kappa_1/\kappa_0$. This mechanism arises in all subsequent periods and drives the economy towards the asymptotic equilibrium described in expression (50) above.[24] Symmetrically, if the initial stock is relatively low, $\kappa_0 < \tilde{\kappa}$, the same self-reinforcing mechanism works in the opposite direction: generic-sector employment is initially so high that savings are discouraged and capital per worker declines,[25] implying further increase in $\ell(\kappa)$ and therefore permanent decumulation. In the very special case $\kappa_0 = \tilde{\kappa}$, there is a permanent steady-state equilibrium: capital per worker and employment shares are constant forever because the saving rate is exactly at the level implying $\kappa_{t+1} = \kappa_t$ in each period. In this situation, however, any small perturbation increasing (decreasing) capital per-worker would drive the economy towards self-reinforcing (accumulation) decumulation: see Figure 4, diagram (a), for a graphical description of this result.

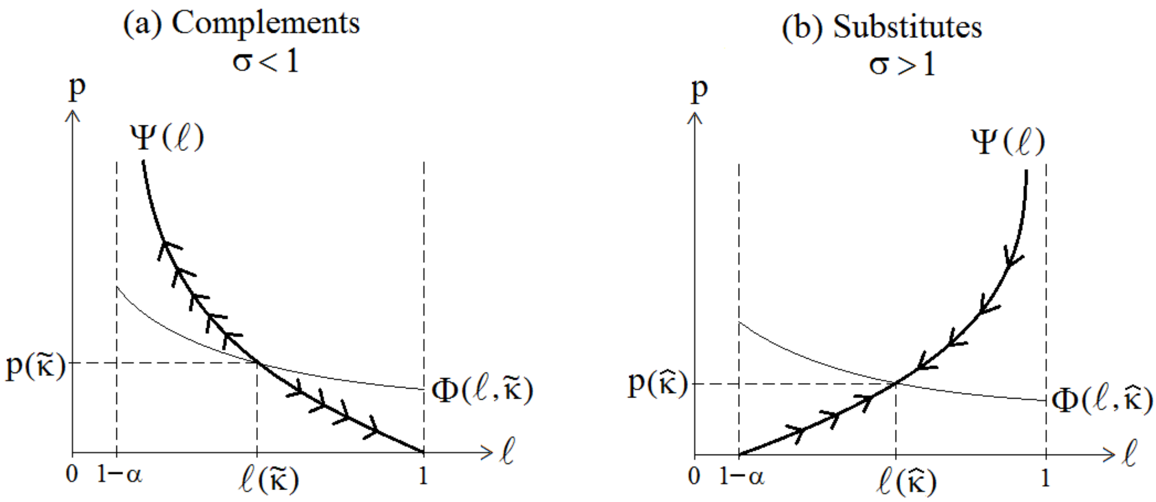

Figure 4: Dynamics of the AK model ($\bar{h} = 0$). If there exists a steady-state level of capital per worker compatible with positive production in both sectors, it is a separating threshold under complementarity, a global attractor under substitutability.

Proposition 4 also shows that, if preference and technology parameters do not satisfy $1-\alpha < \frac{(1+\beta)(1+n)}{A\beta} < 1$, only one of the two paths survives because the critical threshold $\tilde{\kappa}$ cannot be positive and finite. In these cases, there is either self-reinforcing accumulation or self-reinforcing decumulation because the critical threshold on capital per worker is either zero or infinity.[26]

[24] Expressions (50) represent an asymptotic equilibrium that is never reached in finite time. The proof follows from our previous analysis in Figure 3: as κ grows forever, the curve $\Phi(\ell, \kappa; 2)$ permanently shifts upward and the resulting equilibrium share $\ell(\kappa)$ approaches $1-\alpha$ only asymptotically because $\lim_{\ell \to 1-\alpha^+} \Psi(\ell) = +\infty$.

[25] A necessary condition for observing inequality $\ell(\kappa_0) > \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}$ at time zero is that parameters satisfy $\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} < 1$, because $\ell(\kappa_0)/(1-\alpha)$ must be greater than unity in an interior equilibrium. This is why, in Proposition 4, decumulation paths may arise only if $1-\alpha < (1+\beta)(1+n)/A\beta$.

[26] On this point, see the previous two footnotes and the Appendix for details.

Substitutability: Self-Balancing Accumulation and Stagnation

Considering substitutability, the analysis of equation (49) is modified by the fact that $\ell'(\kappa) > 0$. When $\sigma > 1$, the generic-sector employment share increases with capital because old agents respond to higher health-care prices by spending a higher fraction of income on generic goods. This implies that, contrary to the case of complementarity, an initial increase in capital generates *negative feedback effects* on savings through the intergenerational distribution effect: higher generic-sector employment reduces the total income share of young agents and, hence, the economy's saving rate. The consequences for economic growth are summarized in the following

Proposition 5 *(AK model under substitutability) If* $1-\alpha < \frac{(1+\beta)(1+n)}{A\beta} < 1$*, there exists a finite critical level* $\hat{\kappa} > 0$ *satisfying*

$$\ell(\hat{\kappa}) = \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)},$$

and representing a global attractor: if $\kappa_0 > \hat{\kappa}$ *(*$\kappa_0 < \hat{\kappa}$*), the economy follows a self-balancing accumulation (decumulation) path during the transition, and converges from below (above) to the stationary long-run equilibrium featuring* $\lim_{t\to\infty} \kappa_t = \hat{\kappa}$ *and* $\lim_{t\to\infty} \ell(\kappa_t) = \ell(\hat{\kappa})$*. If* $\frac{(1+\beta)(1+n)}{A\beta} < 1-\alpha < 1$*, the economy exhibits positive growth of* κ_t *forever and* $\lim_{t\to\infty} \ell(\kappa_t) = 1$*. If* $1-\alpha < 1 < \frac{(1+\beta)(1+n)}{A\beta}$*, the economy exhibits negative growth of* κ_t *forever and* $\lim_{t\to\infty} \ell(\kappa_t) = 1-\alpha$*.*

The first result established in Proposition 5 may be restated as follows: when generic consumption and old-specific goods are strict substitutes, if there exists a steady-state level of capital per worker that is compatible with positive production in both sectors, *the linear AK model behaves similarly to a neoclassical model.* Starting from relatively low capital, capital per worker grows over time but at decreasing rates, until the economy reaches a stable steady state representing the long-run equilibrium. The result is opposite to the case of complementarity, where the steady-state critical level $\tilde{\kappa}$ acts as a separating threshold: see Figure 4.

Under substitutability, accumulation is not self-reinforcing but rather self-balancing. Still, there is the possibility of permanently positive growth: when $\frac{(1+\beta)(1+n)}{A\beta} < 1-\alpha < 1$, there is no finite steady state $\hat{\kappa}$ (the steady-state level is virtually equal to plus infinity) and the economy grows forever. However, the transitional dynamics of employment shares, saving rates and growth rates are qualitatively opposite to the case of complementarity: workers flow to the generic sector, the saving rate declines and growth decelerates in the short-medium run.

Remarks

Propositions 4 and 5 show that the elasticity of substitution between generic goods and health care bears fundamental implications for economic growth. On the one hand, the separating threshold level $\tilde{\kappa}$ that arises under complementarity recalls several conclusions of the literature on poverty traps in endogenous growth models – an early reference is Azariadis and Drazen (1990) – but is still a specific result of our model as it hinges on the degree of substitutability between generic consumption goods and old-specific consumption goods. On the other hand, the fact that substitutability may "neoclassicize" the linear AK model of endogenous growth – by generating a stable long-run equilibrium with stationary capital per worker $\hat{\kappa}$ – is, to the best of our knowledge, a novel conclusion. The fact that the steady-state levels of capital per worker, $\tilde{\kappa}$ and $\hat{\kappa}$, have opposite characteristics is conceptually linked to the results of Peretto and Valente (2011), who study the existence of pseudo-Malthusian equilibria in a growth model with labor, land, endogenous fertility and endogenous technological progress.[27]

6 Concluding Remarks

To Be Written In this paper we have.

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[27] Peretto and Valente (2011) show that population size – somewhat similarly to the variable "capital per worker" in our model – exhibits an unstable steady state when labor and land are complements, and a stable steady state when inputs are substitutes. Besides the totally different aim of the analysis, the implications of these results for economic growth are somewhat opposite in the two models: in our framework, growing capital is necessary for output growth whereas stationary population in Peretto and Valente (2011) induces permanent economic growth in income per capita generated by innovations.

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