ASPIRATIONS AND INEQUALITY

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ABSTRACT

This paper develops a theory in which society-wide economic outcomes shape individual milestones or aspirations, which affect the investment incentives of individuals. Through its impact on investments, individual milestones in turn affect ambient social outcomes. We explore this two-way link. A central feature is that aspirations that are moderately above an individual's current standard of living tend to encourage investment, while still higher aspirations may lead to frustration and lower investment. When integrated with the feedback effect from investment, we are led to a theory in which aspirations and income evolve jointly, and the social determinants of preferences play an important role. We examine conditions under which growth is compatible with long-run equality in the distribution of income. More generally, we describe steady state income distributions, which are typically clustered around local poles. Finally, the theory has predictions for the growth rates along the cross-section of income, and for aggregate growth rates starting from any initial distribution of income.

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[T]hose made to wait unconscionably long for “trickle-down” — people with dramatically raised but mostly unfulfillable aspirations — have become vulnerable to demagogues promising national regeneration. It is this tiger of unfocused fury, spawned by global capitalism in the “underdeveloped” world, that Modi has sought to ride from Gujarat to New Delhi.

— Pankaj Mishra (2014), *The Guardian*

1. INTRODUCTION

In the 2014 general elections of India, the incumbent United Progressive Alliance, led by the Congress Party, was handed a resounding defeat by the National Democratic Alliance, headed by the Bharatiya Janata Party (BJP). The BJP alone won over half of all the contested seats, the first time since 1984 that a party had taken enough seats to govern on its own. Yet in the decade (2004-2013) that the United Progressive Alliance governed India — over two terms — Indian GDP per capita grew at the impressive rate of 7.6% per year. As Ghatak, Ghosh, and Kotwal (2014) observe, “By many economic indicators, UPA’s record in the last ten years (not just the last two) is good and compares rather favourably against the outcomes under NDA. It is a period during which growth accelerated, Indians started saving and investing more, the economy opened up, foreign investment came rushing in, poverty declined sharply and building of infrastructure gathered pace . . . [But a] period of fast growth in a poor country can put significant stress on the system which it must cope with. Growth can also unleash powerful aspirations as well as frustrations, and political parties who can tap into these emotions reap the benefits.” Ghatak, Ghosh and Kotwal wrote their paper before the general elections, but the same sentiments are echoed by Pankaj Mishra in a post-election article that we’ve quoted above.

This paper formulates a theory of socially dependent “milestones” or “aspirations,” one that incorporates both the inspiration of higher goals and the potential frustrations that can result. Our starting point is that individual goals for themselves or for their children are conditioned in fundamental ways by the lives of others. Existing literature views such “reference” points as drawn from the past experience of the individual herself. In contrast, we argue that they are (also) profoundly affected by her *social* environment. This is a view of individual preferences that isn’t standard in economic theory. But it should be.

At the same time, while social outcomes affect aspirations, those very aspirations influence — via the aggregation of individual decisions — the overall development of a society. As a result, aspirations and income (and the distribution of income) evolve together. An examination of this relationship is the subject of our paper.

Any such theory must address three issues. First, there is the question of how aspirations are formed. Second, we must describe how individuals react to the aspirations that they do have. Finally, the theory must aggregate individual behavior to derive society-wide outcomes. The last of these is a standard exercise; we emphasize the first two features.

We define utilities around "milestones" and interpret those thresholds as an *aspiration vector*. Our approach is inspired by theories of reference points: see, e.g., Kahneman and Tversky (1979), Karandikar et al. (1998), and Kőszegi and Rabin (2006). Our point of departure is that we emphasize the dependence of such reference points on the ambient income distribution, thereby linking observed social outcomes to individual behavior.[1] For instance, individuals may simply use some uniform function of the income distribution (such as mean income, or a weighted average of the incomes) to form their aspirations. In addition, we also allow for multi-step thresholds, with higher thresholds becoming relevant as an individual moves further up the income scale.

The crossing of each milestone is "celebrated" by an extra payoff. These "add-on" payoff functions are defined on the extent to which outcomes exceed the milestone in question, and are exogenous, but their impact is endogenous to the social environment which determines the milestones. The impact is on the individual's incentives to invest and bequeath, so as to achieve these extra payoffs. We argue that the "best" aspirations are those that lie at a moderate distance from the individual's current economic situation standards, large enough to incentivize but not so large as to induce frustration. Our theory draws on Appadurai (2004) and Ray (1998, 2006), who make similar arguments in a more informal setting, as well as our earlier working paper, Genicot and Ray (2009). Our formulation is also in line with evidence from cognitive psychology, sports, and lab experiments (see, e.g., Berger and Pope (2011), Heath, Larrick, and Wu (1999) and Lockwood and Kunda (1997)) that goals that lie ahead — but not too far ahead — provide the best incentives.[2] The argument captures both encouragement and frustration, and on its own can be used to create an aspirations-based theory of poverty traps.

Similar arguments allow us to study the behavior of continuation growth rates along the cross-section of an existing income distribution. Once again, our propositions reflect the idea that aspirations that are too high can serve to frustrate, while aspirations that are too low might breed complacency. It follows that over a zone of incomes that share the same aspirations, individual growth rates should be inverted U-shaped in income.

The above refers to "partial equilibrium" results, in which aspirations are taken as exogenous. Our main contribution is to embed the theory into a simple growth model with inequality. In equilibrium, the overall income distribution influences individual aspirations, which in turn shape the distribution via individual choices.[3]

We study the properties of equilibrium sequences of income distributions. We do so in two environments. The first is a Solow-like environment in which intertemporal production possibilities are strictly concave and satisfy end-point conditions. In this environment we can define steady state income distributions: those in which the implied set of aspirations feed back to individual

[1] See Macours and Vakis (2009) for evidence of the importance of social interactions in the formation of aspirations.

[2] To cite just one example from social psychology, LeBoeuf and Estes (2004) find that subjects score *lower* on trivia questions when first primed by self-listing the similarities between them and Einstein (what we might interpret as raising their aspirations), relative to when not primed; and they score *higher* when asked to list the differences between them and Einstein (what we interpret as lowering their aspirations) relative to when not primed.

[3] This approach develops the ideas laid down in an earlier working paper, Genicot and Ray (2009). In line with that approach, Bogliacino and Ortoleva (2014) also develop a model of socially determined aspirations, while Dalton, Ghosal, and Mani (2014) study a model of internally determined aspirations.

decisions, generating the very same distributions with which we started. We argue that such steady state distributions must, in general, be multimodal, the extent of multimodality depending on the number of aspirational steps. In the important special case of *single-step aspirations*, in which a single (endogenous) standard serves as reference point for everyone, we argue that a steady state income distribution must be bimodal. These results are in line with a literature that explore various arguments underlying the emergence and persistence of inequality, including nonconvexities (Galor and Zeira (1993), Matsuyama (2004)), occupational choice (Banerjee and Newman (1993), Freeman (1996), Mookherjee and Ray (2003)) and endogenous risk-taking (Becker, Murphy, and Werning (2005), Ray and Robson (2012)). In particular, the "twin-peaks" structure closely resonates with the findings of Durlauf and Johnson (1995), and Quah (1993, 1996).

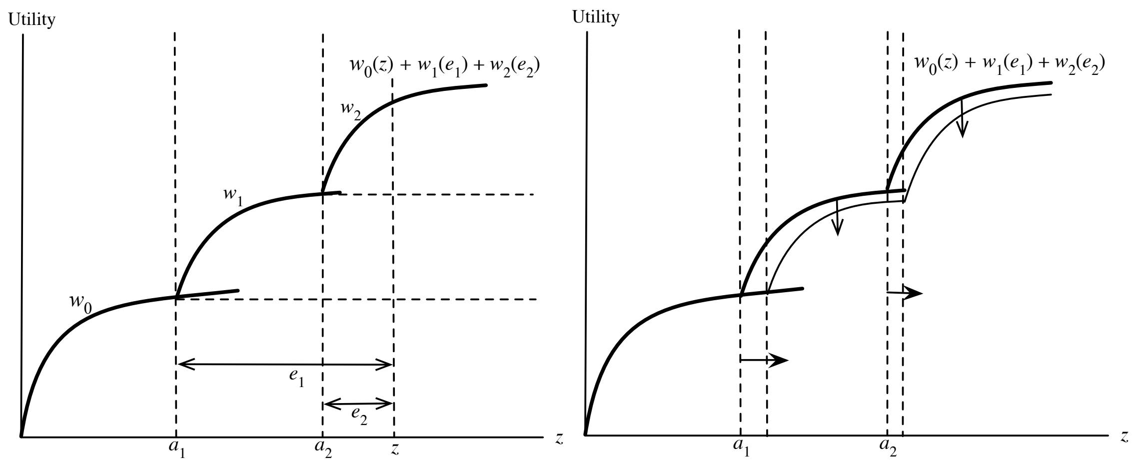
The second environment allows for sustained growth, and we work here under the assumptions of constant-elasticity payoff and a linear "A-K" production technology. Now initial conditions determine the asymptotic behavior of the economy. When the initial income distribution is "equal enough," in a sense that we make precise, the economy converges to perfect equality, with all incomes ultimately growing at the same rate. This is akin to the standard convergence predictions of classical growth models. However, when the initial income distribution is unequal, the economy begins to develop multimodality, and in lines with the findings of Piketty (2014) and others, *inequality must progressively increase*; the distribution never settles down to relative stationarity.

We view this paper as a first step to a theory in which individual goals are socially determined by their economic environment. The explicit description of "milestones" or "aspirations" as the basic building block through which social influences enter make this exercise different from a related literature that emphasizes the effect of the ambient distribution on status-seeking (see, e.g., Clark and Oswald (1996), Corneo and Jeanne (1997), Corneo and Jeanne (1999), Duesenberry (1949), Frank (1985), Hopkins and Kornienko (2006), Ray and Robson (2012), Robson (1992), Schor (1992), Scitovsky (1976), and Veblen (1899)).[4] Yet our formulation is far from being general and comprehensive. Rather, we seek the essential ingredients of a model that is tractable and useful enough in several applications. The particular application we have emphasized here concerns the evolution of income distributions, retaining the endogenous feedback from distributions to aspirations, and the consequent impact of aspirations on investment and income.

2. Aspirations, Wealth Distribution and Equilibrium

2.1. **An Intertemporal Model With Aspirations.** We study a society populated by a large number of single-parent single-child families. Each person lives for a single period. A sequence of individuals in a family forms a dynasty. A typical member of any generation has lifetime income (or wealth) y, and allocates y over her lifetime consumption c and investments to affect

[4]More closely, our approach is related to Karandikar et al. (1998) and Shalev (2000) who endogenize reference points using the realized payoffs of a game. However, the structure we place on aspirations formation as a reference point, and on the "nonlinear" way in which individuals react to the gap between their aspirations and their current standards of living, makes this a distinct exercise, with its own novel distributional and growth implications.



(A) Payoff Function (B) Change in Aspirations

FIGURE 1. ASPIRATIONS AND PAYOFFS

the wealth of her child z, so as to maximize payoff:

$$u(c) + w_0(z) + \sum_{i=1}^{n} w_i(e_i).$$

There are three terms in this payoff. The first is the utility u from own consumption c. The second and third terms pertain to the utility derived from the child's wealth z. The first of these may be viewed as "intrinsic" parental utility derived from the wealth of the child. Together, u and w_0 represent a standard model of intergenerational altruism.[5]

The last set of utility functions $\{w_1, \ldots, w_n\}$ represents "milestone utility," the return that parents receive from the *excess* $e_i = \max\{z - a(i), 0\}$ of their child's wealth z over a collection of certain thresholds $\boldsymbol{\alpha} = (a(1), \ldots, a(n))$, to be thought of as the *aspiration thresholds* or *milestones* of the parent. That is, w_i kicks in each time wealth crosses the aspiration $a(i)$, which is the ith step of $\boldsymbol{\alpha}$. The aspiration vector $\boldsymbol{\alpha}$ is akin to a collection of reference points, similar to Kahneman and Tversky (1979), Karandikar et al. (1998) and Kőszegi and Rabin (2006), and below, we will view $\boldsymbol{\alpha}$ as endogenously determined by the parent's socioeconomic environment.

Throughout the paper, we assume that each of the payoff functions is increasing, smooth and strictly concave, with unbounded steepness at zero.[6]

We interpret the parent as receiving both "intrinsic utility" and "milestone utility" from the wealth of their children. Milestone utility depends on the threshold that the parent seeks to attain: each such threshold $a(i)$ confers additional payoff w_i defined on the extent to which

[5]Because w_0 is exogenously given, we might think of this specification as one of "paternalistic altruism". In contrast, had w_0 been the value function of the child, we would interpret this as a model of "nonpaternalistic altruism". We do not pursue this alternative here.

[6]Formally, we assume that $u'(0) = w_i'(0) = \infty$ for all i.

that milestone has been crossed. Each milestone payoff has the familiar concave shape, but the achievement of fresh milestones confers some local nonconcavity on payoffs.

Observe that the entire vector α is the same for all parents (though in the sequel it will be determined endogenously by the wealth distribution). That doesn't mean that each parent will achieve the same aspirations. Where they place their children will certainly depend on the wealth they have to allocate.

Figure 1 depicts the joint function $w_0(z) + w_1(\max\{z - a(1), 0\}) + w_2(\max\{z - a(2), 0\})$ for a two-step aspiration $\boldsymbol{\alpha} = (a(1), a(2))$; so $n = 2$ (Panel A). Note that an increase in aspirations can never increase overall payoffs (Panel B). Loftier goals per se do not make for immediate happiness. Whether or not a higher $\boldsymbol{\alpha}$ increases the wealth — and the payoffs — of the next generation depends on how *marginal* incentives to accumulate are affected, a subject that we take up below.

2.2. **The Formation of Aspirations.** Two alternative approaches, by no means mutually exclusive, connect aspirations to economic outcomes and so bring the theory full circle. One possibility is to take an entirely *private* viewpoint: one's personal experiences determine future goals, so that each individual can be analyzed as as a self-contained unit. This is the approach taken in Karandikar et al. (1998) and Kőszegi and Rabin (2006) when determining reference points; see also Alonso-Carrera, Caball, and Raurich (2007), Carroll and Weil (1994) and Croix and Michel (2001). In this literature, the loop that runs from reference points to behavior and back to reference points is entirely internal to the individual.

In contrast, economic models of status (see the many references in the Introduction) achieve closure by using *social* outcomes external to the individual. A broad range of possibilities is captured under the specification

$$\boldsymbol{\alpha} = \Psi(F), \tag{1}$$

where $\boldsymbol{\alpha}$ stands for a vector of aspirations (or milestones) for an individual, and F is the society-wide distribution of lifetime incomes in the current generation.[7]

How many steps can $\boldsymbol{\alpha}$ have? Theoretically, there is no reason why $\boldsymbol{\alpha}$ cannot have an infinite number of milestones (especially if the distribution is spread out), but we push towards tractability by treating the *maximum* number of steps n as a parameter. More formally, we assume that:

[A] Ψ maps the going wealth distribution F into a vector $\boldsymbol{\alpha} = \{a(1), \ldots, a(n)\}$ in $\mathbb{R}^n$, for some given integer $n \geq 1$. Ψ is continuous,[8] and $\min F \leq a(1) \leq \cdots \leq a(n) \leq \max F$.[9]

[7] It is, of course, also possible to adopt a specification in which the *anticipated* distribution of wealth over future generations drives aspirations. A previous version of the paper, see Genicot and Ray (2009), discusses and compares the two approaches. We are comfortable with either model, but adopt the current approach for two reasons: (a) it uses the perhaps more satisfying formulation that goals are derived from an *actual* situation rather than an anticipated state of affairs which may or may not come to pass, and (b) the resulting structure is fully recursive and far more friendly to numerical computation.

[8] Continuity is defined with respect to the topology of weak convergence on distributions.

[9] The terms $\min F$ and $\max F$ refer to the minimum and maximum values in the support of the distribution F.

In particular, while n is given, the number of *distinct* aspirations may be smaller than n. (Note that we presume that all aspirations must lie within the range — though not necessarily the support — of the going distribution of wealth.)

The case $n = 1$ is a leading special case, which we call *single-step aspirations*. Everybody in the society has just one milestone; for instance, an aspiration threshold given by some weighted convex combination of incomes in the support of the going distribution. This case fits most closely with the literature on reference points. The advantage of considering a larger-step map is that it allows for individuals with greater wealth to effectively have higher aspirations. As already mentioned, these milestones may all be "simultaneously" present at some abstract cognitive level (as represented by the vector $\boldsymbol{\alpha}$), but nevertheless different aspirations may be salient at different wealth levels.

2.3. **Dynastic Equilibrium With Aspirations.** To describe equilibria, embed this model of aspirations formation into a standard growth model. If the distribution of current wealth is given by F_t, each individual has the aspirations vector $\boldsymbol{\alpha}_t = \Psi(F_t)$. She divides her wealth between consumption c_t and a bequest for the future, k_t:

$$y_t = c_t + k_t.$$

That bequest gives rise to fresh wealth for the next generation:

$$y_{t+1} = f(k_t),$$

where f is a smooth, increasing, concave function. A *policy* ϕ maps current wealth y_t and aspirations $\boldsymbol{\alpha}_t$ to wealth $z_t = y_{t+1}$ for the next generation.

An *equilibrium* from some initial distribution F_0 is a sequence of income distributions $\{F_t\}$ and a policy ϕ such that

(i) At every t, aspirations are given by $\boldsymbol{\alpha}_t = \Psi(F_t)$, and $z = \phi(y, \boldsymbol{\alpha}_t)$ maximizes

$$u\left(y - k(z)\right) + w_0\left(z\right) + \sum_{i=1}^{n} w_i\left(\max\{z - a(i), 0\}\right) \tag{2}$$

over $z \in [0, f(y)]$, where $k(z) \equiv f^{-1}(z)$.

(ii) F_{t+1} is generated from F_t and the policy ϕ; that is, for each $z \geq 0$,

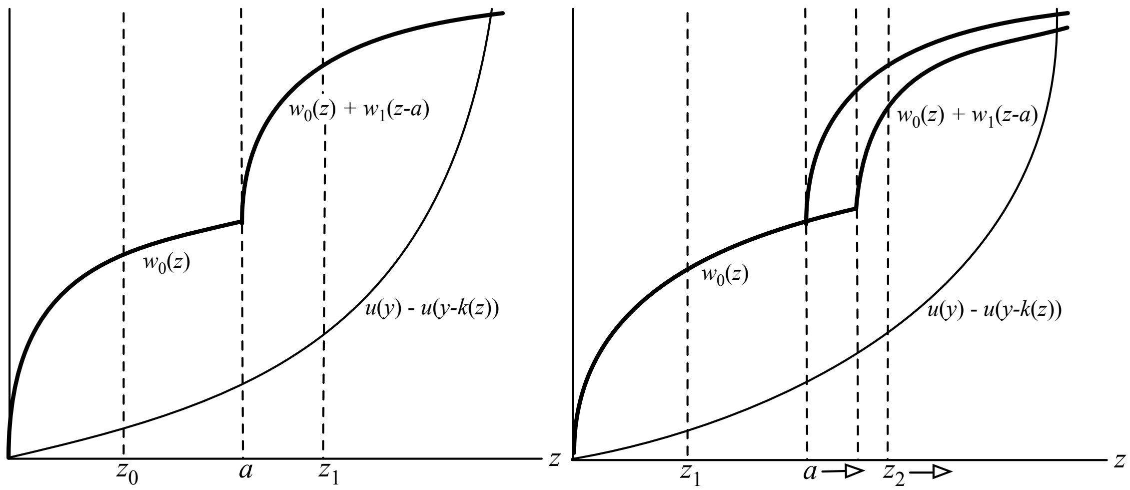
$$F_{t+1}(z) = \text{Prob}_t\{y | \phi(y, \Psi(F_t)) \leq z\},$$

where Prob_t is the probability measure induced by the distribution function F_t.

Note that *given the aspirations*, there is no particular need for the policy function to be time-dependent; the resulting maximization problem (2) is entirely stationary.

Proposition 1. *An equilibrium exists.*

The proof of this proposition is a simple recursive exercise, starting from any initial F_0.



(A) Costs and Benefits

(B) Change in a

FIGURE 2. THE CHOICE OF FUTURE WEALTH.

3. THE EFFECT OF ASPIRATIONS AND WEALTH ON INVESTMENT

We begin with a "partial equilibrium" analysis of the properties of the policy function ϕ. Figure 2 shows us how to graphically think about the maximization problem induced by expression (2). It uses a single milestone, so $n = 1$. The horizontal axis plots the choice of future wealth z, while the vertical axis records various benefits and costs. The benefit that accrues from next generation's wealth is given by

$$w_0(z) + w_1(e_1),$$

where $e_1 = \max\{z - a, 0\}$ is the excess (if any) of wealth over the single threshold $a(1) = a$. The cost is the sacrifice of current utility, which we can write as $u(y) - u\left(y - k(z)\right)$. Panel A of Figure 2 plots both these functions. The "cost function" has a standard shape: it is the convex lower curve. Given income and aspirations, our maximization problem seeks a continuation income z that produces the largest vertical distance between these two curves.

By the concavity of benefits to the left and right of a, there can be at most one "local" solution on either side of a. Finding an optimal solution involves comparing these two local solutions, that is, solving at most two interior first-order conditions

$$w_0'\left(z_0\right) = u'\left(y - k(z_0)\right) / f'(k(z_0)) \tag{3}$$

and

$$w_0'\left(z_1\right) + w_1'(z_1 - a) = u'\left(y - k(z_1)\right) / f'(k(z_1)), \tag{4}$$

the former applicable if $z_0 < a$ and the latter if $z_1 > a$, and picking the one that yields the higher payoff.

This is easily extended to an n-step aspiration vector $\boldsymbol{\alpha}$. For notational convenience, define $a(0) \equiv 0$ and $a(n+1) = +\infty$. Let m be the first index such that $a(m+1) \geq f(y)$. Then there are $m+1$ possible choices for z, given by $z_j \in (a(j), \infty)$ for $j = 0, \ldots, m$, each fully described by the interior first-order conditions[10]

$$\sum_{i=0}^{j} w_i'(z_j - a(i)) = u'\left(y - k(z_j)\right) / f'(k(z_j)). \tag{5}$$

From these $m+1$ choices, the individual picks the value z_j which yields the maximum payoff. Notice that if for any "candidate solution," we have $z_j \geq a(j+1)$, such a solution can never be globally optimal, because it will be surely dominated by the candidate z_{j+1}.

Say that the aspiration $a(i)$ is *satisfied* if the chosen optimal solution exceeds $a(i)$, and *frustrated* if it falls short of $a(i)$. (The slight ambiguity in this definition will be excused as the optimal choice of z will be generically be unique, with multiple solutions possible only for knife-edge values of (y, a). See Proposition 2 below.)

In the single-step case it is possible to refer to "frustrated aspirations" or "satisfied aspirations" without ambiguity, as there is just one possible milestone which is either attained or not. With several milestones, there are some satisfactions and some frustrations, perhaps more cumbersome to write down, but none the less realistic for that.

When is an aspiration threshold satisfied, and when is it frustrated? We can examine this question by varying aspirations for a fixed level of income, or by varying income for some fixed aspiration vector. The *joint* variation of incomes and aspirations will depend on the ambient income distribution, which is itself endogenous. Section 4 takes up this general equilibrium question.

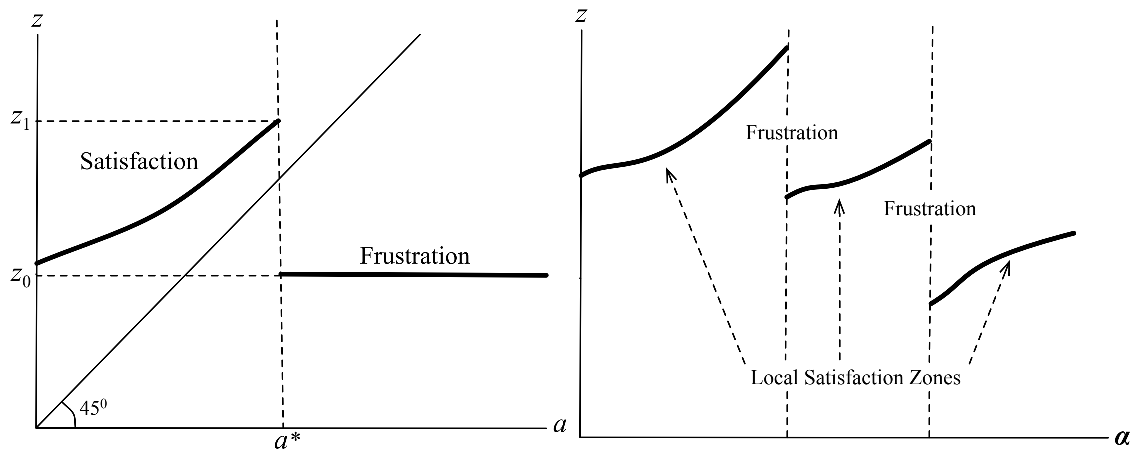
3.1. **Changing Aspirations.** For expositional simplicity, begin with the single-step case (with just a single aspiration). Consider an exogenous change in that aspiration for some individual with given income. Such changes don't just constitute an abstract exercise. For instance, the rise of mass media in developing countries (such as television, advertising or the internet) will bring particular socioeconomic groups into focus, thus affecting the aspirations mapping, typically upward.[11] In addition, a change in aspirations can be fueled by growth or decay in the ambient income distribution.

When the (single) milestone is close to zero, the optimal solution must strictly exceeds the milestone, and so aspirations are satisfied. As the threshold continues to rise, there comes a point when the solution makes a sudden switch from satisfaction to frustration: this switch will arrive with a discontinuous fall in investment, as is evident by consideration of the two first-order conditions (3) and (22). Once in the "frustration zone," investment becomes insensitive to further increases in aspirations.

Proposition 2 formalizes this discussion and adds important details that have been omitted:

[10]The first-order conditions are all guaranteed to hold, by definition of m and the fact that each w_i has unbounded steepness at its origin.

[11]See Jensen and Oster (2009) and Ferrara, Chong, and Duryea (2012) for evidence on how the introduction of cable television can expose people to very different lifestyles, thereby affecting their aspirations and fertility preferences.



(A) Single-Step Aspirations (B) The General Case

FIGURE 3. SATISFACTION AND FRUSTRATION AS ASPIRATIONS CHANGE.

Proposition 2. *Under single-step aspirations, for any given initial wealth, there is a unique threshold value of the milestone below which aspirations are satisfied, and above which they are frustrated. As long as aspirations are satisfied, chosen wealth grows with the milestone. Once aspirations are frustrated, chosen wealth becomes insensitive to the milestone.*

As already noted, the crossover from satisfaction to frustration will come with a discrete fall in investment. This stems from the local non-concavity of payoffs around the milestone. As illustrated in Panel A of Figure 3, there will be a critical aspiration level a^* where there are two solutions z_0 and z_1 that are both optima, with $z_0 < a^* < z_1$. As we've already seen, a further increase in aspirations has no effect on the marginal utility of wealth in the frustration zone and so fails to encourage wealth accumulation.

On the other hand, an increase in aspirations incentivizes growth as long as aspirations remain in the "satisfaction zone." It is true that the *total* payoff of parents falls, but the *marginal* return to investment rises, as illustrated in the second panel of Figure 2. That spurs parents on to greater investment, unless aspirations rise so much that frustration sets in, as discussed already.

With multi-step milestones, the results are similar, with the one change that frustration of some aspirations can coexist with the satisfaction of others. Consider an aspirations vector $\boldsymbol{\alpha}$, and to avoid tiresome enumeration of cases, suppose that the entire vector drifts upward (in each step). The investment behavior of individuals may be partitioned into two types of regions, one in which investment reacts continuously to the change in α, and another with a regime switch, as a particular step shifts from being a satisfied aspiration to a frustrated aspiration. In each regime switch, investment jumps discretely downwards, following which another segment starts off in which investment continues to be positively incentivized, provided that there is still some rising (and satisfied) aspiration threshold attained by the individual's investments. This extended result is depicted in Panel B of Figure 3, with some artistic license ($\boldsymbol{\alpha}$ is a vector). We omit a formal statement, which would closely follow Proposition 2.

Our observations on frustration are consistent with the arguments of Appadurai (2004) and Ray (1998, 2006), and with a recent literature that argues that lowering the aspirations of low income students to more reachable levels reduces the likelihood of dropping out of school in the US (Kearney and Levine (2014)) and in France (Goux, Gurgand, and Maurin (2014)).

It would also be of interest to apply these ideas in social and economic situations that display visible increases in income and wealth, and are yet characterized by a substantial degree of poverty and inequality. Indian liberalization in the 1990s and thereafter present precisely such a picture. At one level, India is a vibrant and growing economy, particularly in sectors that are geared to exports, or contain a sizable foreign exchange component, such as business services. So it is little surprise that the Indian growth story has enjoyed particular visibility in the world at large. Moreover, the veritable explosion of social media, from television to the internet, has undoubtedly raised aspirations everywhere. The rise of an economically powerful urban middle class is certainly consistent with a story of burgeoning aspirations with salubrious effects on investment. But there is a second story to be told, in which large sections of the population are effectively delinked from the growth process. Economic inequality has risen substantially, both across income groups (see Banerjee and Piketty (2005)) and across sectors such as rural and urban, as well as within urban areas; see Deaton and Drèze (2002). A multitude of indicators — literacy rates, infant and child mortality rates, gender imbalances, access to sanitation or electricity — point to India's poor socioeconomic performance, not just relative to the developed world but to other peer groups, such as the BRIC countries or poorer neighbors such as Bangladesh; see, e.g. Drèze and Sen (2013). And along with the success stories that foreign investors so like to hear, there is a subtext of apathy and despair, violence and conflict, driven by increased perceptions of economic inequality coupled with the large displacements of land, capital and labor that are endemic under uneven growth. Whether the potential for frustration caused by rising aspirations plays a central role in this story deserves more investigation and research. But the observations are *prima facie* consistent with such a story, as our discussion in the Introduction also suggests.

3.2. **Changing Wealth.** Our second "partial equilibrium" exercise studies changes in current wealth y, holding aspirations constant. As far as the optimal choice of next generation's wealth is concerned, the argument is intuitive and uncomplicated:

Proposition 3. *For given aspiration vector* $\boldsymbol{\alpha}$ *and each step* $a(i)$*, there is a unique threshold value of current wealth* y_i *above which the milestone* $a(i)$ *is satisfied. Optimally chosen wealth for the next generation is nondecreasing in current wealth.*

The proof of this result follows from an absolutely standard single-crossing argument based on revealed preference, and is therefore omitted.

In what follows we study how the *growth rate* of wealth varies along the cross-section of "starting wealths". To do so, we are going to introduce an important special case of our model that is particularly conducive to the study of endogenous growth. For future reference, we call it the *constant elasticity growth model*. It has the following components.

First, we impose an "A-K" setting in which the production function is linear:

$$f(k) = \rho k \tag{6}$$

where $\rho > 1$ is some constant return on capital holdings.

Next, we assume that utilities are constant-elasticity, with the same elasticity for each utility indicator:

$$u(c) = c^{1-\sigma}, w_0(z) = \delta z^{1-\sigma}, \text{ and } w_i(e_i) = \delta \pi_i e_i^{1-\sigma} \tag{7}$$

for each i, where $\sigma \in (0,1)$, $\delta > 0$ is a measure of discounting, $\pi_i > 0$ is a measure of the additional value of crossing the ith milestone, and e_i is the excess of z over the aspiration level $a(i)$.

Given constant elasticity, the use of a common elasticity term σ for the utility and aspirational components is all but unavoidable (once we incorporate the notion that aspirations move in tandem with income). To see why, imagine scaling up aspirations and income together, which is what will happen in the sequel when incomes are growing and aspirations are growing along with incomes. If the elasticities are not the same, then at least one of these three terms will either become relatively insignificant or unboundedly dominant. To retain the relative importance of both intrinsic consumption and aspirations, we use the same elasticity for each of these functions.

The expositional advantage of constant elasticity growth model is that, in the absence of an aspirations effect, bequests are proportional to wealth and growth rates are constant across the cross-section of current wealths. We can therefore be sure that any cross-sectional variation in the presence of aspirations stems entirely from aspirations alone. We would like to describe the *growth incidence curve*, a relationship that links baseline income to subsequent rates of growth.

For simplicity, consider a model with single-step aspirations, but this will be extended at the end of the section. Suppose, then, that there is a single, fixed milestone given by $a > 0$. An individual with starting wealth y will choose continuation wealth z to maximize

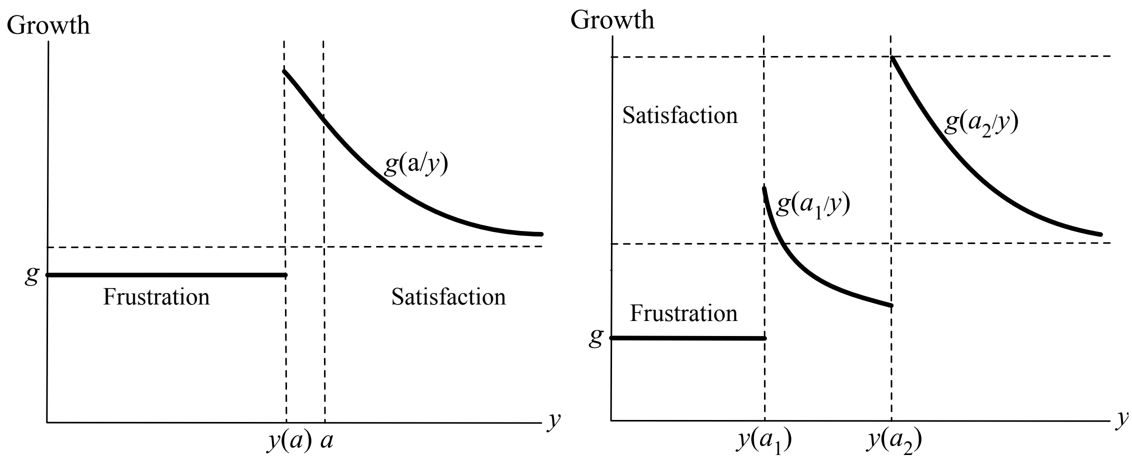
$$\left(y - \frac{z}{\rho}\right)^{1-\sigma} + \delta \left[z^{1-\sigma} + \pi_1 \left(\max\{z - a, 0\}\right)^{1-\sigma}\right]. \tag{8}$$

Let's define the *aspirations ratio* as $r \equiv y/a$: the ratio of the baseline wealth to the milestone. The maximization in (8) is equivalent to choosing a growth factor $g \equiv z/y$ that maximizes

$$\left(1 - \frac{g}{\rho}\right)^{1-\sigma} + \delta \left[g^{1-\sigma} + \pi_1 \left(\max\{g - \frac{1}{r}, 0\}\right)^{1-\sigma}\right]. \tag{9}$$

We solve this problem using a now-familiar method. First write down the first-order condition under the assumption that aspirations are met; that is, $g \geq \frac{1}{r}$. The corresponding growth factor $g(r) \equiv z/y$ is given by the solution to

$$\left(1 - \frac{g(r)}{\rho}\right)^{-\sigma} = \delta\rho \left[g(r)^{-\sigma} + \pi_1 \left(g(r) - \frac{1}{r}\right)^{-\sigma}\right]. \tag{10}$$



(A) Single-Step Aspirations (B) The General Case

FIGURE 4. GROWTH RATES AS A FUNCTION OF INITIAL WEALTH.

Note that there is a unique solution $g(r)$ to this equation in the region $(\frac{1}{r}, \infty)$ as long as this region is "reachable," which it will be provided $\rho > \frac{1}{r}$.[12]

The other option is for the individual to entirely ignore aspirations, which yields (via the usual first order condition), the growth factor $\underline{g}$ that solves:

$$\left(1 - \frac{\underline{g}}{\rho}\right)^{-\sigma} = \delta\rho\underline{g}^{-\sigma} \tag{11}$$

or

$$\underline{g} = \frac{\rho}{1 + \delta^{-\frac{1}{\sigma}}\rho^{-\frac{1-\sigma}{\sigma}}}. \tag{12}$$

If $\underline{g}$ turns out to exceed $\frac{1}{r}$, then it is obvious that the $g(r)$-solution is optimal at y (because the latter "includes" the aspirational payoffs while the construction of $\underline{g}$ does not). On the other hand, if $\underline{g} < \frac{1}{r}$, then the individual must compare payoffs from the two alternative choices given by (10) and (11).[13]

Proposition 4. *Consider the constant elasticity growth model. Fix a single milestone $a > 0$. Then there is a unique threshold $y(a)$ such that for all wealths $y < y(a)$, continuation wealth grows by the factor $\underline{g}$, and for all $y > y(a)$, continuation wealth grows by the factor $g(y/a)$. Once $y > y(a)$, $g(y/a)$ declines in y, but is always strictly larger and bounded away from $\underline{g}$ uniformly for $y \in (y(a), \infty)$.*

[12] If $\rho > \frac{1}{r}$, then starting at income y it will be possible to produce more than a: $\rho y > a$.

[13] That is, she checks if $\left(1 - \frac{g(r)}{\rho}\right)^{1-\sigma} + \delta\left[g(r)^{1-\sigma} + \pi_1\left(g(r) - \frac{1}{r}\right)^{1-\sigma}\right] > \left(1 - \frac{\underline{g}}{\rho}\right)^{1-\sigma} + \delta\underline{g}^{1-\sigma}$, chooses $g(r)$ if this inequality holds, and $\underline{g}$ otherwise.

Proof. By Proposition 3, continuation wealth is nondecreasing in initial wealth, so that if $z \geq a$ at some y, then the same is true at all higher wealths. That proves the uniqueness of the threshold $y(a)$, while existence is assured by the fact that at low income levels, $\rho y < a$, and at high income levels, $\underline{g}y > a$.

Below the threshold $y(a)$, the first order condition (11) therefore pins down the optimal choice of continuation wealth, which proves that incomes must grow at the constant rate $\underline{g}$ solving (11). Beyond $y(a)$, condition (10) pins down the optimal choice, leading to a growth rate of $g(r)$, where $r = y/a$. By comparing (10) and (11), it is easy to see that $g(r) > g$. Moreover, an inspection of (10) shows that $g(r)$ must decline as r rises, which will be the case when y rises with a fixed. At the same time, if we pass to the limit as $y \to \infty$ (so that $r \to \infty$), it still remains true that $g(r)$ stays bounded away from $\underline{g}$ (set $[1/r] = 0$ in (10) and compare the condition with (11)). ■

Figure 4 illustrates the proposition. Look at Panel A. For low levels of initial wealth, growth rates are at their "frustration level" $\underline{g}$. (Depending on the parameters, this rate may imply growth or decay.) As initial wealth climbs, there is a threshold at which the "upper solution" to the first order condition (10) dominates the "lower solution" to the first order condition (11), and the growth rate jumps upward. Thereafter, as growth rates fall again with initial wealth, but because of the additional marginal payoff bestowed by aspirations utility, never come down to the original "frustration rate" $\underline{g}$.

The jump threshold $y(a)$ may or may not lie below a. It surely will lie below a (and that is how we have drawn it in the figure) provided that $\underline{g}$ involves growth and not decay. Otherwise, the relative positioning of a and $y(a)$ is ambiguous.

Panel B of Figure 4 shows us how to carry out the obvious extension of Proposition 4 to the general case of multi-step aspirations. There will now be a succession of thresholds at which the growth rate will repeatedly jump, and one or more milestones will be crossed at each threshold. An individual with income y who crosses m such thresholds, and has an aspirations ratio $r_i = y/a(i)$ with respect to each threshold i that she plans to cross, exhibits a growth factor g that solves

$$(13) \qquad \left(1 - \frac{g}{\rho}\right)^{-\sigma} = \delta\rho \left[g^{-\sigma} + \sum_{i=1}^{m} \pi_i \left(g - \frac{1}{r_i} \right)^{-\sigma} \right].$$

For two individuals with the same set of satisfied aspirations, then, the growth rate declines as wealth rises. However, once wealth passes a threshold at which a new aspiration is met, there is a discrete jump in the growth rate. A multi-step model therefore predicts a rather complicated growth incidence curve, with sudden increases in growth along the curve, followed by more sedate declines. The overall tendency is for growth rates to rise, because each decline is bounded below by a rate that exceeds the lower bound of the previous segment. The single-step aspirations model has a somewhat more unambiguous prediction: that growth rates must initially rise at some threshold, and then fall with initial wealth, though that never takes us back to the frustration growth rate at the lower end.

4. THE JOINT EVOLUTION OF ASPIRATIONS AND INCOMES

In the previous section, we emphasized some partial effects of aspirations and wealth on the subsequent growth of incomes. Once we recognize that aspirations and incomes evolve jointly, these effects intertwine, depending on the precise manner in which aspirations are formed. Section 2.3 formally defines an equilibrium sequence of income distributions $\{F_t\}$. There are several questions that one can ask of such a formulation. For instance:

(i) Does the general equilibrium of aspirations and income foster income inequality in "steady state"?

(ii) What is the relationship between steady state inequality and growth?

(iii) Do initial conditions on income distribution affect subsequent growth paths?

4.1. Steady States.

We begin with the analysis of steady states: distributions of wealth F^* such that the stationary sequence $\{F^*, F^*, F^*, \ldots\}$ is an equilibrium starting from F^*.

Do steady states always exist? A natural setting is a production function which maintains all incomes in some compact support, as in the Solow model. To that end, we make the following assumption, which will be in force throughout this section.

[C] $f(x) > x$ for all x small enough and $f(x) < x$ for all x large enough.

Proposition 5. *Under Assumption C, there exists a steady state distribution.*

Proof. By Assumption C, there is an interval of incomes $[0, Y]$ such that starting from this interval no income (and therefore, by Assumption A, no milestone) can wander out of it. Let $\mathcal{F}$ be the set of all cdfs defined on $[0, Y]$, and equip this space with the weak convergence topology. The mapping Ψ takes elements of F to

$$\mathcal{A} \equiv \{\boldsymbol{\alpha} = (a(1), \ldots, a(n)) \in [0, Y]^n | a(1) \leq a(2) \leq \cdots \leq a(n)\},$$

and it is continuous. We now construct a correspondence from $\mathcal{A}$ to $\mathcal{F}$.

For each $\boldsymbol{\alpha} \in \mathcal{A}$, let $a(0) = 0$, and for each $j \in \{0, \ldots, n\}$, define Y_j as the set of solutions y_j to the equation

$$\sum_{i=0}^{j} w_i'(y_j - a(i))) = u'\left(y_j - k(y_j)\right) / f'(k(y_j)). \tag{14}$$

Under Assumption C, it is easy to see that this set is nonempty for each j. However, there is no guarantee that y_j is the optimal choice of continuation income starting from y_j. That is because the first-order condition (14) is necessary but not sufficient, given the local nonconvexities induced by $\{w_i\}$.[14]

On the other hand, we claim that there is at least one element y of $\cup_j Y_j$ such that y is the optimal choice of continuation income, starting from y.

[14] Besides, as defined, there is not even a guarantee that $y_j \in (a(j), a(j+1))$, failing which it would surely not be the optimal choice of continuation income starting from y_j.

Suppose not. Let z denote an optimal choice from y; then for every j and $y_j \in Y_j$, $z_j \neq y_j$. In particular, given that payoffs are strictly concave for all choices of $z \in [a(0), a(1)] = [0, a(1)]$, it must be that $z_0 > y_0$ for all $y_0 \in Y_0$. It follows from single-crossing and the local strict concavity of payoff on $[a(1), a(2)]$ that $z_1 > a(2)$ for every $y_1 \in Y_1$. Proceeding in this way, we see that $z_{n-1} > a(n)$ for every $y_{n-1} \in Y_{n-1}$, and consequently by single-crossing that $z_n > a(n)$ for every $y_n \in Y_n$ as well. But that contradicts $z_n \neq y_n$, because payoffs are strictly concave when continuation incomes lie in $[a(n), \infty)$, and y_n satisfies (14). It follows that the claim is true.

Let $Y(\boldsymbol{\alpha}) \subseteq \cup_j Y_j$ be the (nonempty) collection of all $y \in \cup_j Y_j$ such that y is the optimal choice of continuation wealth starting from y, given aspirations $\boldsymbol{\alpha}$. To complete the mapping from $\mathcal{A}$ to $\mathcal{M}$, let $\Phi(\boldsymbol{\alpha})$ be the set of all $F \in \mathcal{F}$ with support contained in $Y(\boldsymbol{\alpha})$.

Let Γ be the correspondence $\Phi \circ \Psi$; then Γ maps $\mathcal{F}$ to $\mathcal{F}$. We claim that Γ is nonempty- and convex-valued, and has closed graph. Nonempty-valuedness follows from the fact that $Y(\boldsymbol{\alpha})$ is nonempty for every $\boldsymbol{\alpha} \in \mathcal{A}$, and convex-valuedness is obvious.

To prove that Γ has closed graph, all we need to show is that Φ has closed graph (since Ψ is continuous). To this end, consider any sequence of aspirations $\boldsymbol{\alpha}^k$ with $\boldsymbol{\alpha}^k \to \boldsymbol{\alpha}$. Let F^k be a sequence of cdfs such that for each k, $F^k \in \Phi(\boldsymbol{\alpha}^k)$, and suppose that F^k converges weakly to F. We claim that $F \in \Phi(\boldsymbol{\alpha})$; or equivalently, that the support of F is contained in $Y(\boldsymbol{\alpha})$. To prove this, observe that $\limsup_k Y(\boldsymbol{\alpha}^k) \subseteq Y(\boldsymbol{\alpha})$. This follows from a familiar continuity argument: if y^k is an optimal choice of continuation income starting from y^k under $\boldsymbol{\alpha}^k$, and $y^k \to y$, then y is an optimal choice of continuation income starting from y under $\boldsymbol{\alpha}$. But the supports of F^k are each contained in $Y(\boldsymbol{\alpha}^k)$. By weak convergence, F must have support contained in $Y(\boldsymbol{\alpha})$, and so $F \in \Phi(\boldsymbol{\alpha})$.

Because $\mathcal{F}$ is compact, convex and locally convex and Γ has the properties just proved, the Schauder-Tychonoff fixed point theorem applies, and there is a cdf $F^* \in \mathcal{F}$ such that $F^* \in \Gamma(F^*)$. It is easy to see that F^* satisfies all the conditions of a steady state distribution. ■

4.2. **Inequality in Steady State.** We now turn to the properties of steady states.

Proposition 6. *No steady state can involve perfect equality of wealth.*

Proof. Suppose, on the contrary, that F^* is concentrated on a single point y^*; then, by Assumption A, steady state aspirations must be given by $\boldsymbol{\alpha} = (a, \ldots, a)$, where $a = y^*$. But, we have already noted that there are at most two possible optimal choices at any level of wealth y and aspiration a, and neither of these choices is equal to a itself. (See the discussion around (3) and (22).) That contradicts the presumption that the steady state is concentrated on a single wealth level. ■

The intuition behind the proposition is the convexification of utility caused by the presence of aspirations. When continuation incomes are very close to current aspirations levels, the marginal utility of accumulation is high, and the system pushes away from this neighborhood. Whether it pushes upwards or downwards will depend, as before, on a comparison with the two locally optimal choices on either side of the current aspiration level. But the essential point is that the system cannot stay where it is. Therefore, the only way to have a steady state is to have a multitude of incomes populating that steady state *even without any fundamental uncertainty*. The local convexity of aspirations-based utility (around the aspiration level) precludes convergence.

How seriously we take this result depends on one's intuition about marginal utility as one departs from incomes close to aspirations. Aspiration-fulfillment does imply that an important goal has just been reached, and to the extent that there is some fundamental component of satisfaction that depends on the crossing of that goal, and an important notion of failure on not reaching it, local convexification is not an unrealistic property. The same is true of utility around reference points, as we see in the work of Kőszegi and Rabin (2006). That said, it is nevertheless possible that this convexification is outweighed by some other form of concavity in the system, such as the curvature of the production function. Our model rules out this possibility by juxtaposing a fresh source of utility (the function w) as the aspiration level is crossed on the existing utility from progeny income (the function w_0), thereby creating a kink that "dominates" any degree of (smooth) curvature in f. In short, we do not believe that endogenous separation of incomes is an absolute necessity. But faced with a choice of emphasis, this is the intuitive case to focus on, in our opinion.

There is a second assumption that is important in driving our result on steady state inequality. This is the condition that *parental milestones lie in the range of the parental wealth distribution*. The psychological justification for this is that milestones represent *social* achievements; they are stepping stones that are measured relative to what one's own compatriots are experiencing, and so individuals do not use milestones that are located beyond the full range of whatever they see around them. (Note that we obviously allow for such out-of-range income levels to confer utility; the utility functions are all strictly increasing.) It is this assumption that — coupled with the local nonconcavity of payoffs around the milestone — that forces individuals into (at least) two separate clusters in any steady state.

Proposition 6 is related to different aspects of the literature on evolving income distributions. The closest relationship is to endogenous inequality, in which high levels of equality are destabilized by forces that tend to move the system away from global clustering. In Freeman (1996) and Mookherjee and Ray (2003), this happens because of imperfect substitutes among factors of productions, so that a variety of occupations with different training costs and returns *must* be populated in equilibrium. Together with imperfect capital markets, this implies that in steady state, there must be persistent inequality, even in the absence of any stochastic shocks. In other work, Becker, Murphy, and Werning (2005) and Ray and Robson (2012) argue that endogenous risk-taking can also serve to disrupt equality, as relative status-seeking effectively "convexifies" the utility function at high levels of clustering.

4.3. **More on Clustering.** A steady state must have inequality, but it is possible to place bounds on the number of local poles as a function of the number of aspirational thresholds. To avoid complications that have nothing to do with aspirations, we impose restrictions that ensure a unique steady state income in a standard intergenerational model *without* aspirations. To this end, consider an artificial benchmark model without any aspirations at all, in which the individual chooses z to maximize

$$u(y - k(z)) + w_0(z)$$

where recall that $k(z) = f^{-1}(z)$. A steady state y is interior, and characterized by the condition

$$d(y) \equiv -\frac{u'(y - k(y))}{f'(k(y))} + w_0'(y) = 0. \tag{15}$$

We will assume in what follows that

[D] $d(y)$ is decreasing in y.

Condition D guarantees a unique steady state in the benchmark model. It purges our model of possible inequalities that might arise from a "super-normal" response of child wealth to parental wealth (in a standard setting without changed aspirations). Since the work of Becker and Tomes (1979), it is a well-known restriction, asserting that at least in the standard model of intergenerational altruism, an increase in parental wealth must translate into a smaller increase in progeny wealth. In particular, in that model, the wealth of all families will converge to a uniform limit, independent of initial wealth. To be sure, such a condition is a joint restriction on tastes *and* the return on capital, and is not implied by the other conditions imposed so far. But it is easy enough to write down specific functional forms that satisfy this requirement.[15]

We can now state

Proposition 7. *Under Condition D and n-step aspirations, every steady state consists of mass points, but no more than $n + 1$ of them.*

Proof. Let $\boldsymbol{\alpha} = (a(1), \ldots, a(n))$ be a vector of steady state aspirations. As before, set $a(0) = 0$ and $a_{n+1} = +\infty$. By Proposition 3, there can be no mobility in steady state. So any steady state income y that lies in $(a(j), a(j + 1)]$ must satisfy the condition (14), reproduced here as

$$\sum_{i=0}^{j} w_i'(y - a(i)) = u'\left(y - k(y)\right) / f'(k(y)), \tag{16}$$

where we are using (5) with the steady state condition $y = z_j$ imposed.

But there can only be one such value of y for each j. For (16) can be rewritten as

$$d(y) + \sum_{i=1}^{j} w_i'(y - a(i)) = 0,$$

interpreting the sum as zero if $j = 0$. The uniqueness of y follows directly from Condition D and the assumption that w_i is strictly concave for every i. As there are only $n + 1$ regions of the form $[a(i), a(i + 1))$, the proposition is established. ■

Proposition 7, stark as it is, is not meant to be taken literally. The clustering of incomes is a robust feature of the aspirations model, but it goes without saying that the convergence to *degenerate* poles is an artifact of the assumptions. (That would be akin to stating that the Solow model predicts a *single* income level in steady state.) When there are stochastic shocks, the distribution will always be dispersed, as in the extension of the standard growth model by Brock and Mirman (1972) and others. We could easily introduce such shocks into the model at hand and

[15] For instance, if $f(k) = Ak^{\alpha}$, then [D] holds whenever $-u''(c)c/u'(c) \leq (1 - \alpha)$ for all consumption levels c.

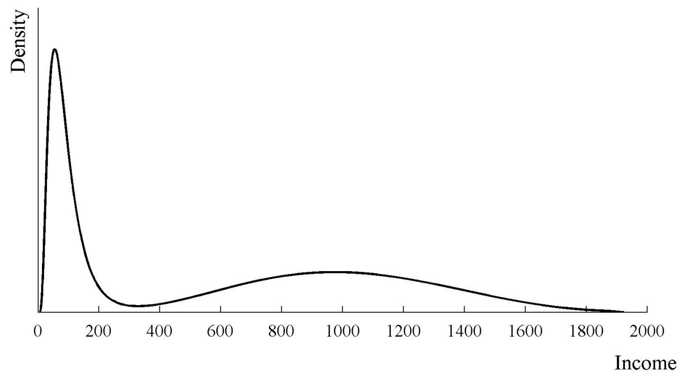


FIGURE 5. POLARIZATION AND SINGLE-STEP ASPIRATIONS.

prove a more nuanced proposition that replaces such clustered distributions by multi-modality. For illustrative purposes, we do so in Example 1; see also Figure 5. But it is unclear what additional conceptual insight that more complex model would yield. The central point is that there is a tendency for the distribution to exhibit local modes. This is adequately conveyed by the Proposition.

There is evidence of multimodality in the income distribution of various countries, including the United States (see Pittau and Zelli (2004), i Martin (2006) and Zhu (2005)), and especially Durlauf and Johnson (1995), and Quah (1993, 1996).[16] These authors make a strong case for local clustering in the world income distribution and argue that convergence is a local phenomenon "within the cluster" but not globally. Quah refers to these local clusters as "convergence clubs." Durlauf and Quah (1999) summarize by writing that there is an "increase in overall spread together with [a] reduction in intra-distributional inequalities by an emergence of distinct peaks in the distribution".

4.4. **Steady States With Single-Step Aspirations.** It is easy to describe steady states in the special case of *single-step aspirations*. Recall that this refers to a situation in which there is a single, distribution-dependent milestone, such as the mean income, or any convex combination of some or all the wealths in the going distribution.

By Proposition 7, if aspirations are single-step, a steady state distribution must have just two incomes in its support. As before, there is no compulsion to treat such a result literally, especially if the model is augmented in a realistic way to include stochastic shocks. The only essential feature is that there is a tendency for the wealth distribution to become bimodal, as illustrated by the following example.

Example 1. For this example, assume that utilities are of the constant-elasticity form introduced in (7). We set $\sigma = 0.8$, $\delta = 0.8$ and $\pi_1 = 1$. In order to get non-degenerate (and therefore more realistic) distributions of income, we introduce some noise in the production function. We take the production function to be $f(k, \theta) = \theta(A/\beta)k^{\beta}$, where $\beta = 0.8$, $A = 4$ and θ is a stochastic

[16]See also Henderson, Parmeter, and Russell (2008), Canova (2004) and Pittau, Zelli, and Johnson (2010).

shock with mean 1.[17] We set aspirations to equal mean income (but any interior specification would work as well), begin with an initial distribution of income that is uniform over a population of 500 individuals, and iterate the distribution over time. The simulated distributions converge rapidly to a bimodal distribution shown in Figure 5 where the only mobility is due to the noise in the production function.[18]

Single-step aspirations admit a particularly clean and tractable description of the set of steady states. For instance, suppose that aspirations are set at the mean income. We know Proposition 7 that a steady state takes the form of a two-point distribution (y_ℓ, y_h, p), where $y_\ell < y_h$ and p is the population weight on y_ℓ. Therefore a is given by

$$a = py_\ell + (1-p)y_h. \tag{17}$$

This constitutes the first of four conditions to determine the set of steady states. The second is the first-order condition under failed aspirations, which pins down the lower income level y_ℓ:

$$d(y_\ell) = 0. \tag{18}$$

The third is the first-order condition at steady state that solves for the income y_h where aspirations are satisfied:

$$d(y_h) + w_1'(y_h - a) = d(y_h) + w_1'\left(p[y_h - y_\ell]\right) = 0, \tag{19}$$

At this stage we still have one degree of freedom left, for these are three equations to determine three out of the four unknowns (y_ℓ, y_h, p, a). The lower income level y_ℓ is fully determined. Now, for each value of $p \in (0,1)$, y_h is obediently pinned down by the second equality in (19), and a can be read off from (17). This is the extra degree of freedom. But there is a further restriction imposed by global optimality choice across the continuation incomes that satisfy the two first-order conditions. An individual located at y_ℓ must prefer to stay at y_ℓ, rather than choose some continuation income $z > a$ that satisfies the first-order condition (22). For this, p has to be smaller than a certain threshold $p_{\text{max}} < 1$ so that the corresponding value of a strictly exceeds and is bounded away from y_ℓ. (Otherwise the individual will want to accumulate beyond y_ℓ.)

At the same time, as p decreases to zero, (17) tells us that a gets progressively closer to y_h. Moreover, the second equality of (19) tells us that y_h is bounded away from and larger than y_ℓ. But when that happens, the individual at y_h must strictly prefer to choose the continuation income below a that satisfies (3) for him. That imposes the restriction that $p \geq p_{\text{min}} > 0$.

Finally, we know that $p_{\text{min}} < p_{\text{max}}$, because at p_{max}, when an individual at the lower income y_ℓ is *just* indifferent between staying where he is and moving up, the individual at y_h will *strictly* prefer to be at y_h, rather than move below the corresponding aspiration level. This is the consequence of a single-crossing argument that drives the existence theorem of Proposition 5.

In short, aspirations must lie within a range that is bounded away from both y_ℓ and y_h, and so must population shares on either income level: they must be bounded away from both 0 and 1 by numbers that depend intimately on the parameters of the model.

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[17]Specifically, we suppose that θ follows a lognormal distribution. The qualitative results do not depend on the magnitude of the noise term, though in general, the degree of clustering must rise as the variance of the shock falls.

[18]In the figures, we smoothed the simulated distribution using the density estimator "ksdensity" for Matlab.

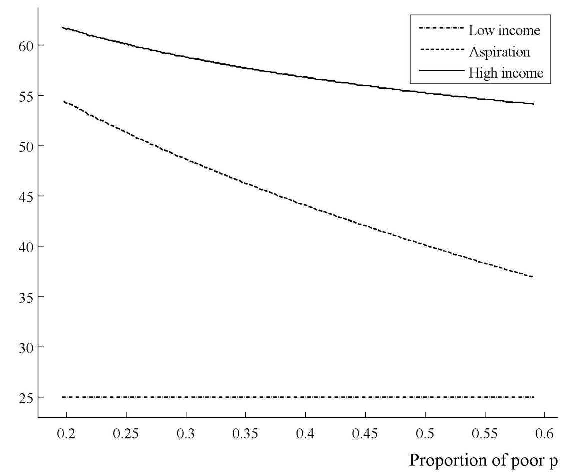


FIGURE 6. INEQUALITY AND SINGLE-STEP ASPIRATIONS.

Within the range of steady states, there is a close relationship between the proportion of low-income earners, and the income gap between high and low incomes. As one moves from a steady state with a high proportion of low-income earners to another with a smaller proportion, the income gap widens (examine condition (19)). A similar feature of widening inequality with growth will reappear more explicitly when we consider dynamic paths with sustained growth.

Example 2. In this example, we assume the same form of preferences and production function as in Example 1 but without noise. We set $\sigma = 0.35$, $\delta = 0.8$ and $\pi_1 = 1$ for the preferences and $A = 4$ and $\beta = 0.55$ for the production function. Aspirations are set at the mean. The lowest level of income in steady state — determined by (18) — is $y_\ell = 25$. There is a range of steady states characterized by combinations of high income y_h values and low income population proportions p. As in the discussion above, a smaller "poor population" is also associated with a higher income for the rich and greater inequality. Notice that only values of p between $p_{\text{min}} = 0.2$ and $p_{\text{max}} = 0.6$ are possible in steady state. The high incomes adjust accordingly. Figure 6 shows the high income y_h and the level of aspiration as a function of p in steady state.

The work of Durlauf and Johnson (1995) and Quah (1993, 1996) is particularly relevant in the present context, as their particular focus is on emerging bimodality in steady state distributions: Quah's 1996 paper has the title "Twin Peaks." Bimodality also speaks to the emphasis placed by Esteban and Ray (1994) and Wolfson (1994) on "polarized distributions," and the connection between such distributions and social tensions. We do not mean at all to suggest that aspirations are the only or even the central driving force behind bimodality (or clustering more generally), only that our model yields outcomes in line with this literature.

4.5. **Aspirations and Inequality with Endogenous Growth.** We now turn to a different scenario which accommodates endogenous growth. To this end, we return to the constant elasticity growth model introduced in Section 3.2. Recall that in that model, all utility indicators are constant-elasticity with the same elasticity, and the production function is linear.

In what follows, we will need to recursively track the economy as the sequence of income distributions $\{F_t\}$ evolves over time. To do this, we impose a mild additional restriction on the milestone formation function.

[G] Aspirations are nondecreasing in income, and linearly homogeneous in all incomes: i.e., if F' is achieved from F by having some incomes increase (and none decrease), then $\Psi(F') \geq \Psi(F)$, and if F' is achieved from F by scaling all incomes by $\lambda > 0$, then $\Psi(F') = \lambda\Psi(F)$.

Condition G merely states that incomes are combined in a scale-invariant way to generate aspirational milestones.

Single-Step Aspirations. We begin with the case of single-step aspirations, and extend the analysis later to the multi-step case. Our starting point is some initial distribution of wealth; call it F_0. Initial aspirations are therefore given by the scalar $a_0 = \Psi(F_0)$. An individual with wealth y will choose continuation wealth z to maximize

$$\left(y - \frac{z}{\rho}\right)^{1-\sigma} + \delta\left[z^{1-\sigma} + \pi_1\left(\max\{z - a_0, 0\}\right)^{1-\sigma}\right]$$

with respect to continuation wealth z. Just as in Section 3.2, two choices need to be compared. The higher of the choices involves the growth rate that solves equation (10). This solution, which we denote by $g(r)$, depends on baseline wealth y but only via the aspirations ratio $r = y/a_0$; see Section 3.2. The lower of the two choices involves a growth rate of $\underline{g}$, which solves (12): this choice is entirely independent of y.

We can dispose of an obvious and uninteresting case right away. It is possible that *every* individual has frustrated aspirations under F_0. Then everyone chooses the growth rate $\underline{g}$ in (12), and next period's wealth distribution will just be a proportional scaling of all incomes in F_0 by the factor $\underline{g}$. By Condition G, a will scale by exactly the same magnitude, *so that Case 1 must apply again*. This state of affairs will continue forever, and it will be accompanied by an unchanged relative distribution of incomes, as well as *a perpetual decay of average incomes at the rate* $\underline{g} - 1$. (The reason why $\underline{g}$ must be less than 1 is because aspirations lie in the range of incomes by Assumption A, so that in particular $r_h \geq 1$. It follows that if $\underline{g} \geq 1$, then the optimal solution for h must be $g(r_h) > \underline{g} \geq 1$, which contradicts our premise that all individuals have frustrated aspirations.)

While this case is logically possible, it is uninteresting because it involves universally frustrated aspirations and perpetual decay. We do not consider it further. We now state

Proposition 8. *Consider the constant-elasticity growth model with single-step aspirations that satisfy Condition G. Then, barring the case in which all individuals have frustrated aspirations at date 0, either there is*

1. Convergence to Perfect Equality: *The lowest-wealth individual has satisfied aspirations under* $F = F_0$, $y_t/g(1)^t$ *converges to a single point independent of* $y_0 \in$ Supp F_0*, and all wealths grow at same asymptotic rate* $g(1) - 1 > 0$*;*

Or there is

2. Persistent Divergence: *F_t "separates" into two components. There is a critical income level y^* in the interior of the support of F_0 such that all incomes below y^* change thereafter by the growth factor $\underline{g}$. All incomes initially above y^* grow by some asymptotic factor $\bar{g} > \underline{g}$, with $\bar{g} - 1 > 0$, and there is normalized convergence of these incomes: $y_t/\bar{g}^t$ converges to the same limit irrespective of y_0, as long as y_0 exceeds y^*.*

In Case 2, the asymptotic growth rate of each group falls short of the asymptotic growth rate under perfect equality, and relative inequality never settles down, it perpetually widens.

Proof. Our proof will rely on the following observation, which we state (and use later) for the multi-step case:

Lemma 1. *if F' is the distribution of wealth achieved from F by multiplying every wealth y in he support of F by a growth factor $g(y)$, then for each aspirational step i, $a(i)' \equiv \Psi_i(F')$ differs from $a(i) = \Psi_i(F)$ by a factor that lies in* $[\inf g(y), \sup g(y)]$.

Proof. To prove that $a(i)'/a(i) \geq \inf g(y)$, consider an intermediate step in which we move to a distribution F'' by multiplying all incomes in F by $\lambda = \inf g(y)$. By the linear homogeneity of Condition G, $\boldsymbol{\alpha}'' = \Psi(F'')$ must equal $\lambda\boldsymbol{\alpha}$. But now observe that F' differs from F'' only by an additional increase of incomes. By Condition G again, this implies $\boldsymbol{\alpha}' \geq \boldsymbol{\alpha}$". It follows that $a(i)' \geq [\inf g(y)]a(i)$ for every step i. The proof that $a(i)' \leq [\sup g(y)]a(i)$ employs a very similar argument, and is omitted. ■

Let r_ℓ and r_h stand for the infimum and supremum aspiration ratios in the distribution. For each of these ratios and indeed for all the aspiration ratios r in the support of F, we can calculate the optimally chosen growth rates.

Case 1. In this case, *every* individual with ratio $r \in [r_\ell, r_h]$ chooses $g(r)$, as given by (10). By Proposition 4, $g(r)$ decreases in r. By Lemma 1, aspirations cannot grow at a rate that exceeds $g(r_\ell)$. It follows that the aspirations *ratio* of the lowest wealth individual must increase. So if his aspirations were satisfied earlier (which they were), then they must be satisfied again. Moreover, by single-crossing (Proposition 3), he remains the lowest-wealth individual in the next period. It follows that *Case 1 must apply again in the next period.*

Because poorer incomes always grow faster than richer incomes, the system must converge to perfect equality and a constant growth factor of $g(1) > 1$. To see this formally, let z_t be the ratio of infimum to supremum incomes at date t. Because lower incomes grow faster in this case, z_t is monotonically decreasing and bounded above by 1, so it must converge. But in that limit the lowest income must be growing at the same rate as the highest income. Because $g(r)$ is decreasing, that can only happen if the limit equals 1.

In particular, all aspiration ratios under the sequence F_t must converge to 1. It follows that growth rate converges to $g(1)$. That $g(1) > 1$ follows from inspecting (10) with r set equal to 1.

Eliminating the case in which all individuals have frustrated aspirations under F_0, the only remaining possibility is:

Case 2. At date 0, there is a threshold r^* such that all "poor" individuals with $r < r^*$ are frustrated and grow at $\underline{g}$, while all "rich" individuals with $r > r^*$ have satisfied aspirations, and grow at rate $g(r) > \underline{g}$. (Persons at r^* can be put into either camp.) By Lemma 1, the aspirations ratio of all poor individuals must fall, and so by Proposition 4, they must *continue* to be frustrated and grow at rate $\underline{g}$ in the next period. In contrast, the "infimum rich person" grows at the highest rate $g(r^*)$ (because $g(r)$ is decreasing), and so again invoking Lemma 1, his aspirations ratio must rise. By Proposition 4, he must continue to have satisfied aspirations in the very next period. Because of single-crossing (Proposition 3), all richer individuals than him must continue to stay richer, so they too will have aspirations that are satisfied in the very next period. Using the same convergence argument as in Case 1, these individuals converge to a single pole over time that grows at some common rate; call it $\bar{g}$.

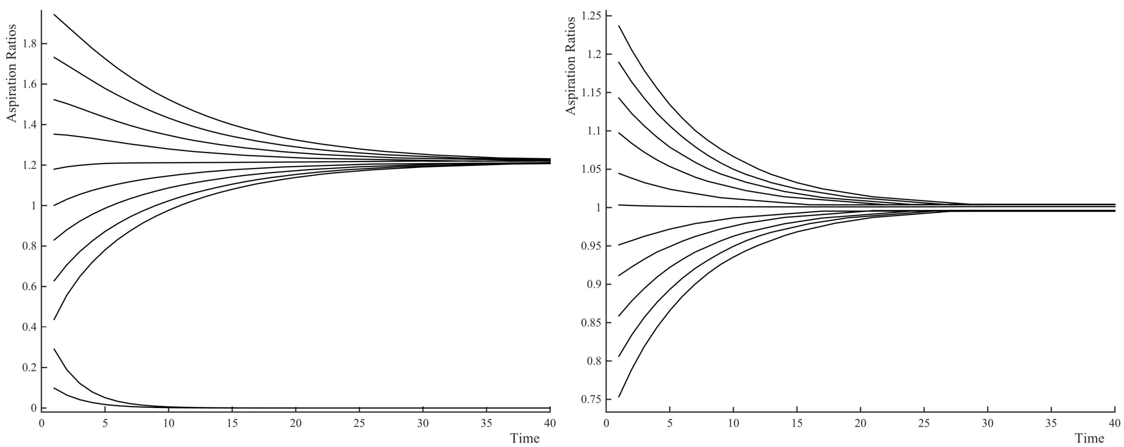
We already know that $\bar{g} \geq \underline{g}$, because the growth rates of all satisfied individuals exceeds $\underline{g}$ throughout. Suppose, contrary to our claim, that equality holds. Then the aspirations ratio of all satisfied individuals converges to some number $\bar{r} \geq 1$, and so $\bar{g} = g(\bar{r})$. But by Proposition 4, $g(r) > \underline{g}$ for all $r \geq 1$, a contradiction.

Finally, note that $g(1) > g(r)$ for any $r > 1$, so $g(1) > \bar{g} \geq \underline{g}$. That proves that the asymptotic growth rate of each group is lower than the asymptotic growth rate under perfect equality. ■

Proposition 8 significantly narrows the way in which the dynamics of an income distribution can evolve. To interpret it, continue to ignore the possibility in which the economy shrinks for all time. Then there are only two remaining possibilities.

In the first of these, every income level ends up with satisfied aspirations. That means that the initial distribution has a high level of equality to begin with, so that even the lowest income level can meet the (single-step) aspiration level for all of society. That may be a tall order, but if it is met, then indeed all incomes converge to perfect equality with sustained growth. Thus the basin of attraction for an equal steady state with growth is a relatively equal society to begin with. If that condition is not met, so that incomes at the lower end fall short of aspirations, the economy turns bimodal and inequality increases. Moreover, that inequality never stops increasing, *even in relative terms*, with the income ratio between the haves and the have-nots steadily rising. This is illustrated by the following example.

Example 3. We use the same preferences as in Example 1, with aspirations given by mean income, but assume a linear production function without noise with $\rho = 2.5$. We consider 1000 individuals and the evolution of their income over time. Figure 7 plots the evolution of the aspiration ratios of the poorest individual and every decile thereafter (the individual at the 10th percentile, 20th percentile, etc) up to the richest individual in the economy. Since there is no relative mobility higher curves represent richer individuals. In Panel A we start with a uniform distribution ranging from 50 to 1000 and end up with bipolar divergence. Individuals with less than 216 choose $\underline{g} = 1.2188$ and their aspiration ratios progressively decline over time. In contrast, individual with an initial income of more than 216 choose a higher growth rate of $g(r)$, as their incomes converge it grows at a rate of 1.89. The aspiration ratio of the poor starts at 0.09 and converges to 0 while the aspiration ratio of the richest individual starts at 1.88 and converges to 1.2.



(A) Unequal Initial Distribution

(B) Equal Initial Distribution

FIGURE 7. AN ILLUSTRATION OF PROPOSITION 8.

In Panel B we start with a much more compressed initial distribution, ranging from 369 to 605, and observe convergence to equality with all incomes and aspirations growing at a rate $g(1) = 1.94$. Notice that this growth rate is higher than the growth experienced by the rich in the unequal society of case (A).

In addition to a more equal initial distribution, technological improvements (higher ρ) and lower initial aspirations (change in Ψ lowering aspirations for any given distribution) make convergence to perfect equality more likely.

We make two more remarks. First, it is of some interest that in the Solow setting, perfect equality cannot be a steady state while a bimodal distribution can. Here, perfect equality can be sustained with a constant rate of exponential growth. On the other, and again in contrast to the Solow setting, an unequal distribution with a *constant* degree of relative inequality cannot persist with growth: that inequality will need to widen over time, in line with recent observations made by Piketty (2014) and his coauthors.

Second, perfect equality exhibits the highest rate of aggregate growth compared to the *asymptotic growth rate* of any other configuration. This latter rate is some convex combination of a growth rate of $g < g(1)$ for frustrated individuals, and an asymptotic growth rate that is *at best* $g(1)$ for the satisfied individuals. (It should be noted, however, that an unequal society is *temporarily* capable of growing faster. For instance, in Case 2, the overall growth rate is a combination of rates for various aspiration ratios $g(r)$, where r begins below 1 and ends above 1. Because $g(r)$ is decreasing, it is easy to construct an example in which this combination exceeds $g(1)$.)

Multi-Step Aspirations. When aspirations are multi-step, similar results apply. Begin again with some initial distribution function F_0 with compact support. Let y_t^ℓ and y_t^h denote infimum and supremum incomes at every date t, where $\infty > y_0^h \geq y_0^\ell > 0$. As with single-step aspirations, there is a case in which all individuals have failed aspirations (relative to every one of the milestones), and this case behaves just as before: it results in universal and proportional decay. As before, we eliminate this case from consideration. We do so by assuming that nonnegative growth is an optimal outcome in the standard model without aspirations. That is, $\underline{g} \geq 1$, or equivalently, by examining (12),

$$\delta^{1/\sigma}\rho^{(1-\sigma)/\sigma}(\rho - 1) \geq 1. \tag{20}$$

The following result is a partial extension of Proposition 8:

Proposition 9. *Consider the constant-elasticity growth model with multi-step aspirations, and suppose that Condition G is satisfied. Assume (20), or equivalently, that $\underline{g} \geq 1$, so that there is nonnegative growth in the standard model without aspirations. Then one of the following must hold:*

1. *Some individuals are frustrated with respect to every milestone at date 0. Then there is persistently widening relative inequality over time, with $y_t^h/y_t^\ell \to \infty$ as $t \to \infty$.*

2. *Every individual is satisfied with respect to some milestone at date 0. Then full convergence to perfect equality, stable relative inequality, and unbounded relative inequality are all possible outcomes.*

Proof. Case 1. (Some individuals are frustrated with respect to every milestone at date 0.) By Condition A, $y_t^h \geq a_t(n)$ for all t. We claim that the optimal continuation wealth z_t^h at y_t^h satisfies

$$z_t^h > a_t(n). \tag{21}$$

To see this, consider any milestone at date t, $a_t(j)$, and let $z(j) \in (a_t(j), \infty)$ be the candidate optimum satisfying (5), reproduced here for the constant-elasticity growth model as

$$\left(y_t^h - \frac{z(j)}{\rho}\right)^{-\sigma} = \delta\rho\left[z(j)^{-\sigma} + \sum_{i=1}^{j}\pi_i(z(j) - a_t(i))^{-\sigma}\right]. \tag{22}$$

Because $\underline{g} \geq 1$, it is easy to see that $z(j) \geq y_t^h \geq a_t(n)$. It follows that *none* of the candidate solutions for $j < n$ can be the globally optimal choice. The only candidate which can be globally optimal is z_n. But $z_n > y_t^h \geq a_t(n)$, which proves (21). So the very highest incomes grow at date t by the factor g_t^h, which is obtaining by dividing through by y_t^h in (22), setting $j = n$, and defining $s_t(i) \equiv a_t(i)/y_t^h$ for every aspirational step $a_t(i)$:

$$\left(1 - \frac{g_t^h}{\rho}\right)^{-\sigma} = \delta\rho\left[(g_t^h)^{-\sigma} + \sum_{i=1}^{n}\pi_i\left(g_t^h - s_t(i)\right)^{-\sigma}\right]. \tag{23}$$

On the other hand, those who are frustrated at date 0 grow at the rate $\underline{g}$ that solves

$$\left(1 - \frac{\underline{g}}{\rho}\right)^{-\sigma} = \delta\rho\underline{g}^{-\sigma}, \tag{24}$$

and we already know that $\underline{g}$ is the lowest of the possible growth factors. By Condition G, all aspirations must grow by at least the growth factor $\underline{g}$, so it is easy to see that those who are frustrated with respect to every milestone at date 0, must remain frustrated with respect to every milestone *at all subsequent dates*. Now compare (23) and (24) to see that $g_t^h > \underline{g}$ and bounded away from $\underline{g}$, so that $y_t^h / y_t^\ell \to \infty$ as $t \to \infty$.

Case 2. (All individuals are satisfied with respect to some milestone at date 0.) Consider two initial levels of income y_ℓ and y_h and two level of aspirations so that at time 0, y_ℓ chooses a growth rate $g_\ell > \frac{a_0(1)}{y_\ell}$. If $g_\ell > \frac{a_0(2)}{y_\ell}$ then the poorest individuals satisfy all aspiration levels and so does the richest. Since $r_\ell(i) < r_r(i)$ for all i, g_ℓ is larger than the growth rate chosen by the rich, and therefore than the aspirations. It follows that this scenario repeats itself and the income of the poorest group converge to the richest.

Now assume that the poorest satisfy only the first level of aspirations but not the second one, while the rich satisfy all aspirations. The growth rate of the poor can now be smaller than the growth rate of the rich. If $g_\ell < g_r$, then aspirations will grow faster that the income of the poor. In the following period, we can be in case 1 where the poorest group is frustrated with respect to all aspirations and inequality is ever expanding. If $g_\ell = g_r$ then aspirations are also growing at that common rate. It follows that the two groups choose the same growth rate again and balanced growth obtains.

■

Example 4. We now illustrate the different possibilities described in Proposition 9. We use the same preferences and linear production function as in Example 3, but assume two-step aspirations. Each aspiration a is a weighted average of the incomes ($a = \sum w_i y_i$). The lower milestone uses as weights the reciprocal of income: $w_i = y_i^{-1} / \sum y_j^{-1}$. The upper milestone uses income shares as weights: $w_i = y_i / \sum y_j$. The economy is composed of three groups who differ in their initial level of income. We consider four sub-examples. For each, the (long run) growth rates are recorded in Table 1, and the evolution of relative incomes and aspirations (relative to the mean) in Figure 8.

Example	Growth Rates		
	Poorest	Median	Richest
refex:grstep$A$	0.001	0.43	0.76
refex:grstep$B$	1.82	1.82	1.82
4C	1.66	1.66	1.76
4D	1.76	1.76	1.76

TABLE 1. Long Run Growth Rates

Example 4A has a return on investment of $\rho = 2.1$. The three groups have equal population proportions and initial incomes of 100, 300 and $1,00$. As can be seen in Figure 8 in the upper-left quadrant, the poorest segment of the population is frustrated with respect to both aspirations. It remains so throughout, in line with Case 1 of Proposition 9. The second segment of the population is satisfied with respect to the first milestone but not the second, while the richest segment of the population satisfies both aspirations. Each segment is growing at a different

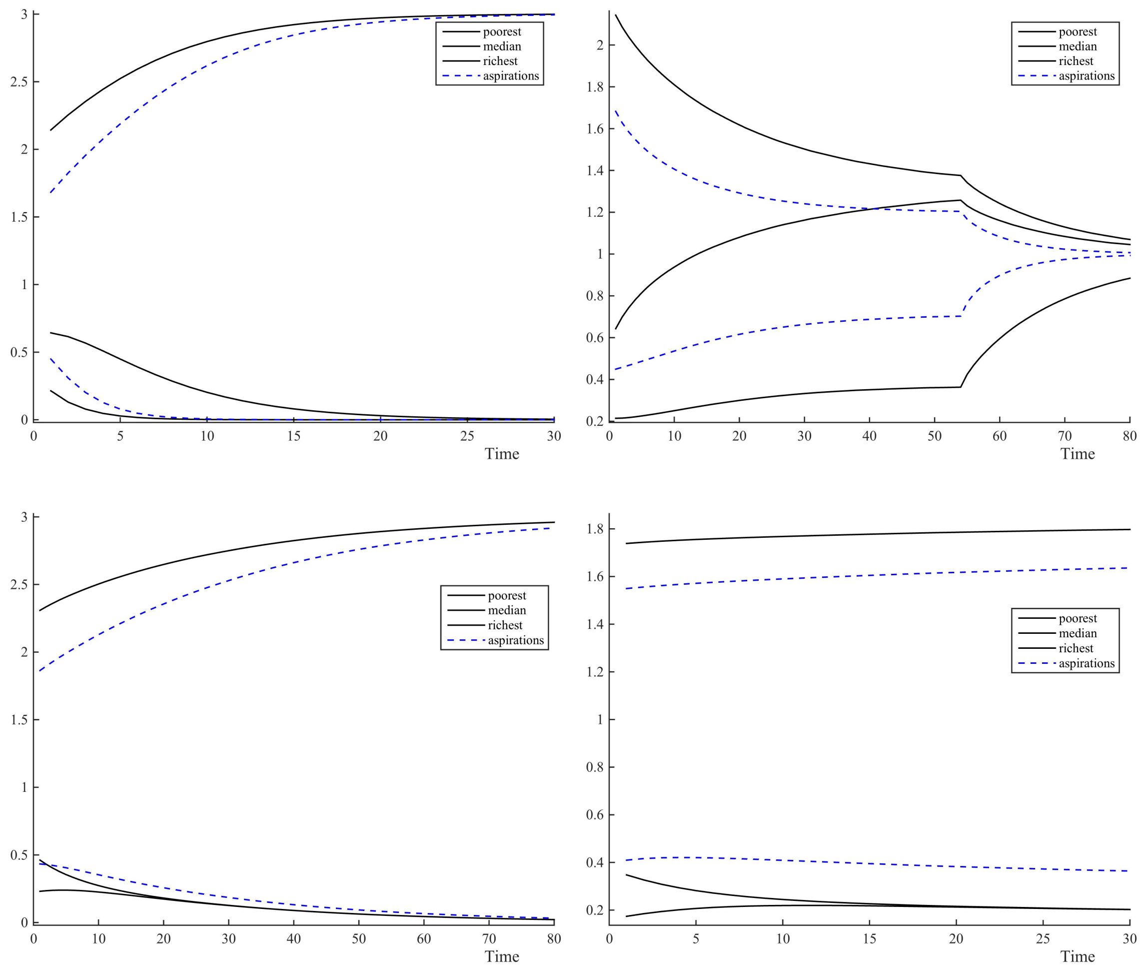


FIGURE 8. Evolution of Relative Incomes and Aspirations.

growth rate (see Table 1) and inequality is ever expanding. In particular, the incomes of the poorest and middle segments converge to zero relative to that of the richest segment.

The next three examples illustrate Case 2 of Proposition 9, in which everyone has some aspiration satisfied initially. To so so we raise the gross return on investment to $\rho = 3.3$.

Example $4B$ considers the same initial distribution with this higher return. Figure 8 in the upper-right quadrant shows that after some fluctuations in relative position, all incomes converge and we end up with full equality. There is balanced growth in the long run.

Example $4C$ alters the initial distribution. There are three groups, again of equal size, thus time with initial incomes of 100, 200 and $1,000$. The poorest and middle segments of the population converge to each other and end up growing at the same long-run rate, while the richest segment grows faster. So there could be ever-expanding inequality (as seen in the lower-let quadrant of Figure 8), even when all groups satisfy some aspirational steps.

Finally, Example $4D$ considers the same levels of initial income as Example $4C$, but alters the relative proportions of these groups. The initial distribution has $1/4$ of the population at 100, $1/2$ at 200 and $1/2$ at $1,000$. We see in the lower-right quadrant of Figure 8 that the two poorest segments of the population converge to each other and always satisfy the first level of aspirations, while the richest segment always satisfies all aspirations. Yet asymptotically all three groups grow at the same rate, and so relative inequality remains constant.

5. An Illustrative Empirical Exercise

We end with an extremely tentative estimation exercise. The reason the exercise is tentative is not because we hesitate to confront the model with the data. Rather, in a full-blown estimation exercise we would need to be far more sensitive to actual policy and regime changes in the economies in question, and control for such changes before applying the details of our model.

With those natural qualifications, and bearing in mind that the exercise to follow is only for illustrative purposes, we ask how much a model with just three aspirational steps can capture of the *actual* variation in growth across observed income distributions. We employ a dataset from the World Bank with 55 growth incidence curves (snapshots of growth rates for every percentile in the income distribution) for 43 distinct countries.[19]

Consider the constant-elasticity growth model with $\sigma = 0.6$ and $\delta = 0.8$. In the absence of aspirations, the constant-elasticity growth model would predict balanced growth across the income distribution. All income percentiles would grow at the same rate. By adjusting the return to capital ρ to match the *actual* aggregate annual rate growth observed in the data we obtain a growth incidence curve for the benchmark model without aspirations.

Next, we assume up to three levels of aspirations a_k and assume that each milestone is a weighted average of the incomes, $a_k = \sum w_p^k y_p$, where the weights are based on an exponential function of income: $w_p^k = y_p^{\alpha_k} / \sum y_p^{\alpha_k}$ and p indexes percentile. Any actual income distribution together with the coefficients α_k will generate milestones a_k for $k \in \{1, 2, 3\}$. Given these milestones a_k, their importance π_k in the society and a return to capital ρ, an individual with wealth y_p would choose continuation wealth z_p to maximize

$$\left(y_p - \frac{z_p}{\rho}\right)^{1-\sigma} + \delta \sum_{k=0}^{3} \pi_k \left(\max\{z_p - a_k, 0\}\right)^{1-\sigma}$$

where $\pi_0 \equiv 1$ and $a_0 \equiv 0$. This maximization gives us a predicted growth incidence curve: a growth rate for each income percentile.

For each of the observations in our dataset (that is, an initial percentile distribution and a growth rate for each percentile), we consider a range of possible values for α_k ($-5 < \alpha_1 < \alpha_2 < \alpha_3 \leq$

[19] Special thanks are due to Claudio Montenegro at the Development Research Group, Poverty Unit, The World Bank.

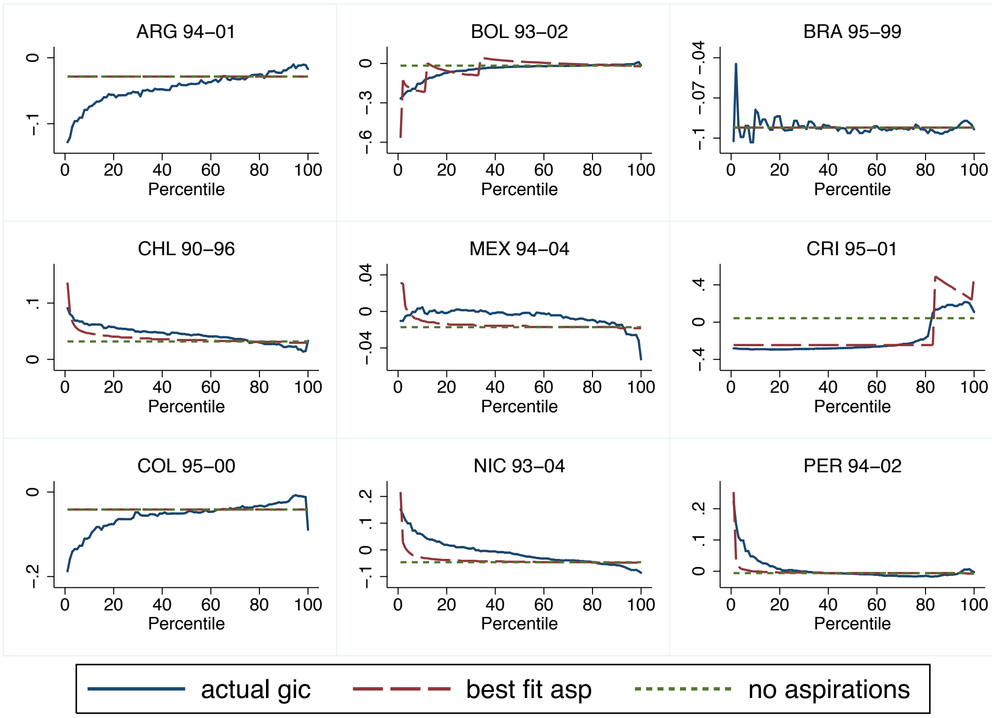


FIGURE 9. GROWTH INCIDENCE CURVES

5) and $\pi_k \in [0, 3]$ for $k \in \{1, 2, 3\}$. And for each combination of these parameters, we find the return to capital ρ that generate in our model the *actual* aggregate annual rate growth observed for each set of parameters. Then, we select among all possible combinations of parameters, the one that minimizes the squared distance between the actual and predicted growth incidence curve (including the benchmark model as one of these combinations). We call the resulting predicted growth incidence curves, our "best fit" growth incidence curves.

Although the limitations of this exercise are obvious, here are a few observations.

For the 34 growth incidence curves (60%) that are mostly flat, the benchmark model does a good job and our best-fit model puts no weight on aspirations ($\pi_k = 0$ for all k). 7 countries have single step aspirations while 14 have multi-step aspirations (10 have 2-step and 4 have 3-step aspirations). When these aspirations matter, the first step is around the first percentile, the second step is around the 12th percentile and the third step around the 17th percentile of the distribution.

Though the pure aspirations-based model is not a perfect match, it does a reasonably good job at fitting the growth rates by percentile. Our specification captures 82% of the observed variation in growth within each distribution.[20] Figure 9 illustrates the "best fit" growth incidence curves, together with the actual growth incidence curve and the benchmark ones (without aspirations),

[20]This is the R^2 of a regression of the actual percentile growth on predicted percentile growth with country-year fixed effects.

for the main nine Latin American countries for which we have observations in the nineties.[21] Among these, a model without aspirations provides the best fit for 3 of them, one country has 3 step-aspirations while for the remaining 5 a 1 or 2-step aspiration model provides the best fit. Overall our exercise is promising and suggests a scope for a rigorous calibration exercise using repeated growth incidence curves for one country and accounting for other factors that might affect the growth profile.

6. Conclusion

This paper builds a theory of aspirations formation. The theory emphasizes the social foundations of individual aspirations, and relates those aspirations in turn to investment and growth. Following a familiar lead from behavioral economics (see, e.g., Kahneman and Tversky (1979), Karandikar et al. (1998), and Kőszegi and Rabin (2006)), we define utilities around "reference points," and interpret these reference points as *milestones* or *aspirations*. We depart from this literature in several ways. First, and most important, we focus on the social determination of aspirations, in contrast to the past experiences of the individual herself, or some self-fulfilling belief about what she expects. We argue that aspirations are as likely to depend on the experience and lifestyle of others.

Second, we allow for multiple reference points, each of which become salient at different living standards. In this way we permit milestones (or at least those milestones that are salient) to depend on one's own living standards.

Finally, aspirations determine an individual's incentives to invest and bequeath. Such behavior can be aggregated across individuals to derive the society-wide distribution of income, thus closing the model. This equilibrium interplay between the individual and the social is a main theme of the paper.

A central feature of our theory is that aspirations can serve both to incentivize and to frustrate. We show that aspirations that are above — but not too far — from current incomes can encourage high investment, while aspirations that are too high may discourage it. Hence, rising aspirations not only decrease individual utilities but can also lack instrumental value. This insight has implications for growth rates across a cross-section of aspirations for a given starting income, as well as for growth rates across a cross-section of incomes, for a given level of aspirations.

A study of society-wide equilibrium leads to additional insights. Steady state distributions must exhibit inequality and be concentrated on a few mass points. With single-step aspirations (just one reference point), steady state distributions are bipolar: with convergence to one level of income associated with failed aspirations and to another associated with satisfied aspirations.

In the canonical linear model, sustained growth is possible. When aspirations are single-step, and initially satisfied by at least some individuals there are only two outcomes possible: either convergence to an equal distribution (with growth) or perennial relative divergence with two components, so that ever-expanding inequality is the result. With multi-step aspirations there is a finer range of predictions; for instance, not only are these two patterns are possible, but so is

[21] We would be happy to provide the graphs for all countries on request.

balanced growth of all incomes at the same rate, which maintaining a constant degree of relative inequality.

The goal of this paper has been to model aspirations as a socially determined set of reference points. Our model has the advantage of being tractable and has allowed us to explore the relationship between aspirations and inequality. We believe that the simplicity of the framework is also conducive to several extensions. We mention a few of interest to us. One direction is a theory of *group-based* aspirations, in which different social or ethnic groups draw their reference points in different ways from society. For instance, Munshi and Myaux (2006) argue that fertility behavior among particular religious groups in Bangladesh influence fertility norms for couples in the same religious groups, but not across groups. This sort of study also suggests a second extension, in which aspirations themselves are multidimensional: not some narrow scalar notion such as wealth, as explored here, but an entire complex that might include education, fertility or social achievement. Developing the model along these lines would tie these ideas in with the notion of capabilities developed by Sen (1985), except that such "capabilities" would, in part, appear as a relativistic construct, inspired by the achievements of others. A third extension, also influenced in part by the notion of group-based aspirations, would link aspirations to frustration and subsequent violence. Presumably these models would extend the simple allocative exercise in this paper to a three-way allocation across consumption, productive investment, and resources spent in social conflict. This sort of theory would tie into recent empirical analyses of uneven growth and conflict, such as Dube and Vargas (2013) and Mitra and Ray (2014). Finally, if one is willing to take these models a bit more literally, it is possible to use growth incidence curves (say, by decile or percentile), along with some controls to account for major policy or regime shifts, to actually estimate the aspirations-formation process for different societies. Section 5 contains the most tentative beginnings of such an exercise.

REFERENCES

Alonso-Carrera, Jaime, Jordi Caball, and Xavier Raurich. 2007. "Aspirations, Habit Formation, and Bequest Motive." *Economic Journal* 117 (520):813–836.

Appadurai, Arjun. 2004. "The Capacity to Aspire." In *Culture and Public Action*, edited by Michael Walton and Vijayendra Rao. Stanford, CA: Stanford University Press.

Banerjee, Abhijit V. and Andrew Newman. 1993. "Occupational Choice and the Process of Development." *Journal of Political Economy* 101 (2):274–298.

Banerjee, Abhijit V. and Tomas Piketty. 2005. "Top Indian Incomes: 1922–2000." *World Bank Economic Review* 19 (1):1–20.

Becker, Gary, Kevin Murphy, and Ivan Werning. 2005. "The Equilibrium Distribution of Income and the Market for Status." *Journal of Political Economy* 113:282–301.

Becker, Gary S. and Nigel Tomes. 1979. "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility." *Journal of Political Economy* 87 (6):1153–1189.

Berger, Jonah and Devin Pope. 2011. "Can Losing Lead to Winning?" *Management Science* 57 (5):817–827.

Bogliacino, Francesco and Pietro Ortoleva. 2014. "The Behavior of Others as a Reference Point." Working paper.

Brock, W. and L. Mirman. 1972. "Reciprocity in Groups and the Limits to Social Capital." *Journal of Economic Theory* 4:479–513.

Canova, Fabio. 2004. "Testing for Convergence Clubs in Income Per Capita: A Predictive Density Approach." *International Economic Review* 45 (1):49–77.

Carroll, Christopher and David Weil. 1994. "Saving and Growth: A Reinterpretation." *Carnegie-Rochester Conference Series on Public Policy* 40:133–192.

Clark, Andrew and Andrew Oswald. 1996. "Satisfaction and Comparison Income." *Journal of Public Economics* 61:359–381.

Corneo, Giacomo and Olivier Jeanne. 1997. "Conspicuous Consumption, snobbism and conformism." *Journal of Public Economics* 66:55–71.

———. 1999. "Pecuniary emulation, inequality and growth." *European Economic Review* 43 (9):1665–1678.

Croix, David de la and Philippe Michel. 2001. "Altruism and Self-Restraint." *Annales d'Économie et de Statistique* (63):233–259.

Dalton, Patricio, Sayantan Ghosal, and Anandi Mani. 2014. "Poverty and Aspirations Failure." *Economic Journal* :forthcoming.

Deaton, Angus and Jean Drèze. 2002. "Poverty and Inequality in India." *Economic and Political Weekly* 37:3729–3748.

Drèze, Jean and Amartya Sen. 2013. *An Uncertain Glory: India and Its Contradictions*. Princeton, NJ: Princeton University Press.

Dube, Oeindrila and Juán Vargas. 2013. "Commodity Price Shocks and Civil Conflict: Evidence from Colombia." *Review of Economic Studies* 80:1384–1421.

Duesenberry, James S. 1949. *Income, Saving and the Theory of Consumer Behavior*. Cambridge, MA: Harvard University Press.

Durlauf, Steven and Paul Johnson. 1995. "Multiple Regimes and Cross-Country Growth Behaviour." *Journal of Applied Econometrics* 10 (2):97–108.

Durlauf, Steven and Danny Quah. 1999. "The New Empirics of Economic Growth." In *Handbook of Macroeconomics*, edited by Michael Woodford John Taylor. Elsevier, 235–308.

Esteban, Joan-Mara and Debraj Ray. 1994. "On the Measurement of Polarization." *Econometrica* 62 (4):pp. 819–851. URL `http://www.jstor.org/stable/2951734`.

Ferrara, Eliana La, Alberto Chong, and Suzanne Duryea. 2012. "Soap Operas and Fertility: Evidence from Brazil." *American Economic Journal: Applied Economics* 4 (4):1–31.

Frank, Robert. 1985. *Choosing the Right Pond: Human Behavior and the quest for status.* Oxford University Press, New York.

Freeman, Scott. 1996. "Equilibrium Income Inequality among Identical Agents." *Journal of Political Economy* 104 (5):1047–1064.

Galor, Oded and Joseph Zeira. 1993. "Income Distribution and Macroeconomics." *Review of Economic Studies* 60 (1):35–52.

Genicot, Garance and Debraj Ray. 2009. "Aspirations and Mobility." Background paper for the lac-regional report on human development 2009, UNDP.

Ghatak, Maitreesh, Parikshit Ghosh, and Ashok Kotwal. 2014. "Growth in the time of UPA: Myths and Reality." *Economic and Political Weekly* 49:April 19.

Goux, Dominique, Marc Gurgand, and Eric Maurin. 2014. "Adjusting Your Dreams? The Effect of School and Peers on Dropout Behaviour." Discussion Paper 7948, IZA.

Heath, Chip, Richard P. Larrick, and George Wu. 1999. "Goals as Reference Points." *Cognitive Psychology* 38:79–109.

Henderson, Daniel J., Christopher F. Parmeter, and R. Robert Russell. 2008. "Modes, weighted modes, and calibrated modes: evidence of clustering using modality tests." *Journal of Applied Econometrics* 23 (5):607–638.

Hopkins, Ed and Tatiana Kornienko. 2006. "Inequality and Growth in the Presence of Competition for Status." *Economics Letters* 93:291–296.

i Martin, Xavier Sala. 2006. "The World Distribution of Income: Falling Poverty and ... Convergence, Period." *The Quarterly Journal of Economics* 121 (2):351–397.

Jensen, Robert and Emily Oster. 2009. "The Power of TV: Cable Television and Women's Status in India." *The Quarterly Journal of Economics* 124 (3):1057–1094.

Kahneman, Daniel and Amos Tversky. 1979. "Prospect Theory: An Analysis of Decision under Risk." *Econometrica* 47:263–291.

Karandikar, Rajeeva, Dilip Mookherjee, Debraj Ray, and Fernando Vega-Redondo. 1998. "Evolving Aspirations and Cooperation." *Journal of Economic Theory* 80:292–331.

Kearney, Melissa S. and Phillip B. Levine. 2014. "Income Inequality, Social Mobility, and the Decision to Drop Out of High School." Working Paper w20195, NBER.

Kőszegi, Botond and Matthew Rabin. 2006. "A Model of Reference-Dependent Preferences." *Quarterly Journal of Economics* 121.

LeBoeuf, Robyn A. and Zachary Estes. 2004. ""Fortunately, I'm no Einstein": Comparison Relevance as a Determinant of Behavioral Assimilation and Contrast." *Social Cognition* 22 (6):607–636.

Lockwood, Penelope and Ziva Kunda. 1997. "Superstars and Me: Predicting the Impact of Role Models on the Self." *Journal of Personality and Social Psychology* 73 (1):91–103.

Macours, Karen and Renos Vakis. 2009. "Changing Households: Investments and Aspirations through Social Interactions: Evidence from a Randomized Transfer Program." Policy Research Working Paper 5137.

Matsuyama, Kiminori. 2004. "Financial Market Globalization, Symmetry-Breaking, and Endogenous Inequality of Nations." *Econometrica* 72:853–884.

Mishra, Pankaj. 2014. "Narendra Modi and the new face of India." *The Guardian* .

Mitra, Anirban and Debraj Ray. 2014. "Implications of an Economic Theory of Conflict: Hindu-Muslim Violence in India." *Journal of Political Economy* 122:719–765.

Mookherjee, Dilip and Debraj Ray. 2003. "Persistent Inequality." *Review of Economic Studies* 70 (2):369–394.

Munshi, Kaivan and Jacques Myaux. 2006. "Social Norms and the Fertility Transition." *Journal of Development Economics* 80:1–38.

Piketty, Thomas. 2014. *Capital in the Twenty First Century*. Cambridge, MA: Harvard University Press.

Pittau, Maria Grazia and Roberto Zelli. 2004. "Testing for changing shapes of income distribution: Italian evidence in the 1990s from kernel density estimates." *Empirical Economics* 29 (2):415–430.

Pittau, Maria Grazia, Roberto Zelli, and Paul A. Johnson. 2010. "Mixture Models, Convergence Clubs, And Polarization." *Review of Income and Wealth* 56 (1):102–122.

Quah, Danny. 1993. "Empirical Cross-Section Dynamics in Economic Growth." *European Economic Review* 37:426–434.

———. 1996. “Twin Peaks: Growth and Convergence in Models of Distribution Dynamics.” *Economic Journal* 106:1045–1055.

Ray, Debraj. 1998. *Development Economics*. Princeton University Press.

———. 2006. “Aspirations, Poverty and Economic Change.” In *What Have We Learnt About Poverty*, edited by R. Bènabou A. Banerjee and D. Mookherjee. Oxford University Press.

Ray, Debraj and Arthur Robson. 2012. “Status, Intertemporal Choice and Risk-Taking.” *Econometrica* 80:1505–1531.

Robson, Arthur J. 1992. “Status, the Distribution of Wealth, Private and Social Attitudes to Risk.” *Econometrica* 60:837–857.

Schor, Juliet B. 1992. *The Overworked American: The Unexpected Decline of Leisure*. New York: Basic Books.

Scitovsky, Tibor. 1976. *The Joyless Economy*. New York: Oxford University Press.

Sen, Amartya. 1985. *Commodities and Capabilities*. Amsterdam and New York: North Holland.

Shalev, Jonathan. 2000. “Loss aversion equilibrium.” *International Journal of Game Theory* 29 (2):269–287.

Veblen, Thorstein. 1899. *The Theory of the Leisure Class.* Viking, New York, 53–214.

Wolfson, Michael C. 1994. “When Inequalities Diverge.” *The American Economic Review* 84 (2):pp. 353–358. URL `http://www.jstor.org/stable/2117858`.

Zhu, Feng. 2005. “A nonparametric analysis of the shape dynamics of the US personal income distribution: 1962-2000.” BIS Working Papers 184, Bank for International Settlements.