



PSY9140 – STRUCTURAL EQUATION MODELLING

Using R/Lavaan

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Monday 5th and Wednesday 7th

- Introduction to R/Rstudio and Lavaan
- Refresh fundamental statistical concepts.
- Components of SEM modelling
 - Path models, identification, estimation, model evaluation,
- Confirmatory factor analysis (CFA)

Friday 9th

- Path analysis and structural regression models

Monday 12th

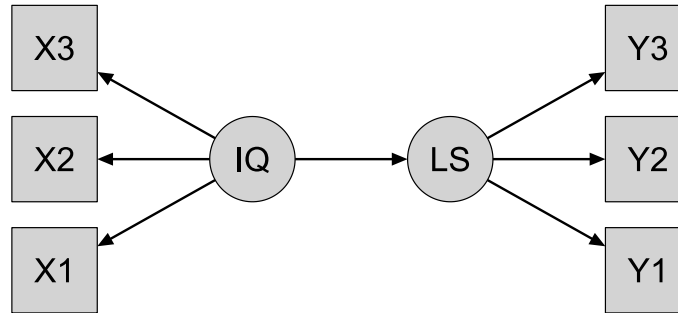
- Longitudinal data analysis using SEM

Wednesday 14th

- Complications
 - Categorical, non-normal, and clustered data.
- Selected topics: Invariance and ESEM

INTRODUCTION TO PSY9140

WHAT IS SEM?



1. Path analysis

- Describes directed dependencies (causal processes) among a set of single indicator variables.
- Can be expressed as a figure, a *path model*.

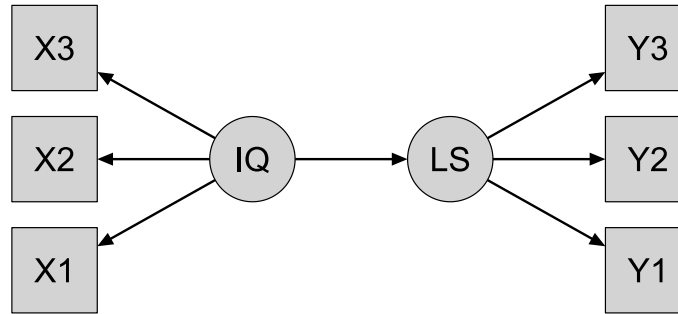
2. Latent variable models

- SEM combines confirmatory factor analysis (CFA) and path analysis.

3. Procedures to estimate parameters and evaluate fit

- A host of estimators and fit statistics.

WHY USE SEM?



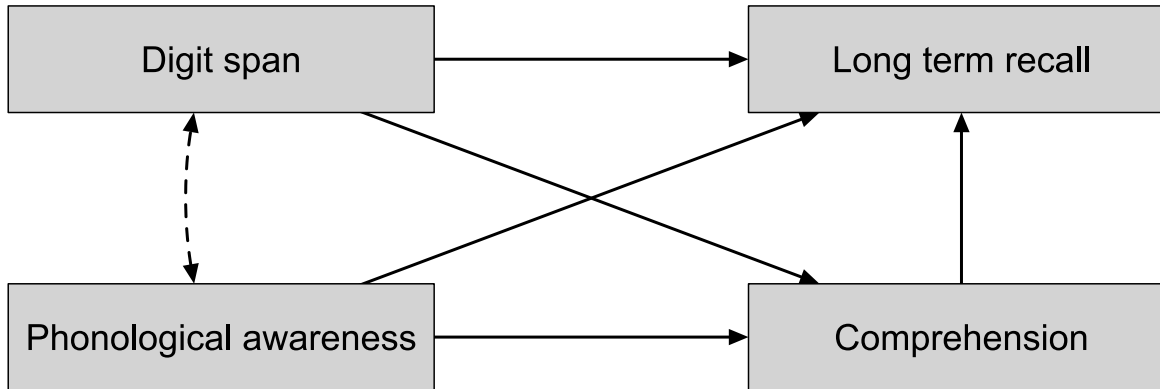
- **SEM is confirmatory in nature**

- The proposed causal explanations are made explicit.
- With SEM you can test competing theoretical models.

- **SEM can handle measurement error and latent variables**

- Less prone to bias when there is measurement error in the independent variables.
- Increased power

WHY USE SEM? : FLEXIBILITY



- **SEM models are flexible**

- Can be used to evaluate *complex* theories.
- Can incorporate both latent and observed variables.

SEM INVOLVES THE FOLLOWING MAJOR STEPS

Specify a model

- The SEM framework allows us to *specify* a "causal" model for the associations between variables.
- Using *path analysis* we can derive the *expected* co-variance matrix for our variables (Σ) given this model.

Estimate parameters

- We choose as parameter estimates ($\hat{\theta}$) values that minimize the distance between the expected ($\Sigma(\hat{\theta})$) and observed (S) co-variance matrices.

Evaluate fit

- With enough data, we can assess the *fit* of the model, or *compare* alternative models.

- **R - computing platform**
 - Various SEM packages can be installed (Lavaan, openMx, SEM).
- **MPLUS**
 - Very powerful SEM package, with latest features.
 - <https://www.statmodel.com/>
- **AMOS**
 - Amos is an SPSS extension.
- **STATA**
- **LISREL**

WHY USE THE LAVAAN SEM PACKAGE?

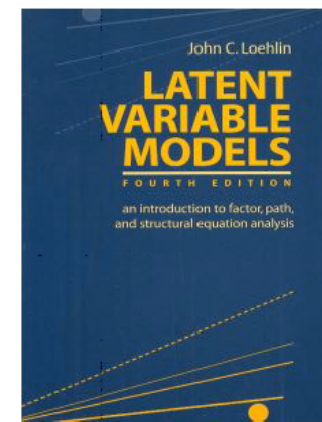
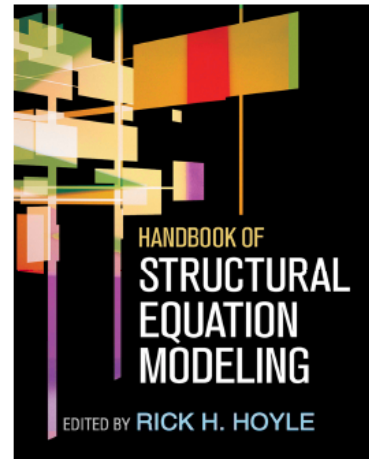
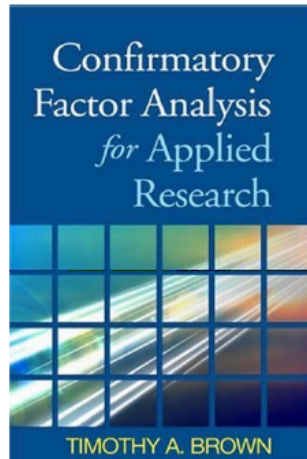
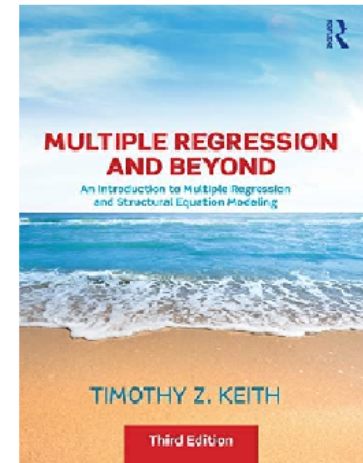
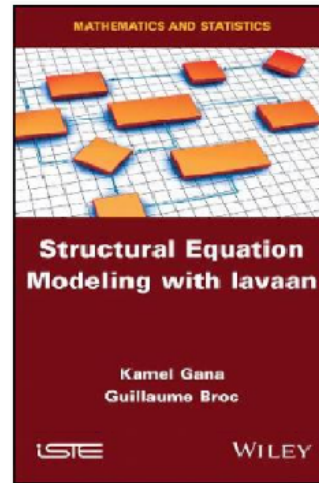
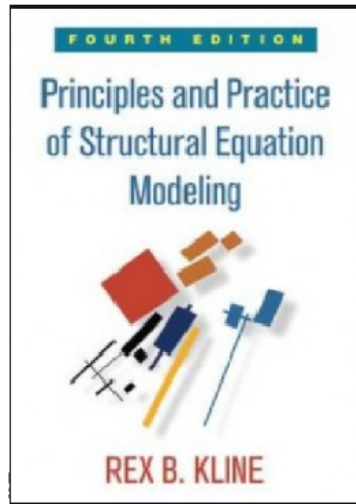
Strengths

- Free and open source.
- Access to all features of the R platform
 - Excellent support for plotting, data management, statistical modelling.
 - R can read all major file formats (SAV, XLSX, DTA).
- Simple model specification syntax (built on Mplus).
- Constantly updated/improved.
- Supports a host of estimators, missing, ordinal and multilevel data.

Limitations

- Black box: Simple syntax hides model complexities.
- Documentation/online resources somewhat limited.

SEM REFERENCES



INTRODUCTION TO R AND RSTUDIO

THE R PROJECT FOR STATISTICAL COMPUTING

- R is a programming language and free software environment for statistical computing and graphics supported by the R Foundation.
 - <https://www.r-project.org/>
- The R language is widely used platform for data analysis and data science across all sciences.
- Comes with a small core of functions, but can be extended from a vast library of Several thousands *packages*.



[https://en.wikipedia.org/wiki/R_\(programming_language\)](https://en.wikipedia.org/wiki/R_(programming_language))



- RStudio is an integrated development environment (IDE) for R, a programming language for statistical computing and graphics.
 - <https://rstudio.com/>
 - Greatly simplifies work in large projects.
- RStudio, Inc. is a commercial enterprise, and has no formal connection to the R Foundation.

SOME R RESOURCES

Official (brief) R documentation:

- <http://cran.r-project.org/doc/manuals/r-release/R-intro.pdf>

If you plan to use R in your work, read this book!:

- <https://r4ds.had.co.nz/>

R-bloggers is a popular portal for R users:

- <https://www.r-bloggers.com/2017/03/the-5-most-effective-ways-to-learn-r/>

- R is case sensitive.
- Commands are separated either by a semi-colon (;), or by a newline.
- Comments start with a hashmark (#), everything to the end of the line is a comment.
- Run highlighted text with «Ctrl-R»

The tidyverse is an opinionated collection of R packages designed for data science. All packages share an underlying design philosophy, grammar, and data structures.

- <https://www.tidyverse.org/>

REGRESSION AND ASSOCIATED CONCEPTS

READING DATA INTO THE R ENVIRONMENT

```
# Read a comma-separated text file
myData.csv = read.csv("my_data_file.csv")

# the rio package reads files from virually any platform
# (spss, stata, excel ...)
library(rio)
dat = import("my_data_file.sav")

# Type in data manually though editor
xnew ← edit(data.frame())

View(dt)
```

THE DATASET (N=1061)

- **Right-wing authoritarianism (RWA):**
 - Right-wing authoritarianism is characterized by obedience to authority, moral absolutism and punitiveness towards dissidents and deviants.
 - 15 items
- **Symptom Checklist (SCL8)**
 - Items measuring depression and anxiety
 - 8 items
- **Justice Sensitivity (JS):**
 - People differ systematically in their inclination to perceive injustice and the strength of their cognitive, emotional, and behavioral reactions.
 - 9 items
- **Big-Five inventory (BFI)**
 - Extraversion (BFI_E), Agreeableness (BFI_A), Conscientiousness (BFI_C), Neuroticism (BFI_N) and Openness (BFI_O).
- Other variables of interest: male, age, edu (education)

RIGHT-WING AUTHORITARIANISM (RWA)

1. Our country needs a strong leader to defeat radical and immoral currents.
2. Our country needs free thinkers who dare to stand up to traditional ways of behaving, even if it offends many.
3. The "old-fashioned ways and values" still show the best way to live.
4. Our society would be better off if we showed tolerance and understanding for non-traditional values and views.
5. God's laws of abortion and marriage must be followed carefully and transgressions must be punished.
6. Society must show openness to people who think differently.
7. It would be best if newspapers were censored so that people did not have access to destructive and reprehensible material.
8. Many good people challenge the state and ignore the "normal lifestyle".
9. We should honor our ancestors more for how they built our society, and at the same time we should put an end to the forces that destroy it.
10. People should pay less attention to the Bible and religion and rather develop their own moral standards.
11. There are many radical, immoral people who try to destroy things; society should stop them.
12. It is better to accept immoral literature than to censor it.
13. Facts show that we must be tougher on crime and sexual immorality in order to maintain law and order.
14. Today's society would be better off if troublemakers were met with reason and compassion.
15. If society so desires, it is the duty of every citizen to help eliminate the poisonous evil in our country.

SYMPTOM CHECKLIST (SCL-8)

During the last 2 weeks, have you been bothered by any of the following? (Not bothered, a little bothered, fairly bothered, very bothered)

1. Constantly scared or anxious (A)
2. Nervousness, inner turmoil (A)
3. Feeling of hopelessness about the future (D)
4. Depressed, melancholy (D)
5. Very worried or uneasy (D)
6. Feeling that everything is a struggle (D)
7. Feeling tense or hurried (A)
8. Sudden fear for no reason (A)

JUSTICE SENSITIVITY ITEMS

1. It makes me angry when others are undeservedly better off than me
2. It worries me when I have to work hard for something that others easily achieve
3. If others are treated better than me, it is difficult for me to stop thinking about it
4. I get upset when some are undeservedly worse off than others
5. It worries me when someone has to work hard for something that others easily get
6. I feel guilty when I am better off than others for no reason
7. It bothers me when I easily get to something that others have to work hard for
8. I feel guilty when I enrich myself at the expense of others
9. It bothers me when I use tricks to achieve something that others must strive for

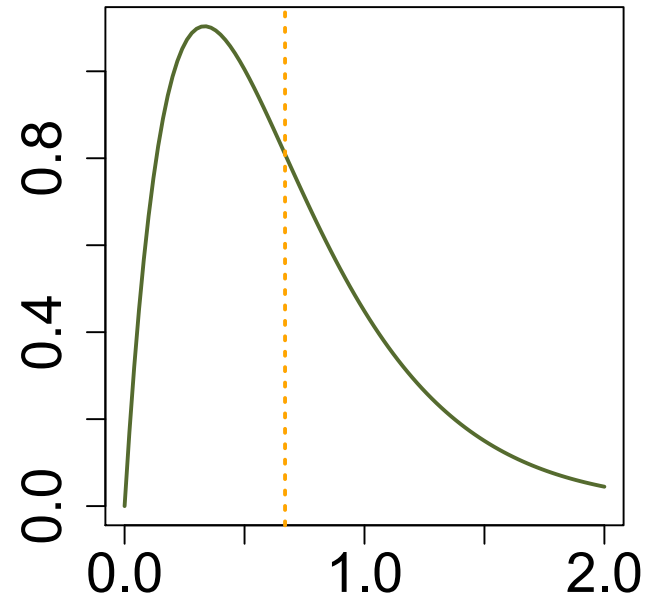
AUTISM SPECTRUM QUOTIENT (ASQ)

The Autism-Spectrum Quotient Test, first published in 2001 by Simon Baron-Cohen, is a diagnostic questionnaire designed to measure the expression of Autism-Spectrum traits in an individual.

1. I often notice small noises when others do not
2. I usually concentrate more on the whole picture, rather than on the small details.
3. I find it easy to do more than one thing at once.
4. If there is an interruption, I can switch back to what I was doing very quickly.
5. I find it easy to “read between the lines” when someone is talking to me.
6. I know how to tell if someone listening to me is getting bored.
7. When I’m reading a story, I find it difficult to work out the characters’ intentions.
8. I like to collect information about categories of things (e.g., types of cars, birds, trains, plants).
9. I find it easy to work out what someone is thinking or feeling just by looking at their face.
10. I find it difficult to work out people’s intentions.

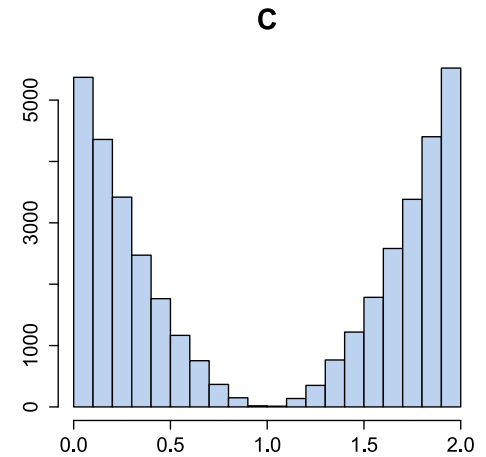
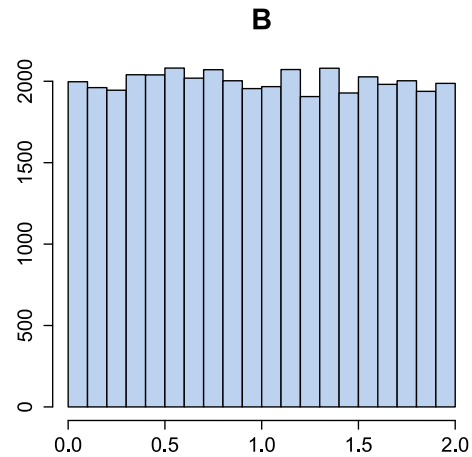
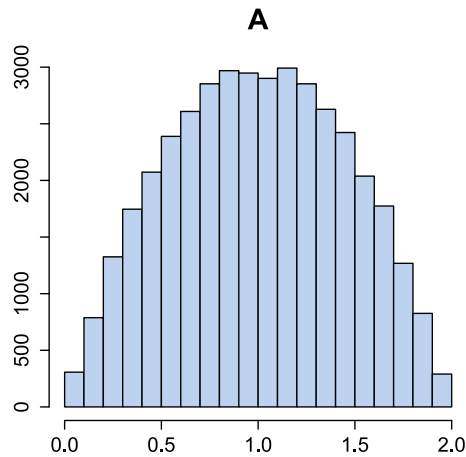
ARITHMETIC MEAN

$$\bar{X} = \frac{\sum X_i}{n}$$



The **mean** is one of several measures of the *central tendency* of a distribution.

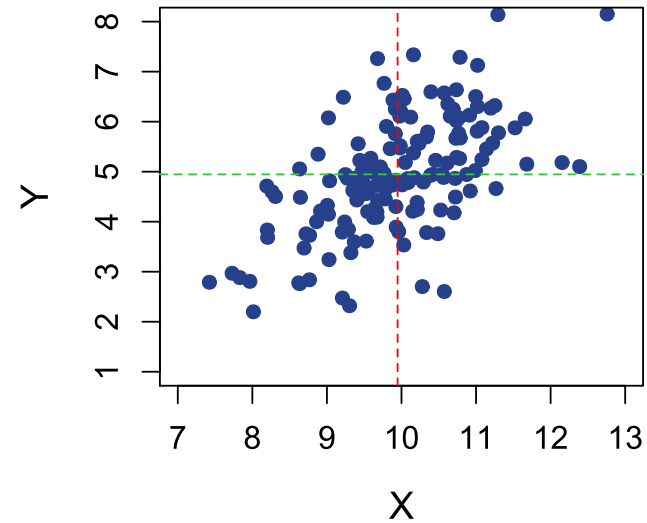
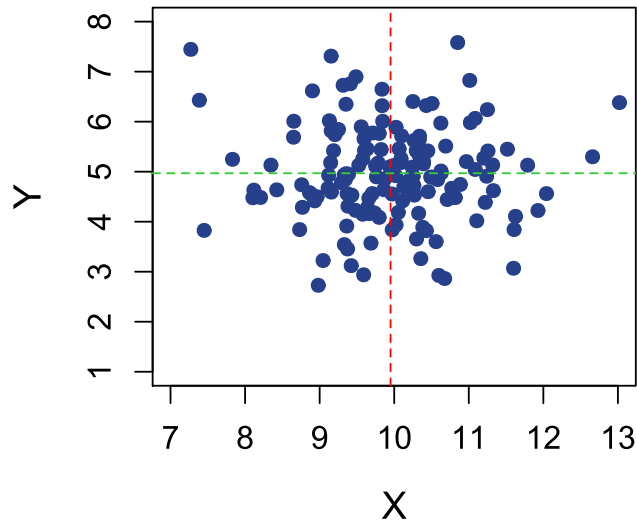
VARIANCE



$$\text{Var}(X) = s_X^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1}$$

Variance is a measure of the variability around the expected value (mean) of a distribution.

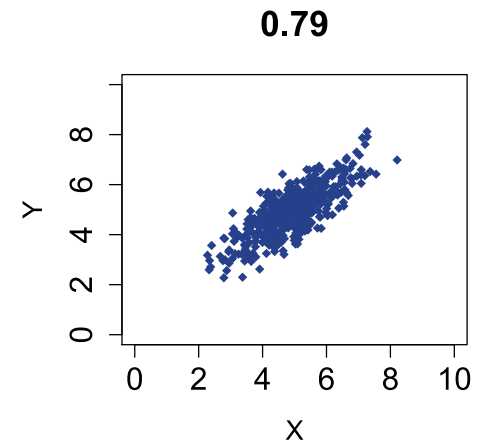
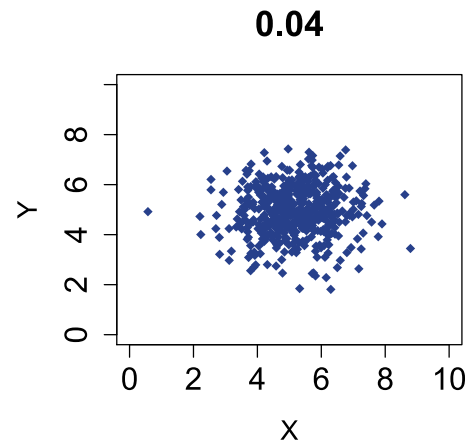
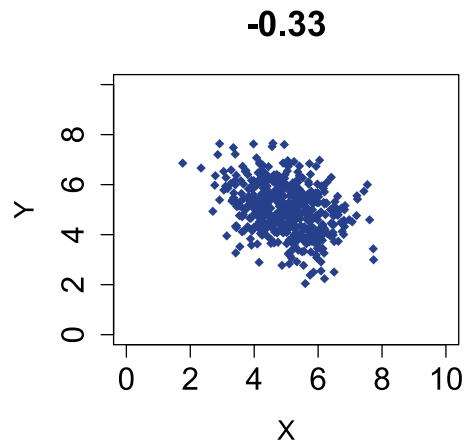
COVARIANCE



$$\text{Cov}(X, Y) = s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

Covariance is a measure of the joint variability of two random variables.

CORRELATION

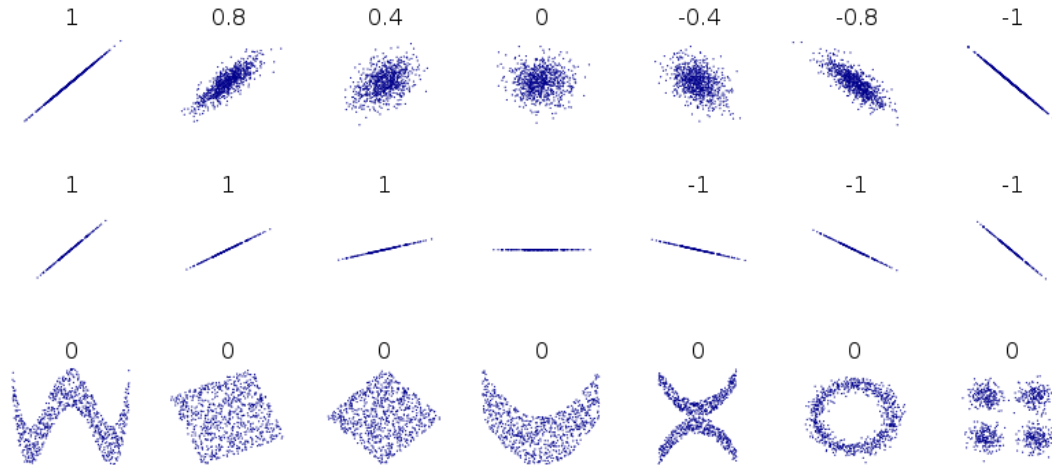


$$r_{xy} = \frac{s_{xy}}{s_x s_y}$$

Pearson correlation is the standardized covariance.

- Ranges between -1 and +1

CORRELATION AND LINEARITY



Pearson correlation quantifies the degree of *linear* relationship between two variables.

[1] See [Wikipedia](#) .

In mathematics, a **matrix** is a rectangular arrangement of numbers.

$$M = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 6 & 8 \end{bmatrix}$$

We refer to an *element* in a matrix (typically lowercase) m_{ij} , where i refers to the row, and j to the column.

- $m_{12} = 3$
- $m_{23} = 8$

If X_1 , X_2 and X_3 are three observed measures then the **covariance matrix** consisting of the following arrangement of variance and covariance statistics:

$$\begin{bmatrix} Var(X_1) & Cov(X_1, X_2) & Cov(X_1, X_3) \\ Cov(X_2, X_1) & Var(X_2) & Cov(X_2, X_3) \\ Cov(X_3, X_1) & Cov(X_3, X_2) & Var(X_3) \end{bmatrix}$$

If there are k observed measures, the covariance matrix contains $k(k + 1)/2$ *unique* values.

EXAMPLE: COVARIANCE MATRIX FOR BFI EXTRAVERSION

Extraversion	BF11 Is talkative	BF16 Is reserved	BF111 Is full of energy	BF116 Generates a lot of enthusiasm	BF121 Tends to be quiet	BF126 Has an assertive personality	BF131 Is sometimes shy, inhibited	BF136 Is outgoing, sociable
BF11 Is talkative	0,78	,450	,319	,334	,646	,190	,532	,516
BF16 Is reserved	0,45	1,027	,482	,456	,678	,060	,719	,679
BF111 Is full of energy	0,32	,482	,890	,453	,400	,269	,269	,545
BF116 Generates a lot of enthusiasm	0,33	,456	,453	,739	,362	,143	,387	,512
BF121 Tends to be quiet	0,65	,678	,400	,362	1,022	,177	,689	,675
BF126 Has an assertive personality	,190	,060	,269	,143	,177	,925	,217	,179
BF131 Is sometimes shy, inhibited	,532	,719	,269	,387	,689	,217	1,315	,642
BF136 Is outgoing, sociable	,516	,679	,545	,512	,675	,179	,642	,972

INTERPRETING A COVARIANCE MATRIX

$$S = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 9 & 0 \\ 2 & 0 & 16 \end{bmatrix}$$

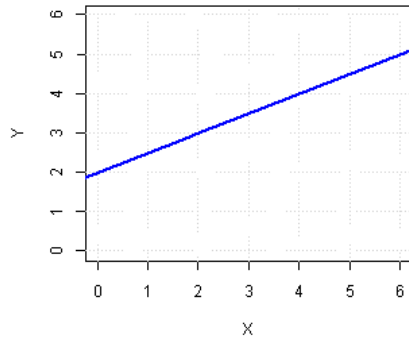
- What is the variance of item 2?
- What is the covariance between item 1 and item 2?
- What is the standard deviation of item 3?
- What is the correlation between item 1 and item 2? $r = \frac{cov(X,Y)}{sd(X)sd(Y)}$
- How many *unique* statistics are there in the matrix?

QUICK REFRESHER OF REGRESSION

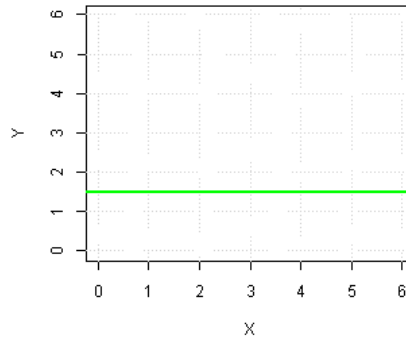
REPRESENTING LINES MATHEMATICALLY

A straight line is defined uniquely by two parameters.

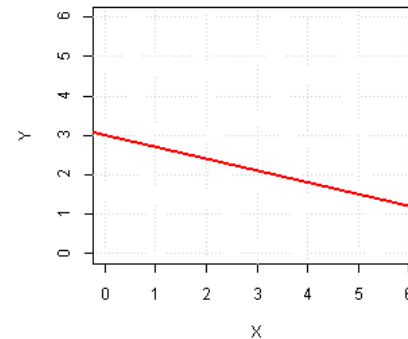
$$\hat{Y}_i = b_0 + b_1 \times X$$



$$b_0 = 2$$
$$b_1 = 0.5$$



$$b_0 = 1.5$$
$$b_1 = 0$$



$$b_0 = 3$$
$$b_1 = -0.5$$

In **bivariate regression** we have one dependent variable (Y) and one independent variable (X).

The process underlying the observed data is thought to be:

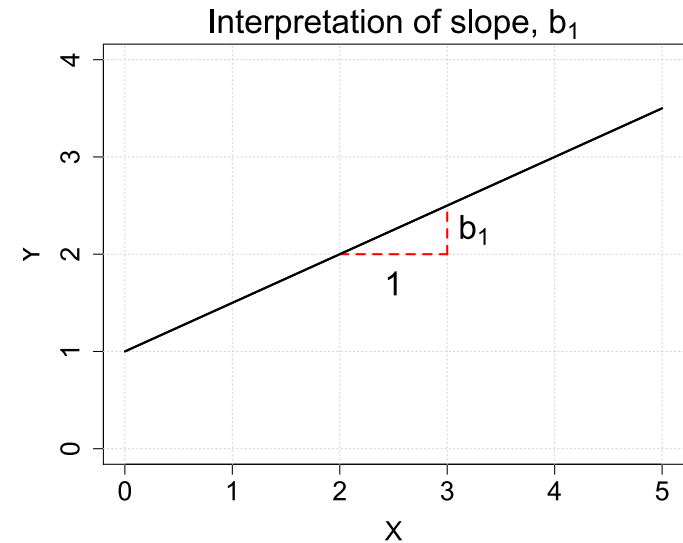
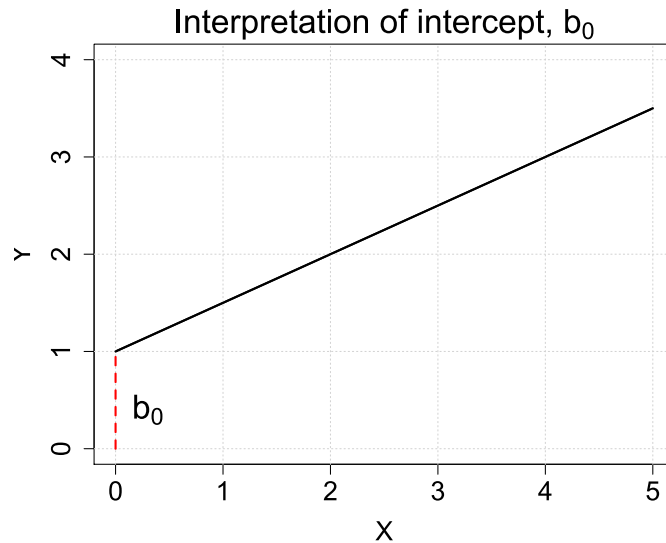
$$Y_i = b_0 + b_1 \cdot X_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2)$$

The *expected* Y value given a level of X based on estimated regression coefficients is given by:

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 \cdot X_i$$

UNSTANDARDIZED REGRESSION COEFFICIENTS

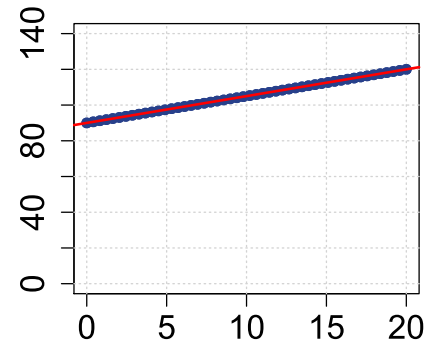
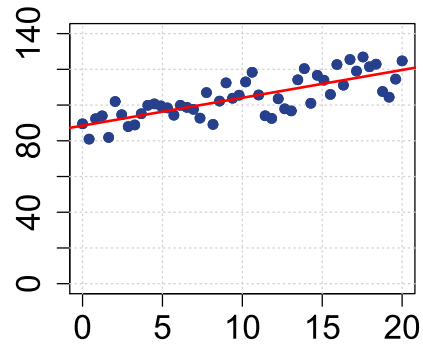
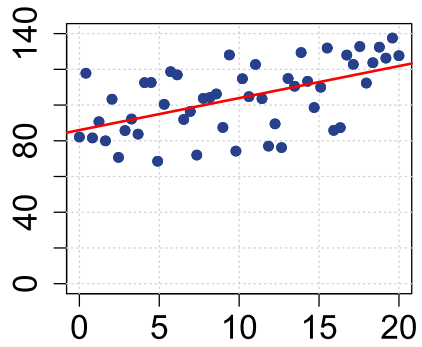
$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 \cdot X_i$$



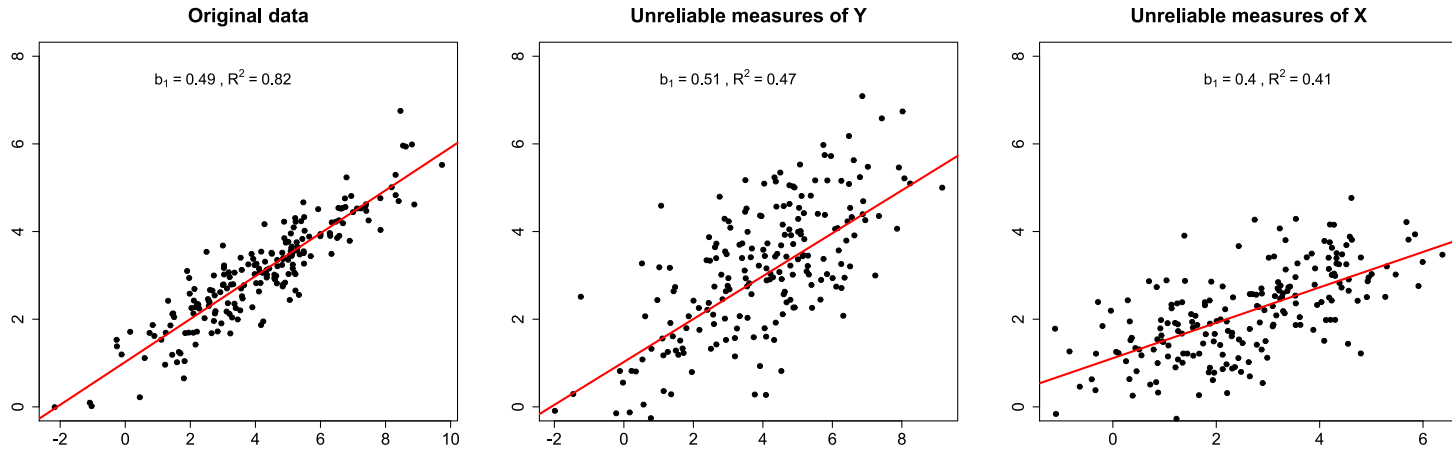
Intercept: value of \hat{Y} when X equals zero.

Slope: The change in \hat{Y} that follows a one-unit change in X .

EXPLAINED VARIANCE



THE EFFECT OF MEASUREMENT ERROR



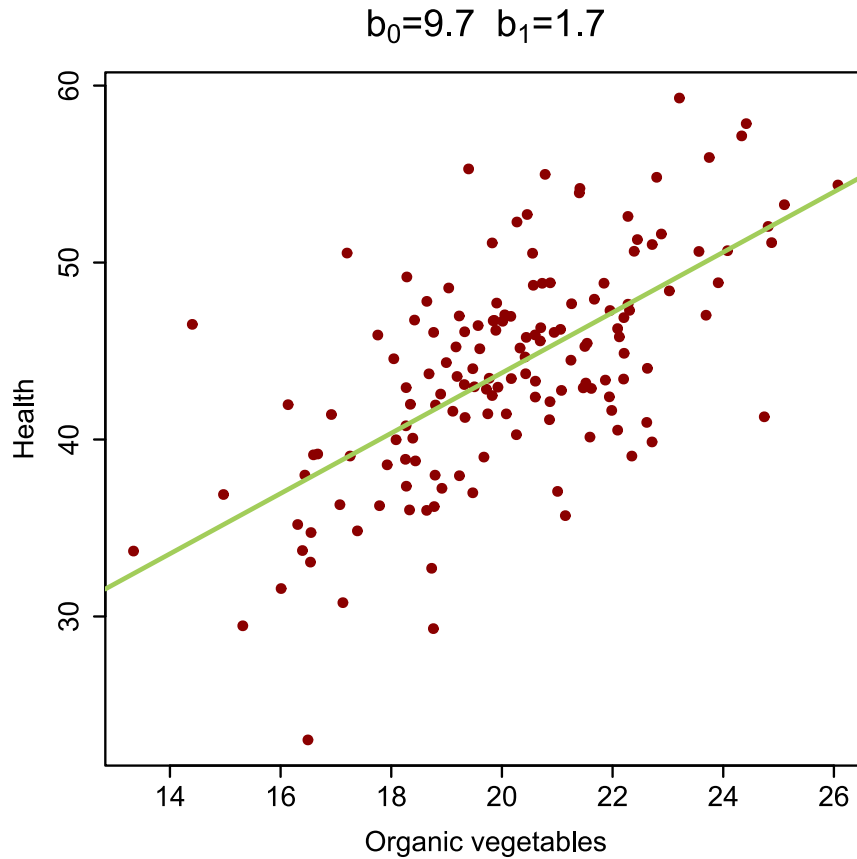
$$Y_i = b_0 + b_1 \cdot X_i + \epsilon_i \quad \epsilon_i \sim N(0, \sigma)$$

- When the **dependent variable (Y)** is measured unreliably, the slope is usually estimated without bias, but the amount of explained variance decreases.
- When the **independent variables (X)** are measured unreliably, regression coefficients (slopes) will be underestimated, increasing the risk of Type II errors.

$$Y_i = b_0 + b_1 \cdot X_{1i} + b_2 \cdot X_{2i} + \dots + b_p \cdot X_{pi} + \epsilon_i$$

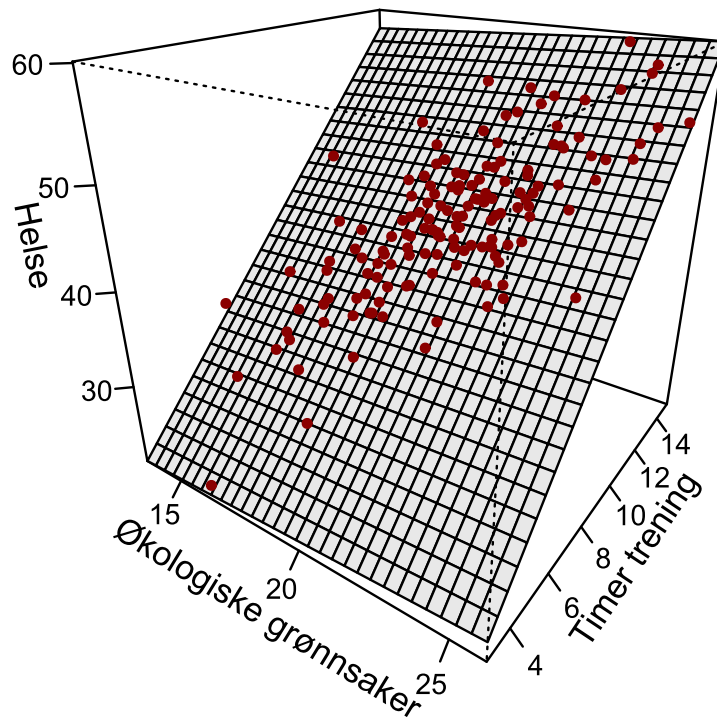
- The parameters (b) are called **partial regression coefficients**.
- Partial regression coefficients quantify the expected increase in Y that is associated with a one unit increase in X_j , *given that all other independent variables are held constant*.

HEALTH EFFECT OF ORGANIC FOOD?



Is there a health effect of eating organic vegetables?

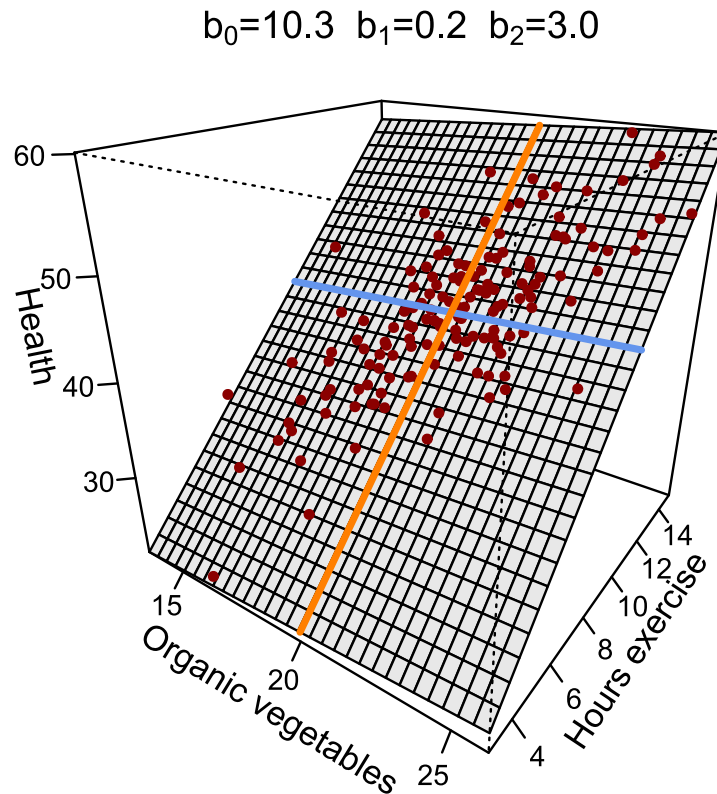
$$b_0=10.3 \quad b_1=0.2 \quad b_2=3.0$$



People who eat more organic vegetables typically also exercise more.

- Among people who exercise the same amount (for example 10 hours), are those who exercise more also healthier?

ORGANIC FOOD IN MULTIPLE REGRESSION

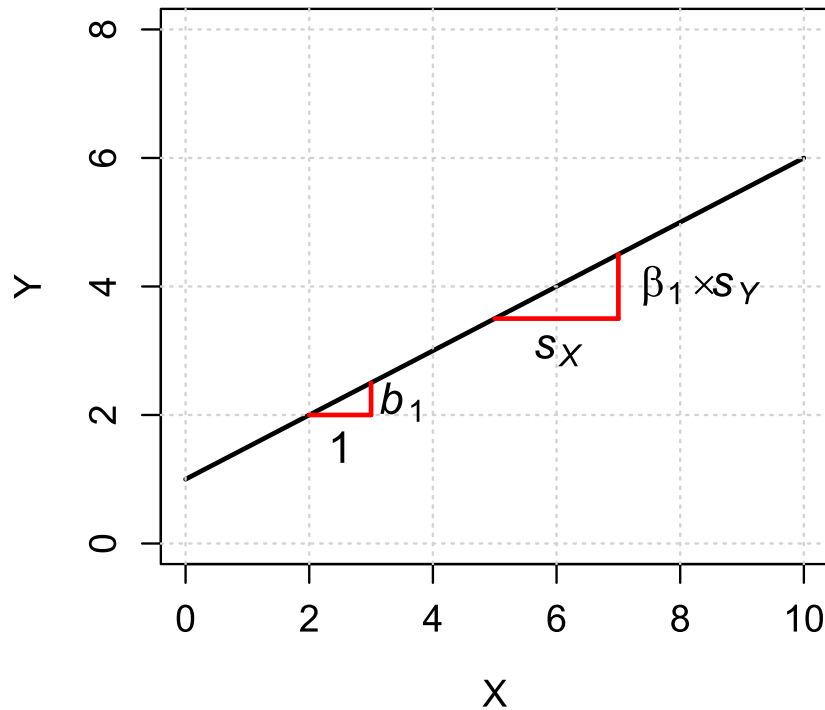


- Orange: People who eat 20 organic vegetables, and exercise from 4 to 14 hours.
- Blue: People who exercise 10 hours a month and eat from 14 to 26 organic vegetables.

In both cases we see what the model predicts when we hold the other variable *constant*.

STANDARDIZED VS UNSTANDARDIZED REGRESSION COEFFICIENTS

Standardized regression coefficients



$$\beta_1 = b_1 \cdot \frac{s_x}{s_y}$$

(b_1) : Number of units change in Y for each unit change in X .

(β_1) : Number of standard deviations Y changes for each standard deviation change in X .

LAVAAN

- The lavaan R-package is free open-source software (no warranty).
 - <http://lavaan.ugent.be>
- Lavan is in *beta*
 - Syntax is mature, but features will be added, and function internals updated.
- We'll specify models using Lavaan Syntax, but graphical tools (ala Amos) interface with Lavaan
 - <http://onyx.brandmaier.de/>

No support for:

- Discrete latent variables (mixture models).
- Bayesian estimation (though see *blavaan* package)
- support for variable types other than continuous, binary and ordinal (for example: zero-inflated count data, nominal data, non-Gaussian continuous data)
- Supports two-level cfa/sem with random intercepts only, for continuous and complete data (random slopes under development).

- **semTools** contains many useful tools
 - Power,
- **simsem** simulate SEM models
- **semPlot** plot path models
- **blavaan** bayesian estimation

... and many more

- The official tutorial is a good place to start:
 - <http://lavaan.ugent.be/tutorial/tutorial.pdf>
- The lavaan discussion forum is a good place to search for help:
 - <https://groups.google.com/forum/#!forum/lavaan>
- Citing lavaan
 - Yves Rosseel (2012). lavaan: An R Package for Structural Equation Modeling. Journal of Statistical Software, 48(2), 1-36. URL <http://www.jstatsoft.org/v48/i02/>

INSTALLING THE PACKAGE

```
install.packages("lavaan", dependencies = TRUE)  
library(lavaan)
```

THE MODEL SYNTAX

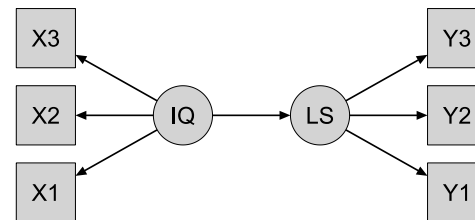
In the R environment, a regression formula has the following form:

```
y ~ x1 + x2 + x3 + x4
```

In this formula, the tilde (~) is the regression operator.

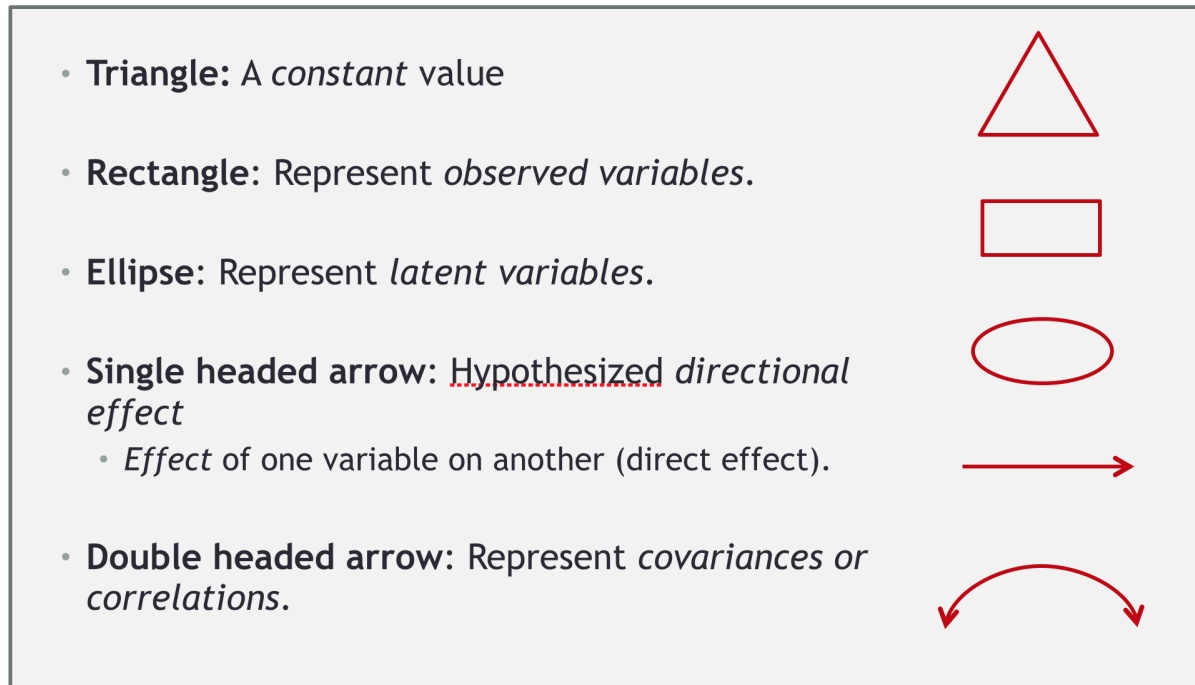
In lavaan, a typical model is simply a set (or system) of regression formulas, where some variables may be latent.

```
IQ =~ X1+X2+X3  
LS =~ Y1+Y2+Y3  
LS~ IQ
```



PATH DIAGRAMS

ELEMENTS OF PATH DIAGRAMS



A **path model** depicts the causal relations between a set of variables. The model constitutes a graphical representation of a set of regression equations.

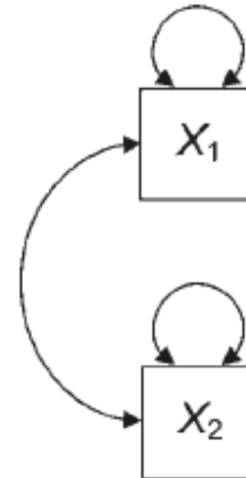
STRAIGHT ARROWS



- Straight arrows are **partial regression coefficients** (path coefficients for short).
- What extent is a change on the variable at the tail transmitted to the variable at the head of the arrow.
- Because they are *partial* regression coefficients, all other variables in the diagram held constant.

DOUBLE HEADED (CURVED) ARROWS

Double headed arrows represent variances and covariances.



EXOGENEOUS AND ENDOGENOUS VARIABLES



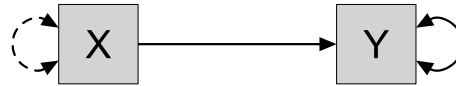
Exogenous variables (Greek: "of external origin")

- These variables represent *causal sources* in the diagram, and are not caused by the other variables in the model.

Endogenous variables ("of internal origin")

- These are *causally dependent* on at least some other variables in the model, and all such relationships are assumed defined (else specification error).

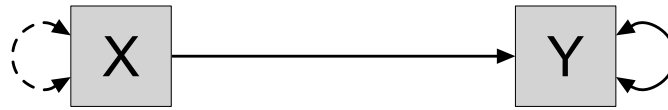
DISTURBANCE (ERROR TERM)



- Every endogenous variable has a **disturbance**.
 - These represent all omitted causes, plus measurement error.
- Disturbances can be conceptualized as unmeasured (latent) exogenous variables, and are often also represented by a circle.
- They allow us to compute the proportion of variance explained for each endogenous variable.
- We will refer to variance of observed variables with θ^2 (theta), and variance of latent variables with ψ^2 (psi).

PATH MODEL OF A BIVARIATE REGRESSION

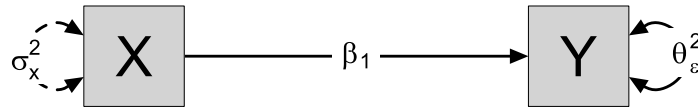
$$Y_i = b_0 + b_1 \cdot X_i + \theta_i$$



- Which parameters are estimated?
- If this depicts a regression, are any parameters omitted from the diagram above?

PATH MODEL OF A BIVARIATE REGRESSION

$$Y_i = b_0 + b_1 \cdot X_i + \theta_i$$



- Which parameters are estimated?
- Are any parameters omitted from the diagram above?

- **Lavaan has three fitting functions**

- *cfa()* / *sem()* are very similar, and greatly simplify the implementation because they have many defaults.
- *lavaan()* require a much more detailed specification, and is useful when non-standard models need to be specified.

```
mod1 ← 'RWA~edu'  
mod1_fit ← sem(model = mod1, data=dt)
```

INSPECTING THE RESULT

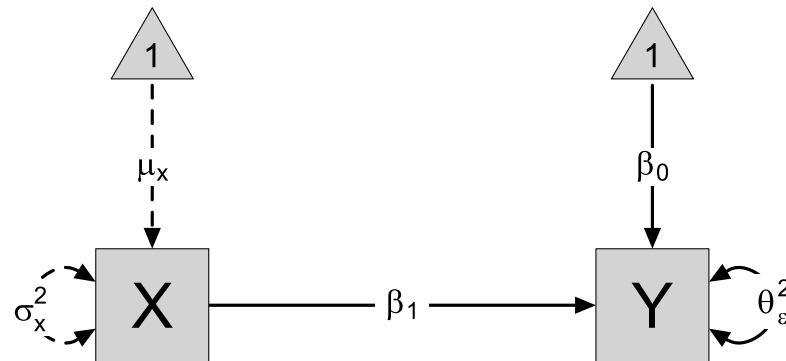
```
mod1 ← 'RWA~edu'  
mod1_fit ← sem(model = mod1, data=dt)  
  
summary(mod1_fit)  
semPaths(mod1_fit, what="est")
```

ADDING THE INTERCEPT

```
mod1 ← 'RWA~edu'  
mod1_fit ← sem(model = mod1, data=dt, meanstructure = TRUE)  
  
summary(mod1_fit)  
semPaths(mod1_fit)
```

PATH MODEL WITH INTERCEPTS

$$Y_i = b_0 + b_1 \cdot X_i + \theta_i$$

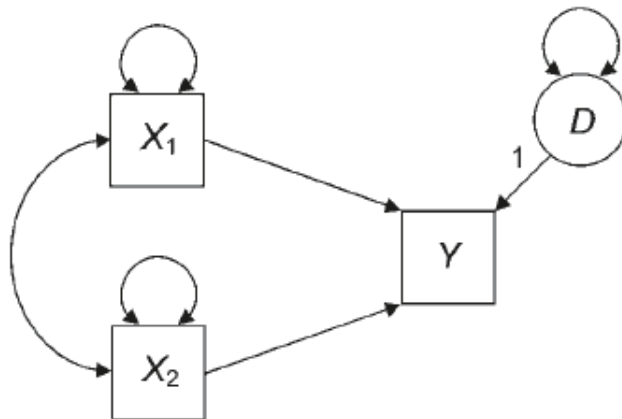


- Triangles represent *constant values* (here 1), and consequently they have no variance.
 - This also allows us to represent the *mean structure* of the variables in our model.

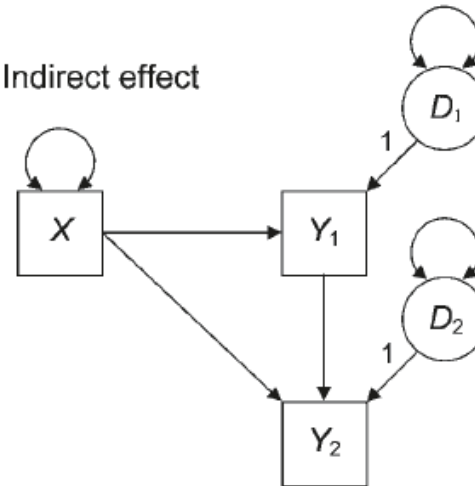
PRACTICAL: LINEAR REGRESSION USING LM AND LAVAAN

EXAMPLES OF EXOGENOUS / ENDOGENOUS VARIABLES

(b) Correlated causes

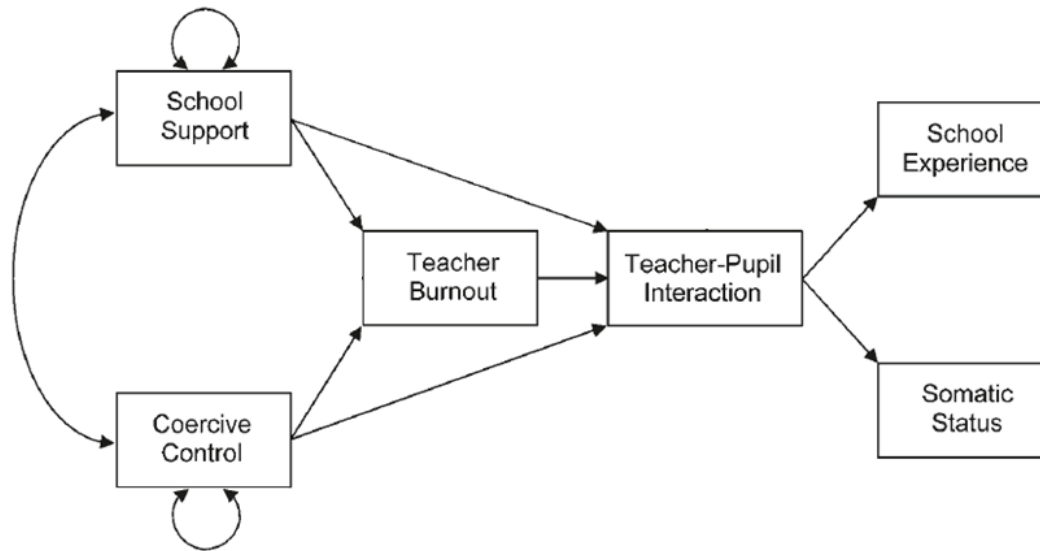


(c) Indirect effect



Which variables are endogenous and exogenous?

EXAMPLES OF EXOGENEOUS / ENDOGENEOUS VARIABLES



Which arrows are missing?

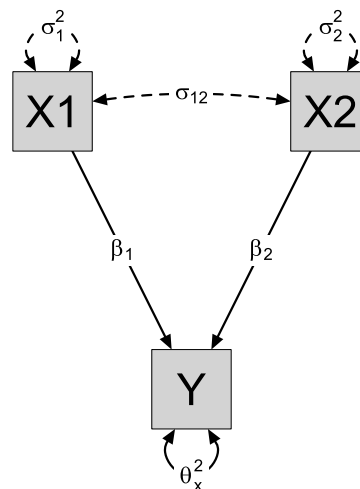
- **Causal relationships are linear**
- **Causes are unitary** (except for residuals).
 - Variables may not be composed of components that behave in different ways with different variables in the system.
- **Causal closure**
 - All direct influences must be included in the path diagram. Absence of path means absence of a relationship.
 - Any variance not accounted for by a variable in the diagram must be expressed as a residual.

PATH TRACING RULE

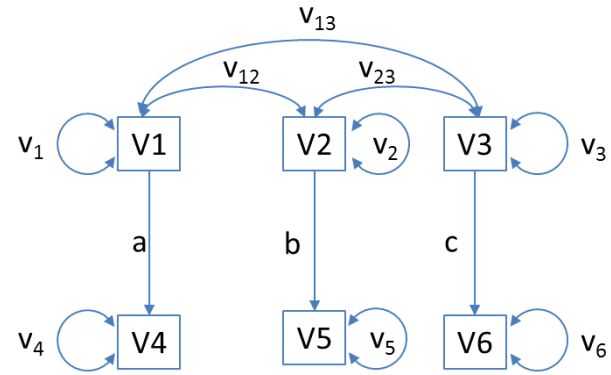
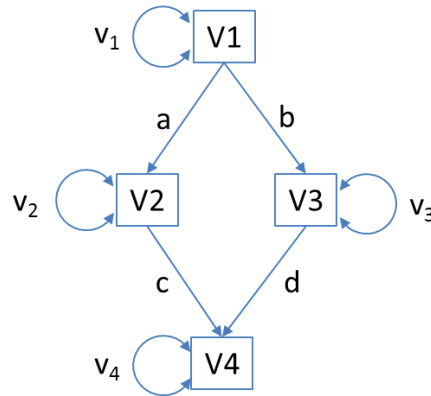
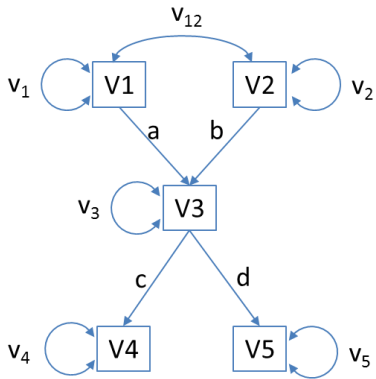
- The covariance between any two variables in the diagram can be found using the *tracing rules*.
- Sum all distinct routes, i.e. routes that contain different coefficients, or encounter those coefficients in a different order.
- **Trace backward, change direction at a two-headed arrow, then trace forward.**

Legal paths are those where you:

1. Pass through each variable only once (in each chain)
2. Trace backward along an arrow, then forward (never forward then back)
3. Trace through one (not more) two-way arrows



PATH TRACING RULES



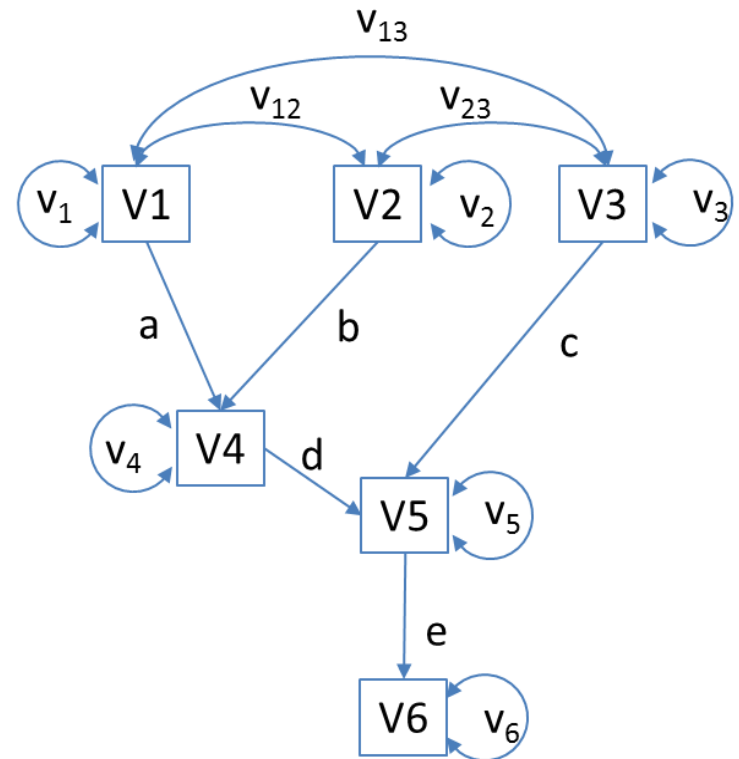
Legal paths are those where you:

1. Pass through each variable only once (in each chain)
 - Left: $c \times a \times v_{12} \times b \times d$?
2. Trace backward along an arrow, then forward (never forward then back)
 - Center: $a \times b$? $c \times d$?
3. Trace through one (not more) two-way arrows
 - Right: $a \times v_{13} \times c$? $a \times v_{12} \times v_{23} \times c$?

TRACING RULE EXERCISES

What is:

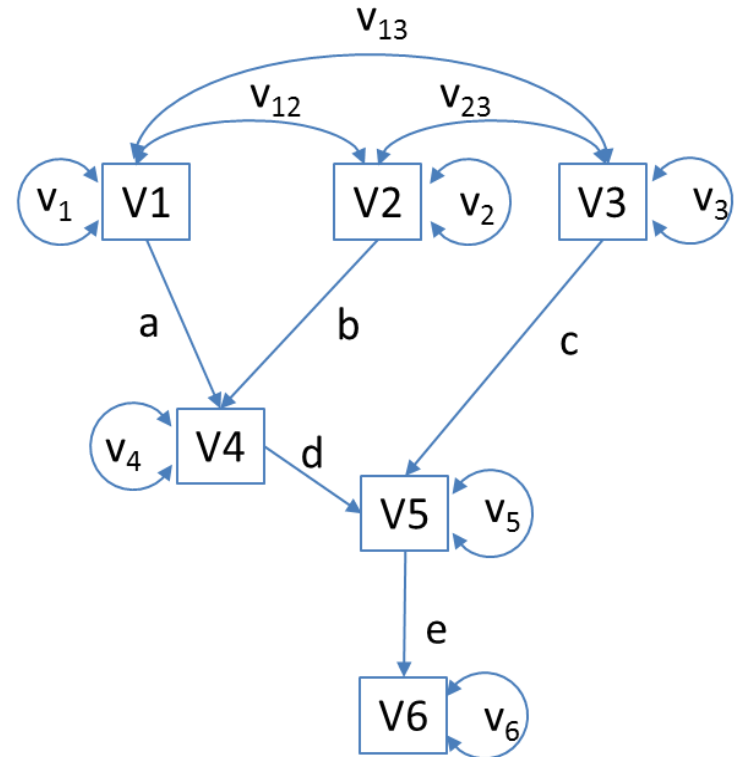
- $\text{Cov}(V1, V2)$?
- $\text{Cov}(V1, V4)$?
- $\text{Cov}(V1, V5)$?
- $\text{Var}(V4)$?



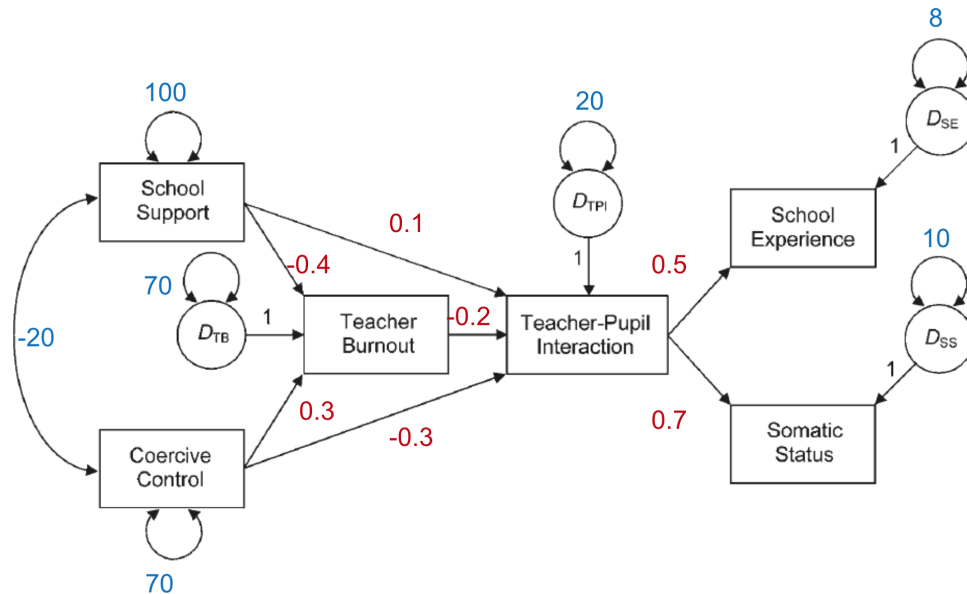
TRACING RULE EXERCISE (2)

What is:

- Cov (V1,V2)?
 - v_{12}
- Cov (V1,V4)?
 - $v_1a + v_{12}b$
- Cov (V1,V5)?
 - $v_1ad + v_{12}bd + v_{13}c$
- Var (V4)?
 - $a^2v_1 + b^2v_2 + 2abv_{12} + v_4$

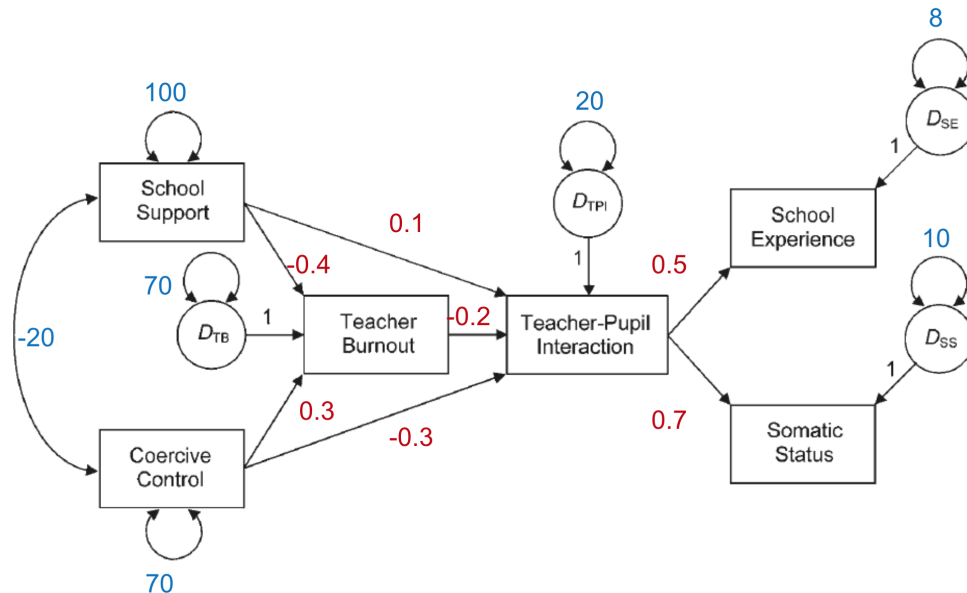


APPLYING TRACING RULES ON AN ESTIMATED MODEL



- How much variance in Teacher Burnout can be accounted for by School Support and Coercive Control?
- What proportion of the variance in Teacher Burnout can be accounted for by these variables?

APPLYING TRACING RULES ON AN ESTIMATED MODEL



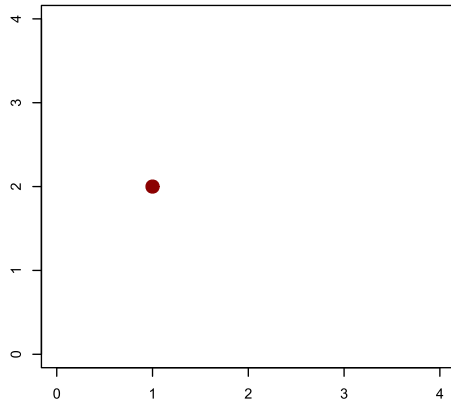
- How much variance in Teacher Burnout can be accounted for by School Support and Coercive Control?
 - $(-0.4)^2 \times 100 + (0.3)^2 \times 70 + 2 \times (-0.4) \times (-20) \times 0.3 = 27.1$
- What proportion of the variance in Teacher Burnout can be accounted for by these variables?

$$\circ R^2 = \frac{\text{explained var}}{\text{total var}} = \frac{27.1}{27.1+70} = 0.279$$

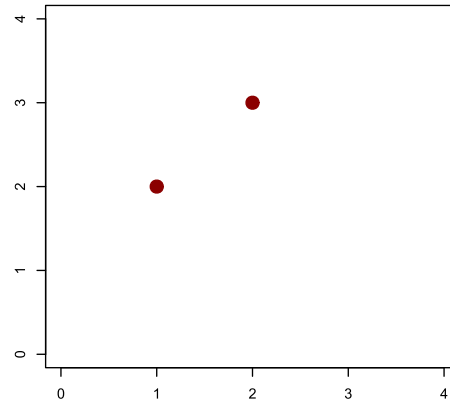
MODEL IDENTIFICATION

MODEL IDENTIFICATION

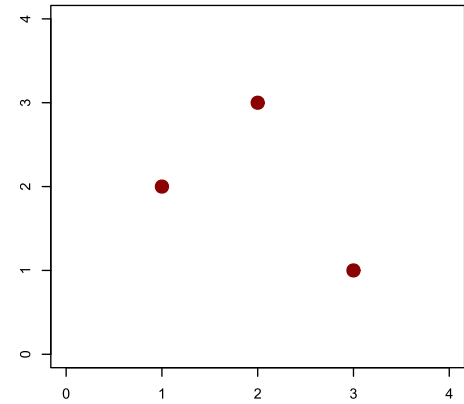
Underidentified



Just identified



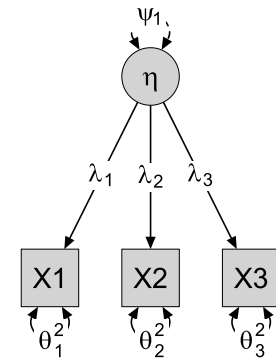
Overidentified



- Almost no psychological phenotypes are observable.
 - There are observable correlates, but the traits themselves must be inferred.

Manifest variables (indicators):

- Variables we observe/measure are referred to as manifest variables.
- Typically a number of indicators are necessary to fully reflect the latent construct.

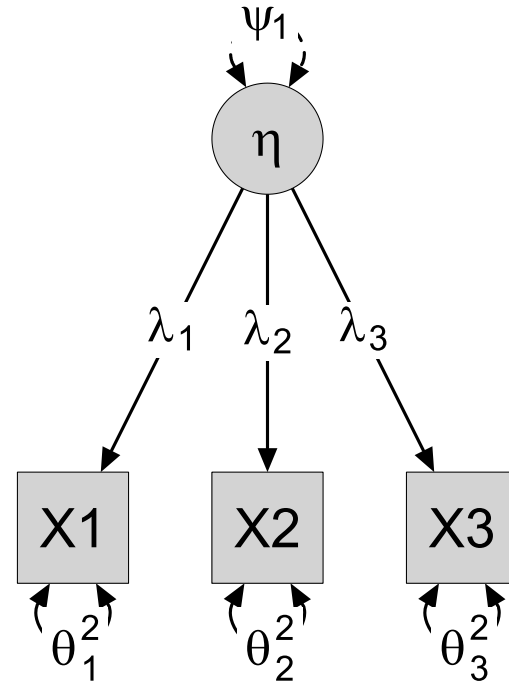


Latent variables (factors):

- Are assumed cause of scores in the observed variables.
- Indicators are correlated because they share a common cause.

LATENT VARIABLE MODEL

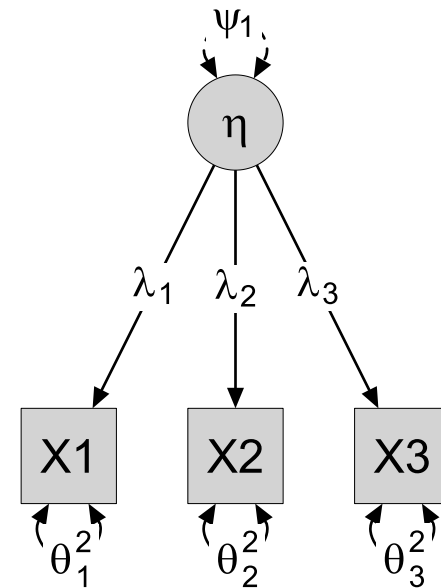
- What causes variance in X1?
- What causes X1 and X3 to correlate?
- If someone has a score of 2 on the latent variable, what does this contribute to X1?
- What is the expected correlation between X1 and X3?



CONFIRMATORY FACTOR ANALYSIS (CFA)

CFA aims for a parsimonious understanding of the covariation among a set of indicators. A CFA partitions the variance of each indicator into two parts:

1. **Common variance** is the variance accounted for by the latent factor, estimated on the basis of variance shared with other indicators in the analysis.
2. **Unique variance** is a combination of reliable variance that is specific to the indicator and random error variance (i.e., measurement error or unreliability in the indicator).



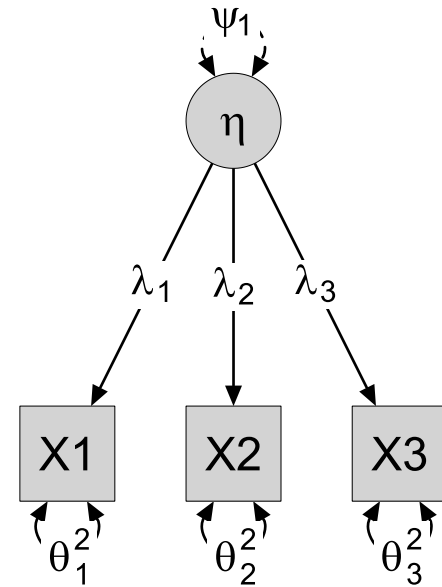
REGRESSION EQUATIONS UNDERLYING A CFA

The path model represents an underlying set of regression expressions.

$$X_1 = b_{01} + \eta \times \lambda_1 + \theta_1$$

$$X_2 = b_{02} + \eta \times \lambda_2 + \theta_2$$

$$X_3 = b_{03} + \eta \times \lambda_3 + \theta_3$$

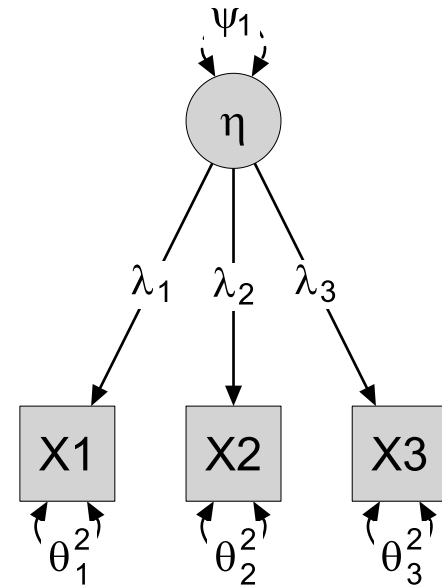


MATRIX EXPRESSION FOR EXPECTED COVARIANCE MATRIX IN CFA

The expected covariance matrix for a CFA model can be compactly expressed in matrix notation:

$$\Sigma = \Lambda\Psi\Lambda^T + \Theta$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} [\psi_1] \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} + \begin{bmatrix} \theta_1^2 & 0 & 0 \\ 0 & \theta_2^2 & 0 \\ 0 & 0 & \theta_3^2 \end{bmatrix}$$



Types of parameters in your SEM model:

- A **free** parameter is one whose value is unknown, but is to be estimated on the basis of the data.
- A **fixed** parameter is specified by the researcher have a specific value
 - Usually either 1 or 0.
- A **Constrained** parameter is estimated, but subject to some restriction.
 - *Equality constraints*: two parameters are restricted to have equal value.
 - *Proportionality constraint*: One parameter is proportional to another ex. Twice as large.
 - *Nonlinear constraint*

formula type	operator	mnemonic
latent variable definition	=~	is measured by
regression	~	is regressed on
(residual) (co)variance	~~	is correlated with
intercept	~ 1	intercept

SPECIFYING CFA MODELS IN LAVAAN

Models in Lavaan are specified with using three commands:

Measurement model

$F1 \sim X1 X2 X3$

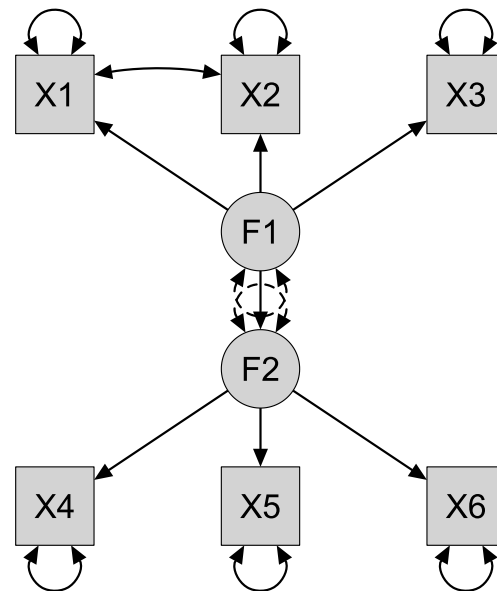
$F2 \sim X4 X5 X6$

Regression paths

$F2 \sim F1$

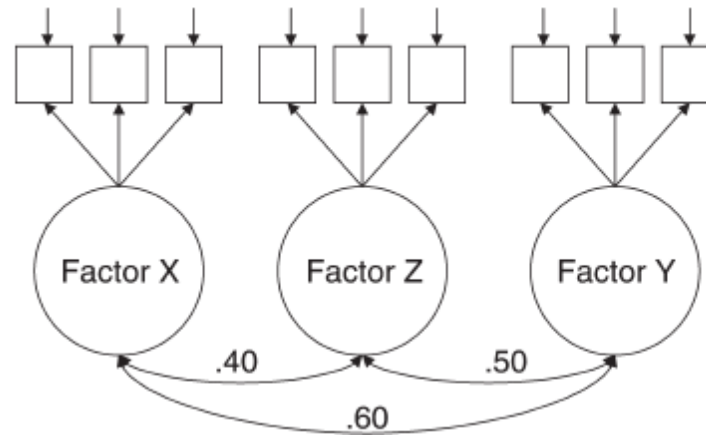
Variances/covariances:

$X1 \sim X2$



IMPORTANT CFA() AND SEM() DEFAULTS

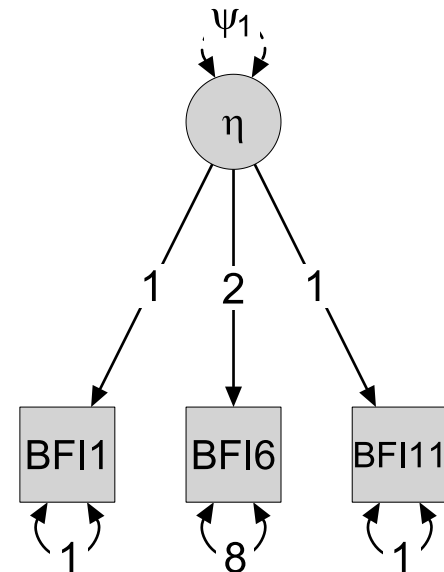
Model A: Measurement Model



- Factor loading of first variable is fixed to one.
- Factor loadings of other variables are estimated.
- Residual variances are estimated (need not be specified).
- Residual covariances are fixed to zero.
- Variances of factors are estimated.
- Covariance between the exogenous factors is estimated. If this is not what you want, you have to set these to zero yourself.

STANDARDIZED SOLUTIONS

- In a standardized solution, all latent and observed variables are rescaled to have a variance of 1.0.
- Standardized variables are useful in determining the relative importance of each variable.
- Standardized coefficients are *sample specific* and may vary across samples due to changes in the variance of the variables.



PRINTING PARAMETER OUTPUT

```
# Set up model
myModel='F =~ X1 + X2 + X3 + X4'

# Fit the model to data
myModel_fit ← cfa(myModel, data=dt2)

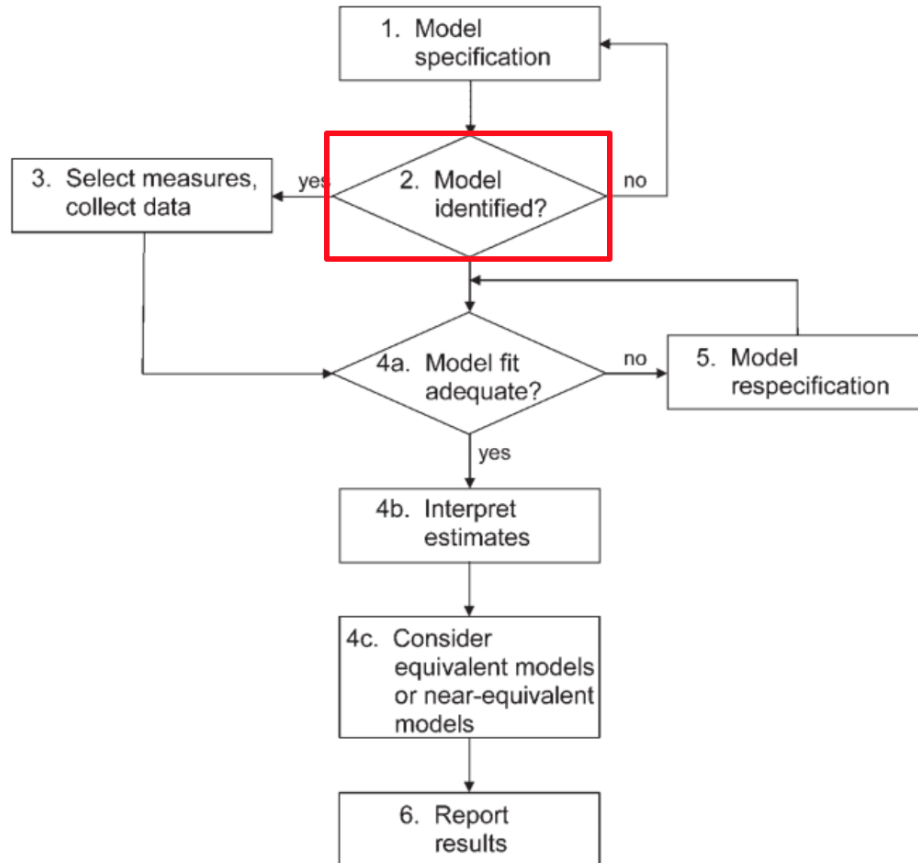
# The "summary" function is nicely formatted
summary(myModel_fit, fit.measures=TRUE)

# inspect function is better for computer processing
inspect(myModel_fit, what="est")
inspect(myModel_fit, what="std")
inspect(myModel_fit, 'R2')

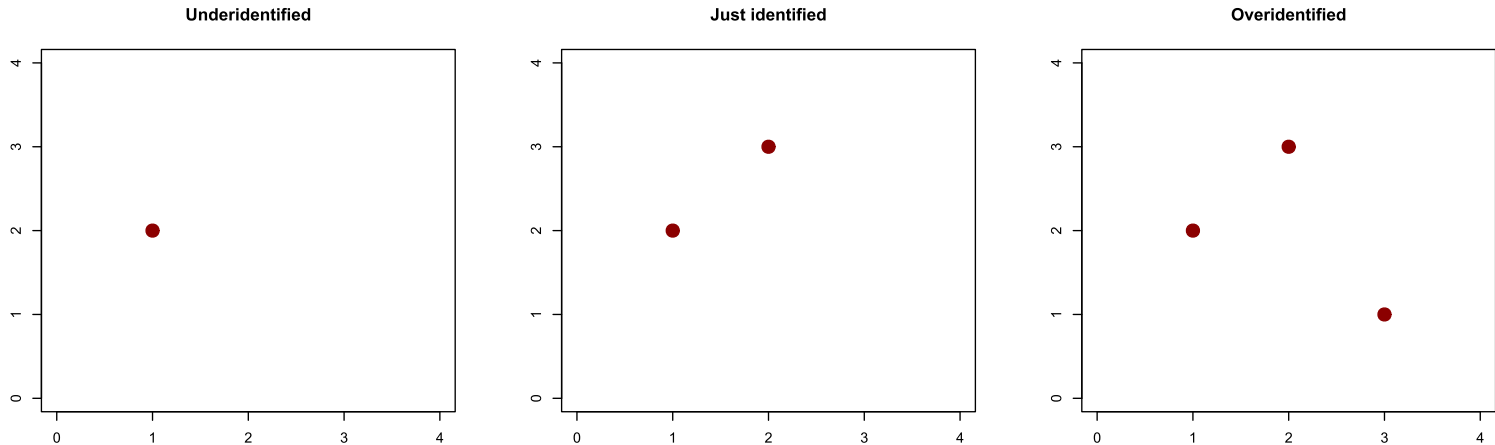
coef(myModel_fit) # Print only coefficients
parameterEstimates(myModel_fit)
```

MODEL IDENTIFICATION

MODEL SPECIFICATION



$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 \cdot X_i$$



- In *underidentified* systems there is not enough information to uniquely determine values for the model parameters.
- In *just identified* systems there is a unique set of estimates.
- In *overidentified* systems you can find a unique solution, but not one that fits perfectly.

NECESSARY (NOT SUFFICIENT) REQUIREMENTS FOR SEM IDENTIFICATION

- A model is *identified* if a unique estimate can be found for every free parameter.
1. Every latent variable (including the residual terms) must be assigned a scale.
 2. Degrees of freedom must be at least zero ($df_M \geq 0$).

1. LATENT VARIABLE SCALING

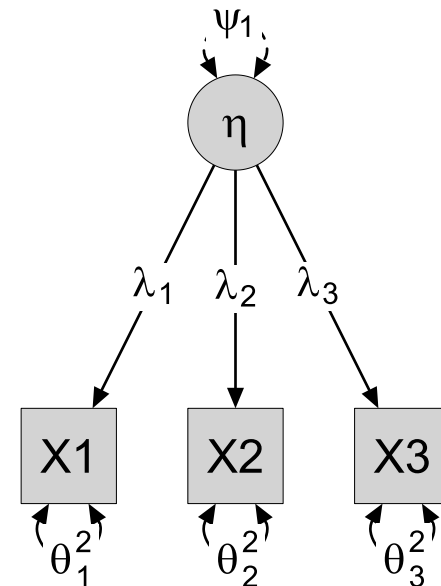
Latent variables have no units of measurement (scale), but the factor model is unidentified without a scale. Two approaches to scaling latent variables:

Reference variable method:

- Constrain one factor loading (typically to 1, hence reference).
- Default method in Lavaan

Fixed factor method:

- Constrain factor variances (typically to 1).



2. IDENTIFICATION AND DF

In SEM the number of observed statistics is given by the number of elements in the covariance matrix ($k(k+1)/2$).

$$\begin{bmatrix} Var(X_1) & Cov(X_1, X_2) & Cov(X_1, X_3) \\ Cov(X_1, X_2) & Var(X_2) & Cov(X_2, X_3) \\ Cov(X_1, X_3) & Cov(X_2, X_3) & Var(X_3) \end{bmatrix}$$

df_M = observed statistics - estimated (free) parameters.

Underidentified SEM Model ($df_M \leq 0$)

- The number of parameters to be estimated exceeds the number of statistics.

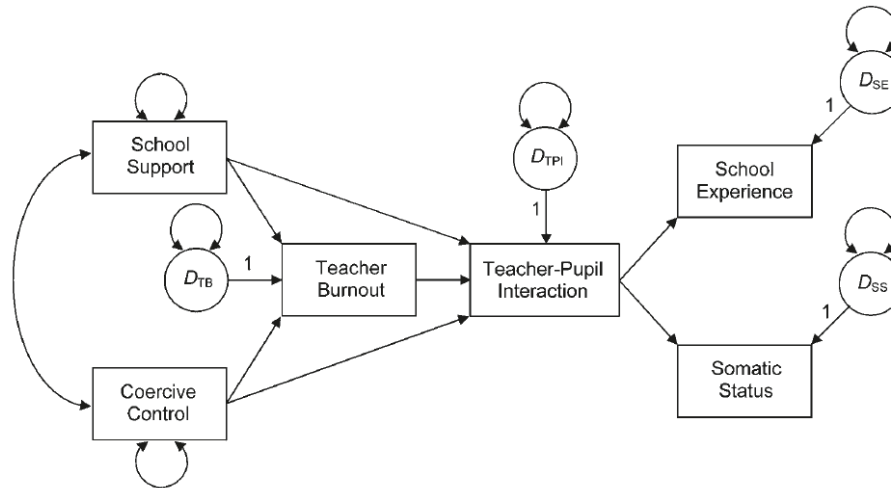
Just-identified model ($df_M = 0$)

- Equal number of free parameters and observations (fits perfectly, by construction).

Overidentified model ($df_M \geq 0$)

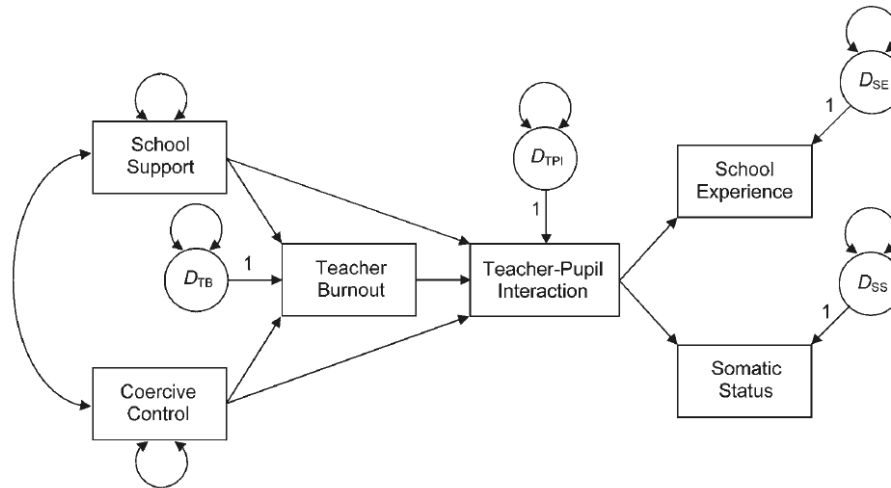
- Fewer free parameters than observations.

EXERCISE: IDENTIFICATION OF A PATH MODEL

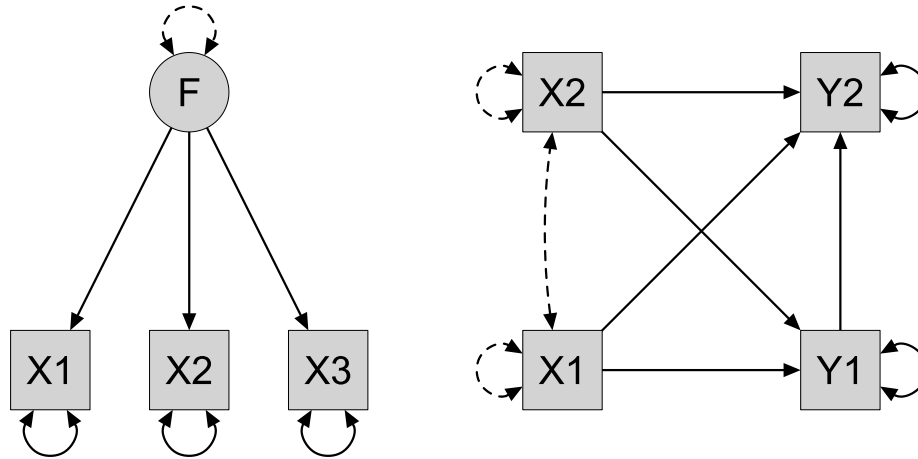


- How many parameters are estimated?
 - All path coefficients, covariances and residual variances.
- How many observed statistics can we base our estimates on?
- How many degrees of freedom do we have?

EXERCISE: IDENTIFICATION OF A PATH MODEL



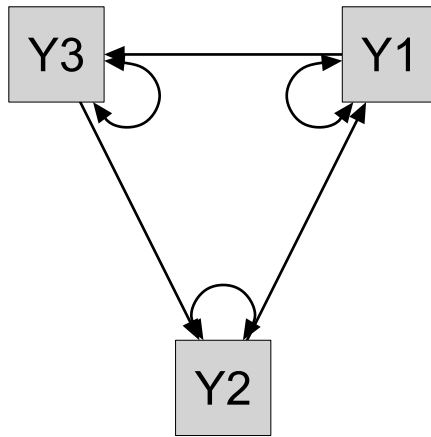
- How many parameters are estimated?
 - 14
- How many observed statistics can we base our estimates on?
 - $6 * (6 + 1) / 2 = 21$
- How many degrees of freedom do we have?
 - $df = 7$



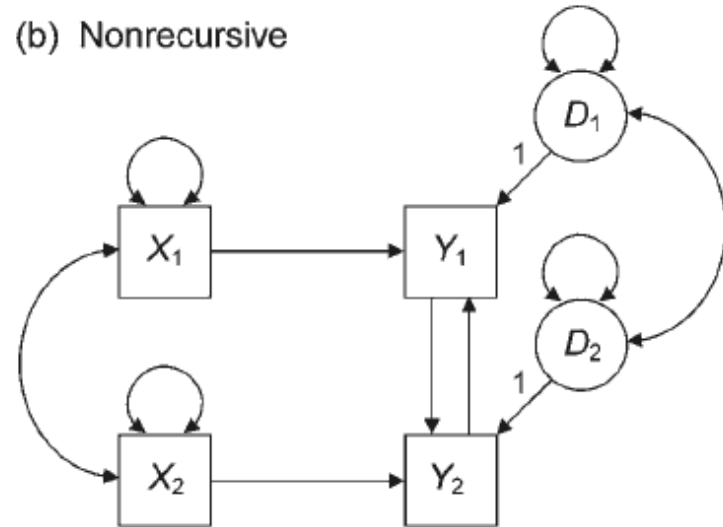
Recursive models are the most straight forward, and are characterized by:

- Disturbances are uncorrelated
- All causal effects are unidirectional

NON-RECURSIVE MODELS



(b) Nonrecursive

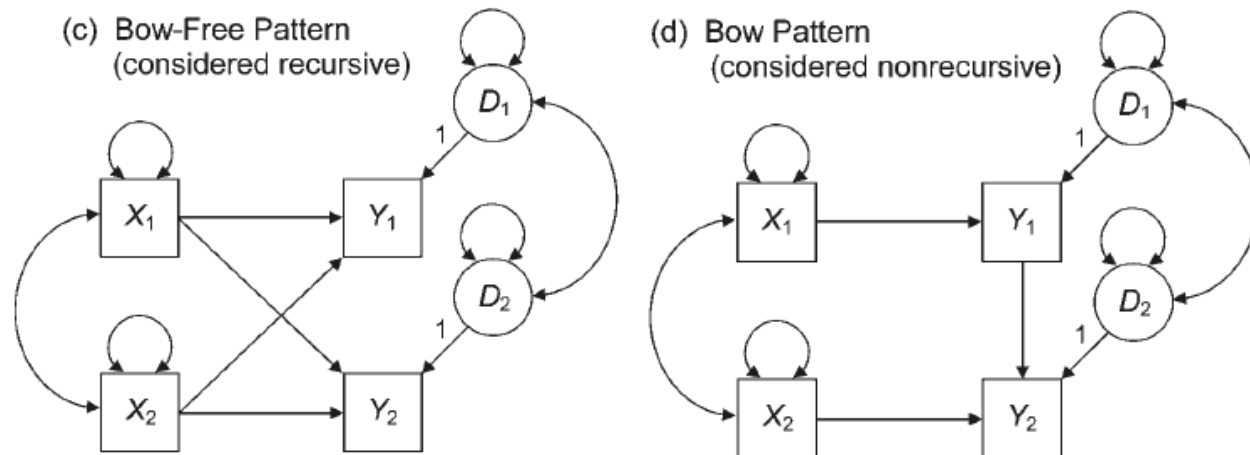


Non-recursive models is one which allows for reciprocal or feedback terms, and may have:

- Direct or indirect feedback loops
- Correlated disturbances

PARTIALLY RECURSIVE MODELS

- **Partially recursive models:** Models that have unidirectional effects and correlated disturbances.
- **Bow pattern** means that a disturbance correlation occurs with a direct effect between that pair of endogenous variables.



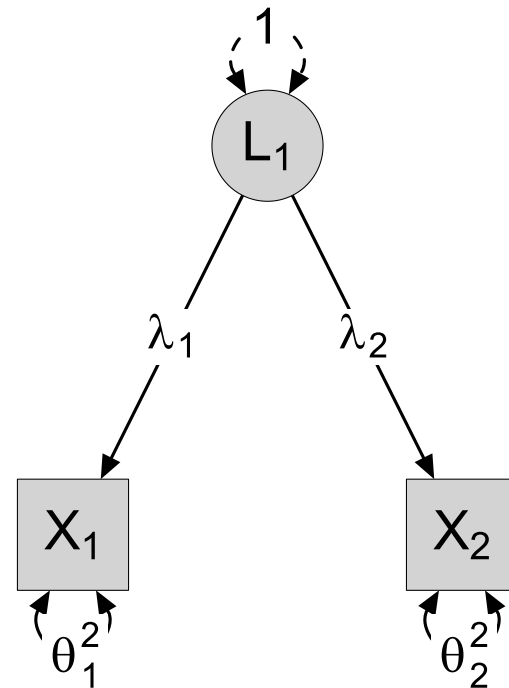
Partially recursive models with a bow-free pattern can be treated in the analysis just like recursive models.

IDENTIFICATION OF CFA MODELS

What is the expected covariance matrix based on the parameters of the model?

$$\textit{Observed } (S) = \begin{bmatrix} s_1^2 & \\ s_{21} & s_2^2 \end{bmatrix}$$

$$\textit{Expected } (\Sigma) = \begin{bmatrix} \lambda_1^2 + \theta_1^2 & \\ \lambda_1 \lambda_2 & \lambda_2^2 + \theta_2^2 \end{bmatrix}$$

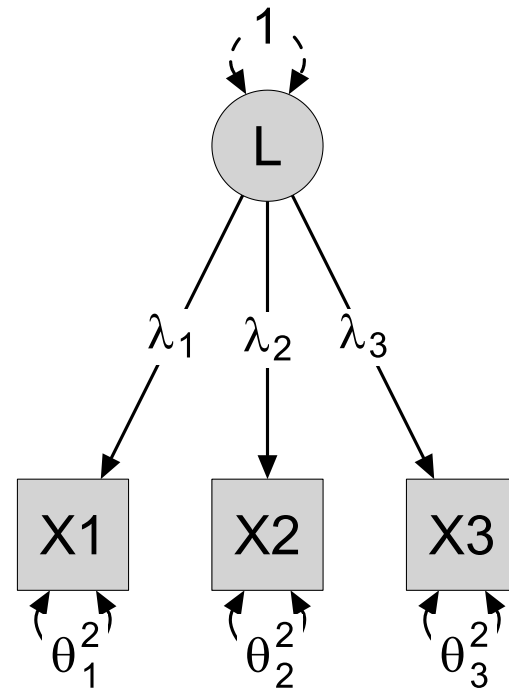


Is this model identified?

IDENTIFICATION OF CFA MODELS (2)

What is the expected covariance matrix, based on the parameters of the model?

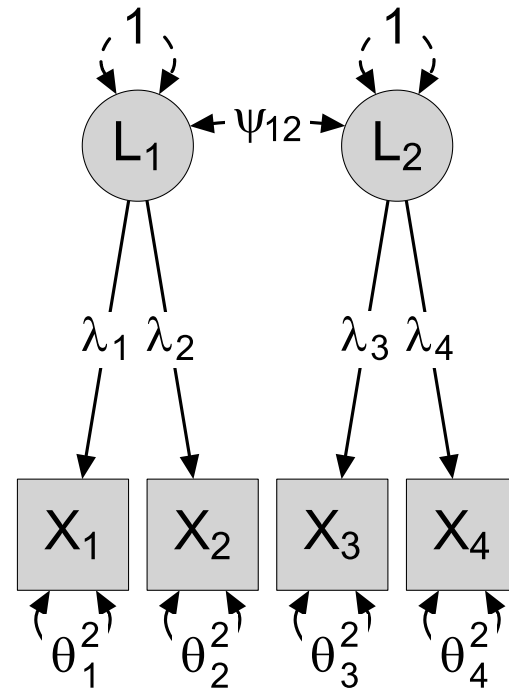
$$S = \begin{bmatrix} s_1^2 & & \\ s_{21} & s_2^2 & \\ s_{31} & s_{32} & s_3^2 \end{bmatrix}$$



Is this model identified?

IDENTIFICATION OF CFA MODELS (3)

- Is this model identified?
- How many observed statistics do we have?
- How many parameters are we estimating?



Recursive models

- Assuming the two necessary requirements for identification ($df_M \geq 0$; latent variables scaled) are satisfied, recursive structural models are identified.

Partially recursive models with a bow-free pattern

- Assuming ($df_M \geq 0$; latent variables scaled), these can be treated in the analysis just like recursive models.

Non-recursive structural models given

- Be careful, only identified under certain circumstances

CONSTRAINTS IN LAVAAN

```
# To set two parameters equal, pre-multiply with the same label
RWA_1F =~ RWA1 + v2*RWA2 + v2*RWA3

# Another way of setting parameter value equal
RWA_1F =~ RWA1 + RWA2 + equal("visual=~x2")*RWA3

# To fix a parameters, pre-multiply with a value
RWA_1F =~ RWA1 + 1*RWA2 + RWA3

# To remove a constraint, pre multiply with NA
RWA_1F =~ NA*RWA1 + RWA2 + RWA3

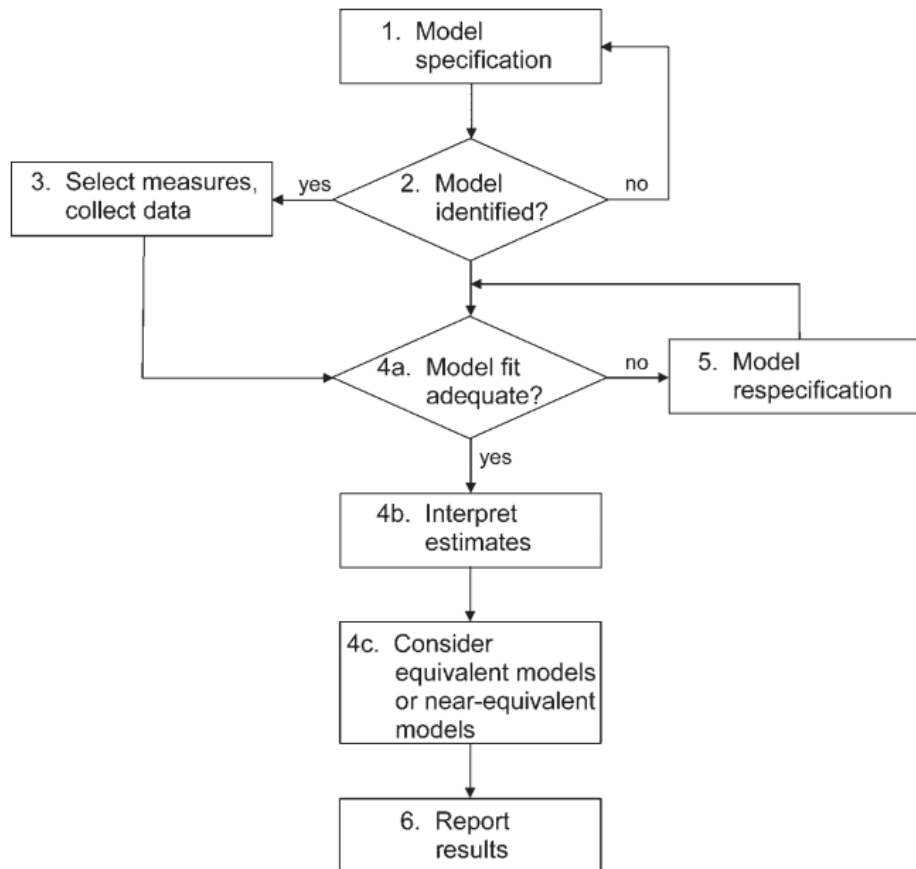
# To set a residual variance to zero
RWA1 =~ 0*RWA1

# To constrain a residual correlation to 0.5
RWA1 =~ 0.5*RWA2

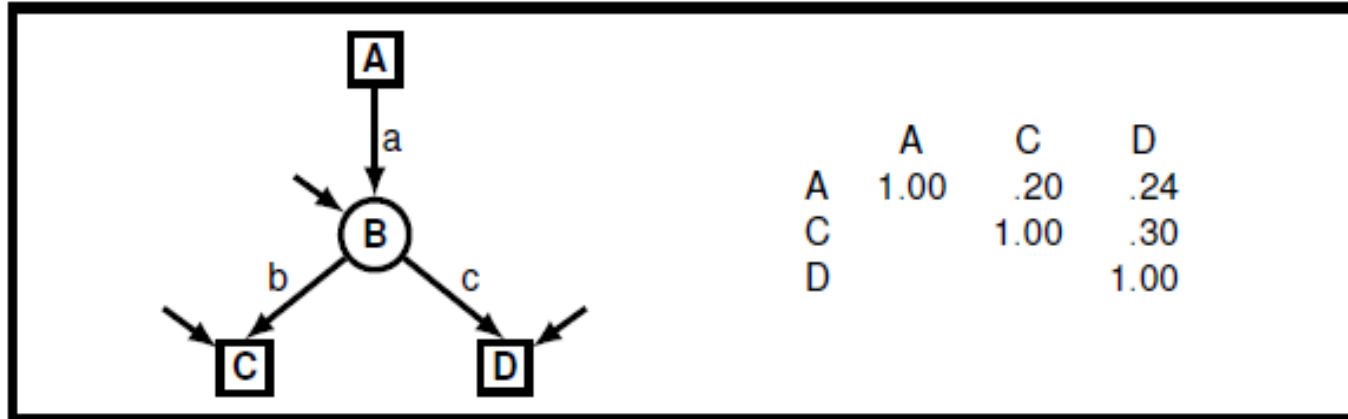
# Non-linear constraints
RWA_1F =~ ba*RWA1 + b2*RWA2 + b3*RWA3
b1 = (b2 + b3)^2
```

PARAMETER ESTIMATION

ESTIMATION



JUST IDENTIFIED MODELS CAN BE SOLVED



The equations are:

$$r_{AC} = ab$$

$$r_{AD} = ac$$

$$r_{CD} = bc.$$

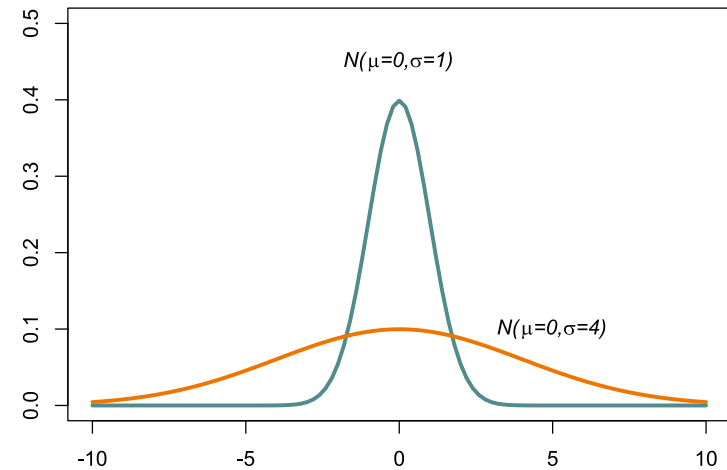
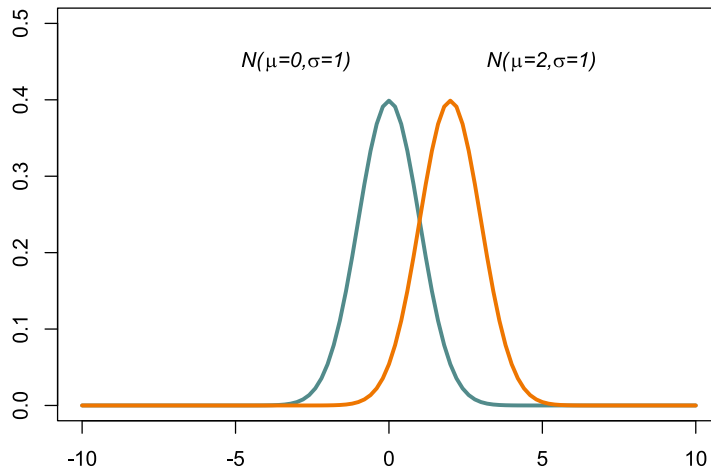
A solution is:

$$\frac{r_{AC} \times r_{CD}}{r_{AD}} = \frac{ab \times bc}{ac} = b^2 = \frac{.20 \times .30}{.24} = .25 \implies b = .50$$

$$a = \frac{r_{AC}}{b} = \frac{.20}{.50} = .40$$

$$c = \frac{r_{AD}}{a} = \frac{.24}{.40} = .60.$$

THE NORMAL DISTRIBUTION



The normal probability density function is given by:

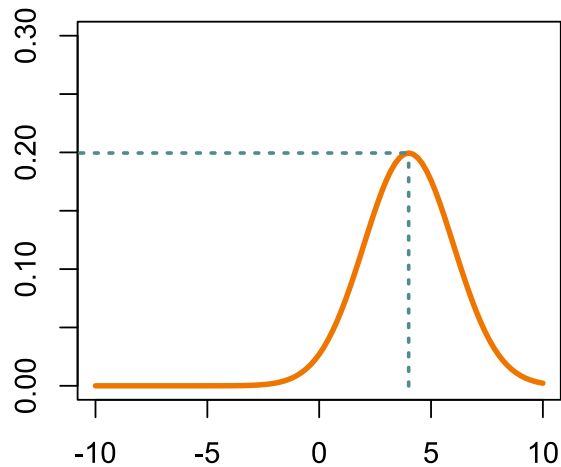
$$f(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

A random variable is drawn from a normal distribution with a variance of 2, but with unknown mean, $N(\mu, 2)$.

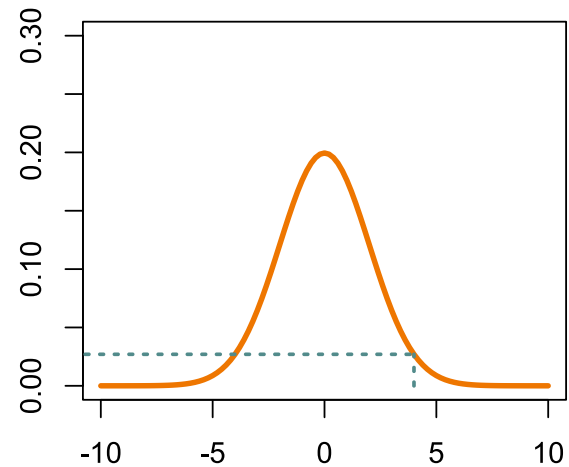
- *Assume you have 1 observation, and this has the value 4. What would be you estimate (guess) as to the value of the unknown parameter μ ?*

LIKELIHOOD OF A SINGLE OBSERVATION WITH VALUE 4

A random variable is drawn from a normal distribution with known variance (sd=2), but unknown mean.



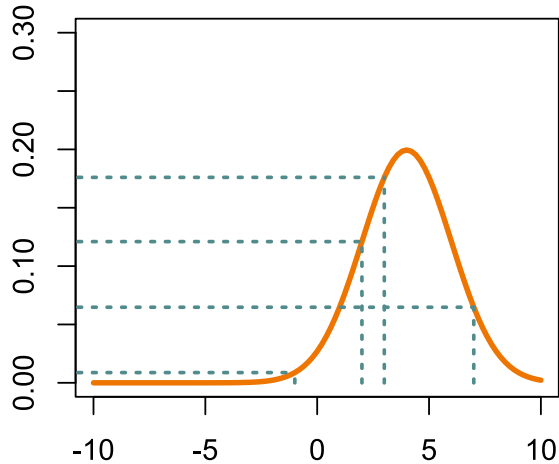
Population is $N(4,2)$
 $l(4)=0.199$



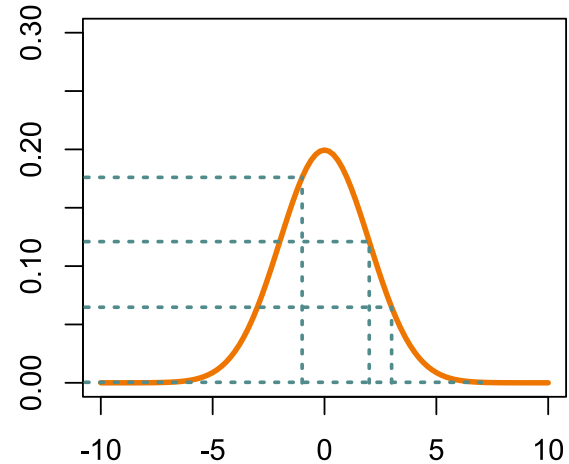
Population is $N(0,2)$
 $l(4)=0.027$

LIKELIHOOD OF FOUR OBSERVATIONS

Assume you had 4 observations (2,3,-1,7). If the observations are *independent*, the joint probability is equal to the product of the individual probabilities.

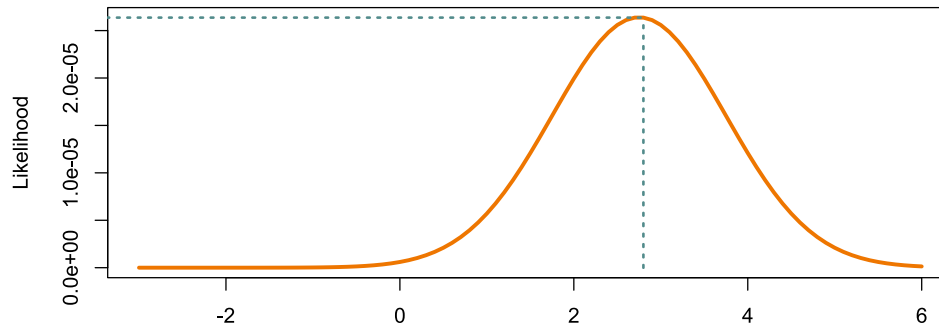


Population is $N(4,2)$
 $l()=1.2087e-05$



Population is $N(0,2)$
 $l()=6.0179e-07$

THE LIKELIHOOD FUNCTION



$$likelihood(\mu|X, \sigma) = \prod_{i=1}^n f(X|\mu, \sigma)$$

In statistics, the *likelihood function* (likelihood) is formed from the joint probability of a sample of data given a set of model parameter values; it is viewed and used as a function of the parameters given the data sample.

https://en.wikipedia.org/wiki/Likelihood_function

ASSUMPTIONS IN MAXIMUM LIKELIHOOD

Some key assumptions of ML in are:

- The sample size is large.
- The distribution of the indicators is multivariate normal (kurtosis particularly problematic).

Although the actual parameter estimates may not be affected, non-normality in ML analysis can result in biased *standard errors*.

NUMERICAL OPTIMIZATION OF THE LIKELIHOOD FUNCTION

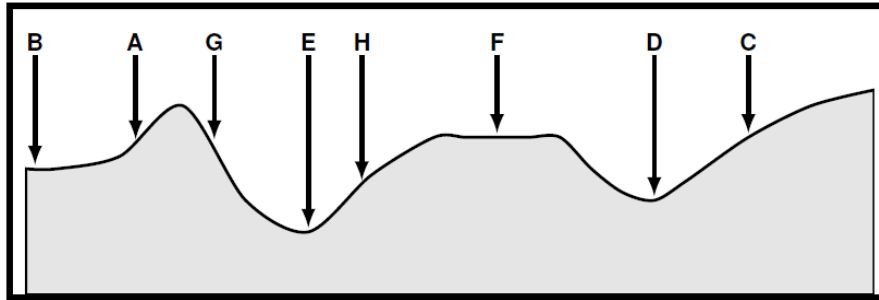


Figure 2.3 Cross section of a less hospitable search terrain.

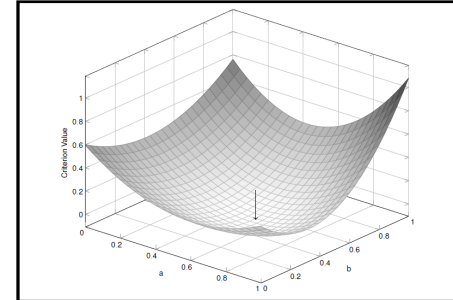


Figure 2.2 Graphical representation of search space for Fig. 2.1 problem, for values 0 to 1 of both variables.

Likelihood is a function of the parameter values.

$$F_{ML} = \ln|\Sigma| + \text{trace}[S\Sigma^{-1}] - \ln|S| - k$$

- Maximum likelihood *estimates* of the parameters are the values that maximize the likelihood function.
- In practice, parameter estimates are found *numerically*. The software tests a lot of parameter values, and determines which ones results in the highest likelihood.

SETTING STARTING VALUES IN LAVAAN

```
# To set two parameters equal, pre-multiply with the same label
visual =~ x1 + start(0.8)*x2 + start(1.2)*x3

# Checking starting values in the model
inspect(fit)
inspect(fit, what="start")
```

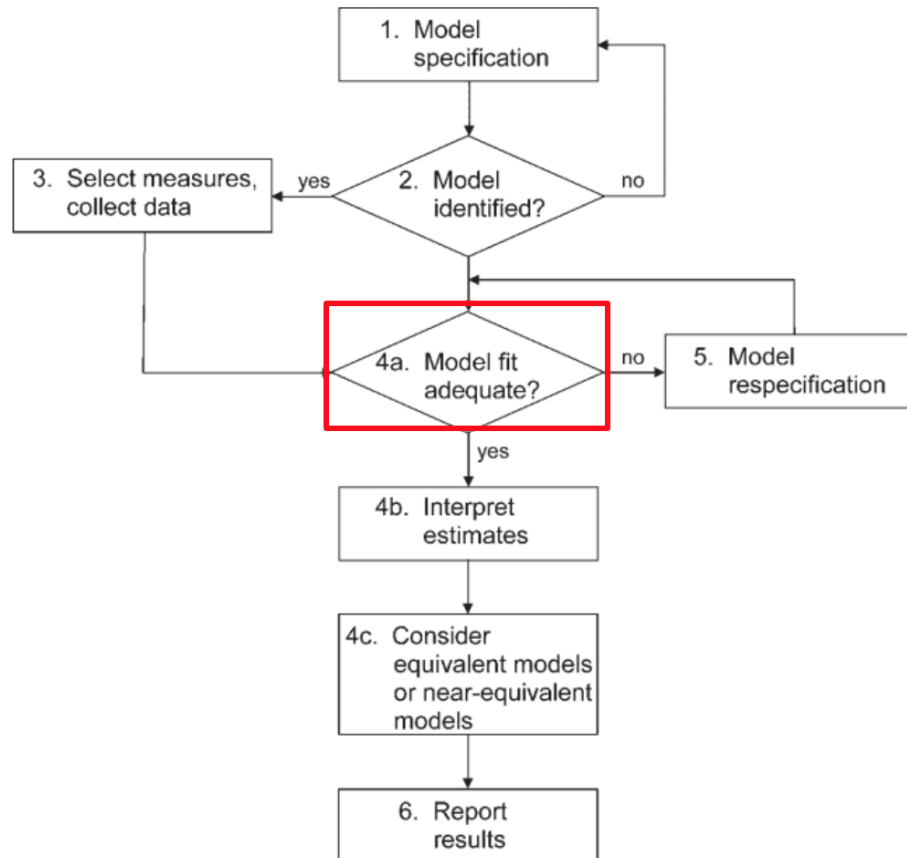
By default, calling *inspect()* on a fitted lavaan object returns a list of the model matrices that are used internally to represent the model. The free parameters are nonzero integers.

PRACTICAL: SINGLE FACTOR MODEL OF SCL

See the syntax file

EVALUATING MODEL FIT

FITTING AND EVALUATING MODELS}



Good fit is a necessary, but not sufficient condition for model validity.

Absolute fit indecies

- Determined by the distance between the observed (S) and expected covariance/correlation matrix (Σ).
 - χ^2 , RMSEA, SRMR

Comparative fit indecies

- Relative improvement in fit of your model compared with a statistical *baseline* model where all variables are uncorrelated.
 - TFI, CFI

Parsimony corrected indecies

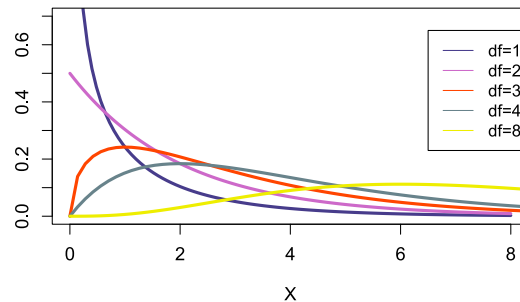
- Relative fit indices which are adjusted so as to penalize models that are less parsimonious.
 - AIC, BCI

CHI-SQUARE DIFFERENCE TESTS

OVERALL CHI-SQUARE (χ^2) TESTS

$$F_{ML} = \ln|\Sigma| + \text{trace}[S\Sigma^{-1}] - \ln|S| - k$$

$$\chi^2 = F_{ML}(n - 1)$$



- A significant value indicates a statistically significant difference between the observed and implied covariance matrices
- The most frequently cited index of absolute fit, but rarely reported alone.
- Most other fit statistics are functions χ^2
- $df_M = (\text{nr statistics}) - (\text{nr free parameters})$.

OVERALL CHI-SQUARE (χ^2) TESTS, ISSUES

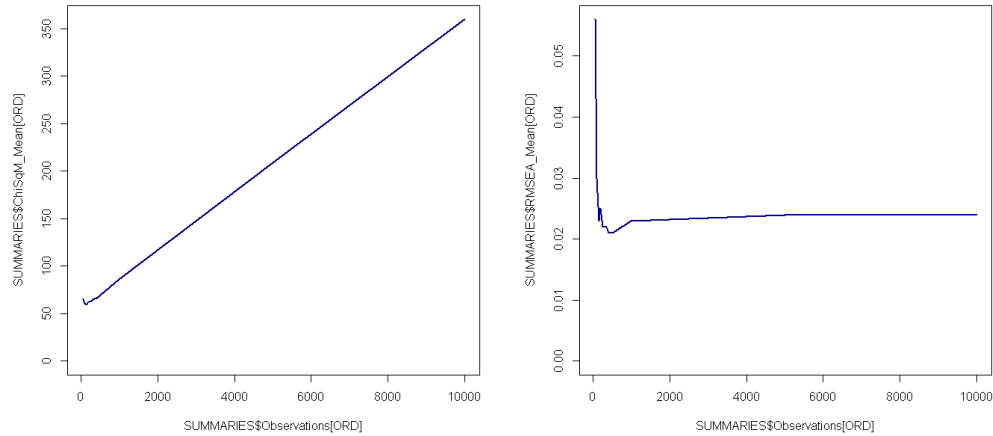
1. If the sample is small, and the data strongly non-normal, the distribution of the fit statistic is not truly chi-squared.
2. Chi-square value depends strongly on sample size:
 - In small samples we can fail to reject models that are very discrepant from the data.
 - In large samples we can reject model where the difference between S and Σ is very small, but *significantly different* may not equal *substantively different*.

ROOT MEAN SQUARE ERROR OF APPROXIMATION (RMSEA)

$$RMSEA = \sqrt{\frac{\chi_M^2 - df_M}{df_M(n - 1)}}$$

- RMSEA is a measure of *discrepancy per degree of freedom* in a model.
- Based on chi-square yet takes parsimony into account.
- Lavaan generates confidence intervals and significance tests of RMSEA.
- **Rules of thumb:** ≤ 0.08 indicates adequate fit, ≤ 0.05 indicates good fit.

COMPARING χ^2 AND RMSEA



The plots above illustrate the effect of a tiny residual correlation as the sample size grows on Chi-square (left) and RMSEA (right).

THE STANDARDIZED ROOT MEAN-SQUARE RESIDUAL (SRMR)

- Can be viewed as the average discrepancy between the correlations observed in the input matrix and the correlations predicted by the model
- A single residual can be large even though the average is small
 - tends to be smaller as sample size increases and the number of parameters increases, but no penalties for model complexity
- **Rules of thumb:** As with RMSEA, ≤ 0.08 indicates adequate fit, ≤ 0.05 indicates good fit

OVER VS. UNDERFITTING



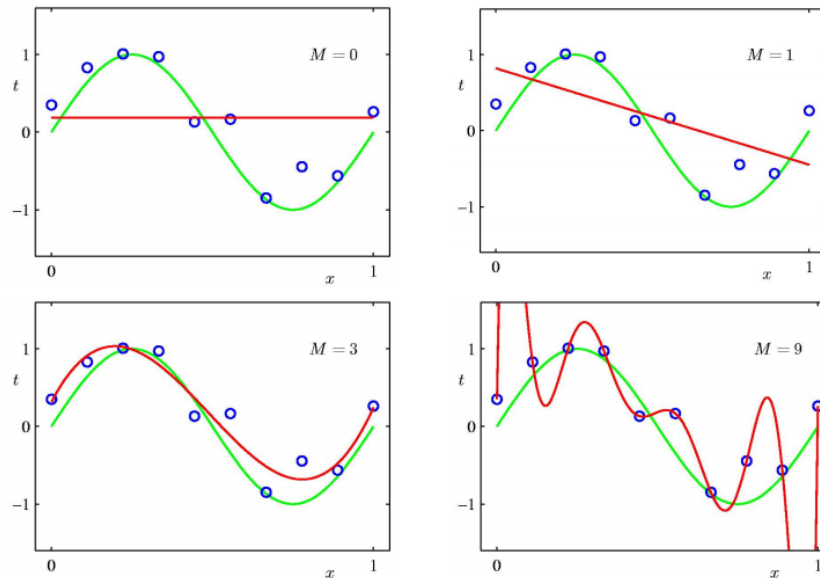
- You can always get better fit by including more predictors.
- While more predictors fit the current data better, they often predict *new* data worse, as there are both regular and irregular features of every sample.

Overfitting: Poor prediction by learning too much from your data.

Underfitting: Poor prediction by learning too little from your data.

<https://xcelab.net/rm/statistical-rethinking/>

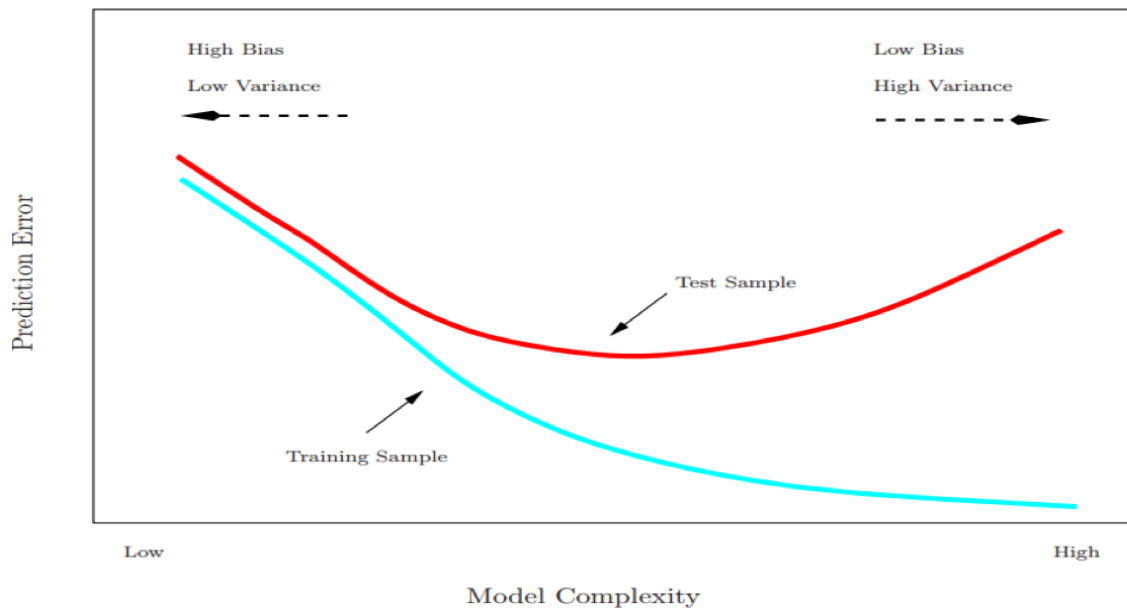
Fit using Degree 0,1,3,9



The above data were generated by adding error to the green curve. Higher order functions adapt both to the regular features as well as the error.

ILLUSTRATING THE DIFFICULTY WITH PREDICTION

Training- versus Test-Set Performance



Larger models may be overfitted to the idiosyncrasies of the data, and, thus, more poorly predict new, independent observations.

<http://faculty.marshall.usc.edu/gareth-james/ISL/>

INFORMATION CRITERIA (IC)

- IC contain explicit penalties for model complexity.
- The model with the lowest AIC/BIC value is considered the best fitting.
- Used in SEM to select among competing nested and non-nested models estimated with the same data.
 - Only meaningful when multiple models are estimated.

$$AIC = -2 \times \text{Loglikelihood} + 2 \times p$$

$$BIC = -2 \times \text{Loglikelihood} + p \times \log(N)$$

- Information criteria contain explicit penalties for model complexity.
 - AIC has a penalty of 2 per parameter estimated, BIC increases the penalty as sample size increases (SABIC places a little less).
- Lower values indicate a better fit, and the model with the lowest value is selected.
- When erring, AIC tends to select an overly complex model, and BIC an overly simple.

COMPARATIVE FIT INDEX(CFI) AND TUCKER LEWIS INDEX (TLI)

$$CFI = 1 - \frac{\chi_M^2 - df_M}{\chi_B^2 - df_B}$$

- Compares the observed model to a base model (with diagonal covariance matrix).
- A value of zero indicates having the worst possible model and a value of one indicates having the best possible.
- **Rules of thumb:** CFI and TLI should be > 0.95 (ideally >0.97)

WHICH MEASURE OF FIT SHOULD YOU REPORT IN YOUR PAPER?

Index	Studies	
	<i>n</i>	%
Chi-square (χ^2)	173	89.2
Degrees of freedom (<i>df</i>)	173	89.2
χ^2/df ratio	42	21.6
Root-mean-square error of approximation (RMSEA)	126	64.9
Root-mean-square residual (RMR)	29	14.9
Standardized RMR	45	23.2
Goodness of fit index (GFI)	66	34.0
Adjusted GFI	39	20.1
McDonald's centrality index	2	1.0
Normed fit index	46	23.7
Tucker–Lewis index (TLI)	90	46.4
Comparative fit index (CFI)	152	78.4
Relative noncentrality index (RNI)	4	2.1
Bollen's rho 1 (relative fit index)	3	1.5
Bollen's delta 2 (incremental fit index)	17	8.8
Akaike information criteria (AIC)	20	10.3
Consistent AIC	3	1.5
Schwarz's Bayesian criteria (SBC)	1	0.5
Expected cross-validation index (ECVI)	13	6.7
Absolute standardized residuals	3	1.5
Null model χ^2	14	7.2
Null model <i>df</i>	14	7.2

Note. The frequency and percentage reporting each measure are based on the study le

<https://psycnet.apa.org/record/2009-02702-005>

GETTING FIT-STATISTICS FROM LAVAAN

```
summary(RWA_1F_fit, fit.measures=TRUE)  
fitMeasures(RWA_1F_fit, c("chisq", "df", "cfi", "rmsea", "srmr"))
```


SUMMARIZING FIT STATISTICS

- A single statistic reflects only a particular aspect of fit, a favorable value need not indicate overall acceptable fit.
 - Include different classes of fit-statistics in your paper (χ^2 , RMSEA, CFI)
- Values of fit statistics indicate only the average or overall fit of a model.
 - Some parts may fit poorly, even if the overall fit is acceptable.
- There is little direct relation between values of fit statistics and the degree or type of misspecification (Millsap, 2007).

REQUESTING LATENT VARIABLE SCALED MODELS

```
HS.model ← ' visual  =~ x1 + x2 + x3  
            textual =~ x4 + x5 + x6  
            speed   =~ x7 + x8 + x9 '  
fit ← cfa(HS.model,  
          data = HolzingerSwineford1939,  
          std.lv = TRUE)
```

- **semTable** is an R package to make tables CFA and SEM models estimated with the 'lavaan' package,
 - Provides functions to create model summary tables and modelcomparison tables for hypothesis testing.
 - If saved as html, tables can be opened for editing in Word
 - <https://cran.r-project.org/web/packages/semTable/semTable.pdf>
 - <https://cran.rstudio.com/web/packages/semTable/vignettes/semtable.pdf>

```
setwd("M:/__Undervisning/_PSY9140/PSY9140_2020/")  
  
semTable(JS_4F_fit, type="html",      file="CFA_4F_JS_Table")  
semTable(JS_4F_fit, type = "html",  
         paramSets = c("loadings", "latentvariances", "latentcovariances"),  
         file="CFA_4F_JS_Table", print.results = FALSE)
```

- A model is **nested** under another if it can be obtained from the model with more free variables by fixing one or more parameters.
- The **chi-square difference** statistic can be used to test the significance of the decrement in overall fit as free parameters are eliminated (*trimming*) or the improvement in fit as free parameters are added (*building*).

CALCULATING CHI-SQUARE DIFFERENCE TESTS

- Determine df of (1).
- Compare (2) to (1). Would you accept or reject (2)?
- How would you compare (2) and (6)?

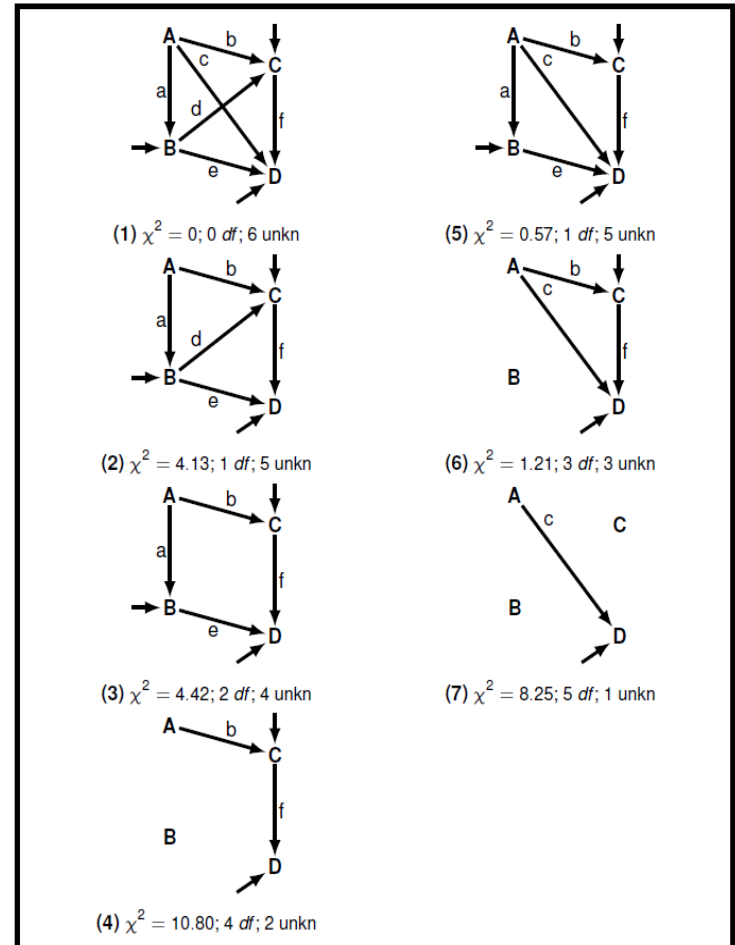
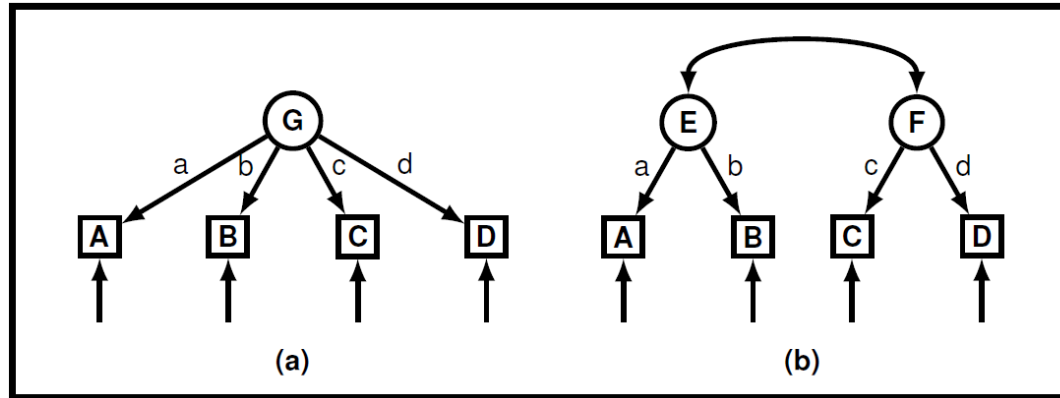


Figure 2.9 Hierarchical series of path models (χ^2 values hypothetical).

CALCULATING CHI-SQUARE DIFFERENCE TESTS



		Model											
		(a)	(b)		Difference								
S	1.0 .3 .2 .1	$\chi^2, N = 120$	4.64	.75	3.89								
	.3 1.0 .2 .2					$\chi^2, N = 240$	9.31	1.51	7.80				
	.2 .2 1.0 .3									<i>df</i>	2	1	1
	.1 .2 .3 1.0												

Use the chi-square test to compare the models.

COMPARING MODELS IN LAVAAN

```
# Using the base anova() function  
anova(model1, model2, ... ,modelk)  
  
# The semTools compareFit() function  
compareFit(model1, model2, ... ,modelk)
```

STATISTICAL SIGNIFICANCE AND HYPOTHESIS TESTING

SE, P-VALUES AND CONFIDENCE INTERVALS

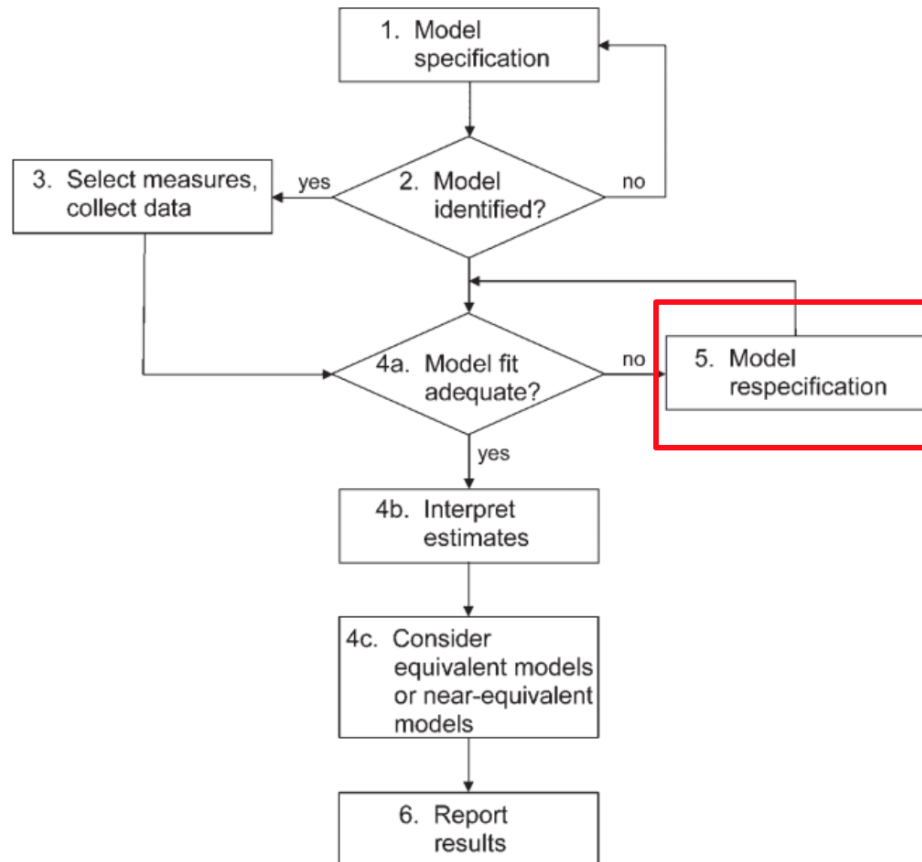
- One reason why ML is widely used in SEM is that it provides standard errors (SEs) for each of the model's parameter estimates.
- SEs are a measure of how stable the estimates would be if we were able to fit the model repeatedly in new samples from the population.
- SEs are used for conducting statistical significance tests of the parameter estimates. and for determining the precision of these estimates
 - 95% confidence interval (CI): $\text{parameter} \pm [1.96 * \text{SE}]$

CONFIDENCE INTERVALS IN LAVAAN

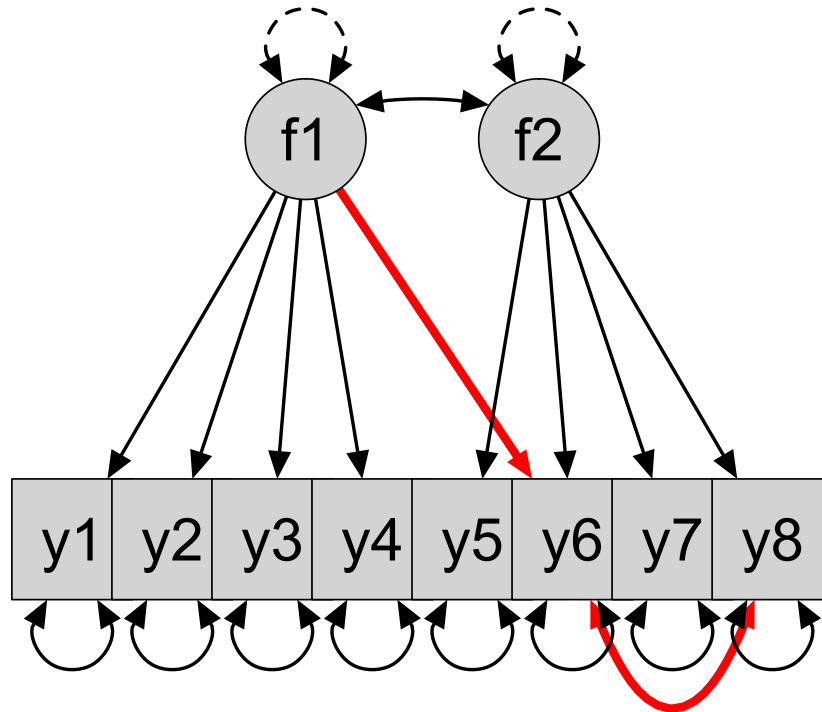
```
parameterEstimates(fit, ci=TRUE)
```

POST HOC ANALYSES

RESPECIFYING MODELS



POSSIBLE MODIFICATIONS TO A MULTIDIMENSIONAL CFA



MODIFICATION INDECIES

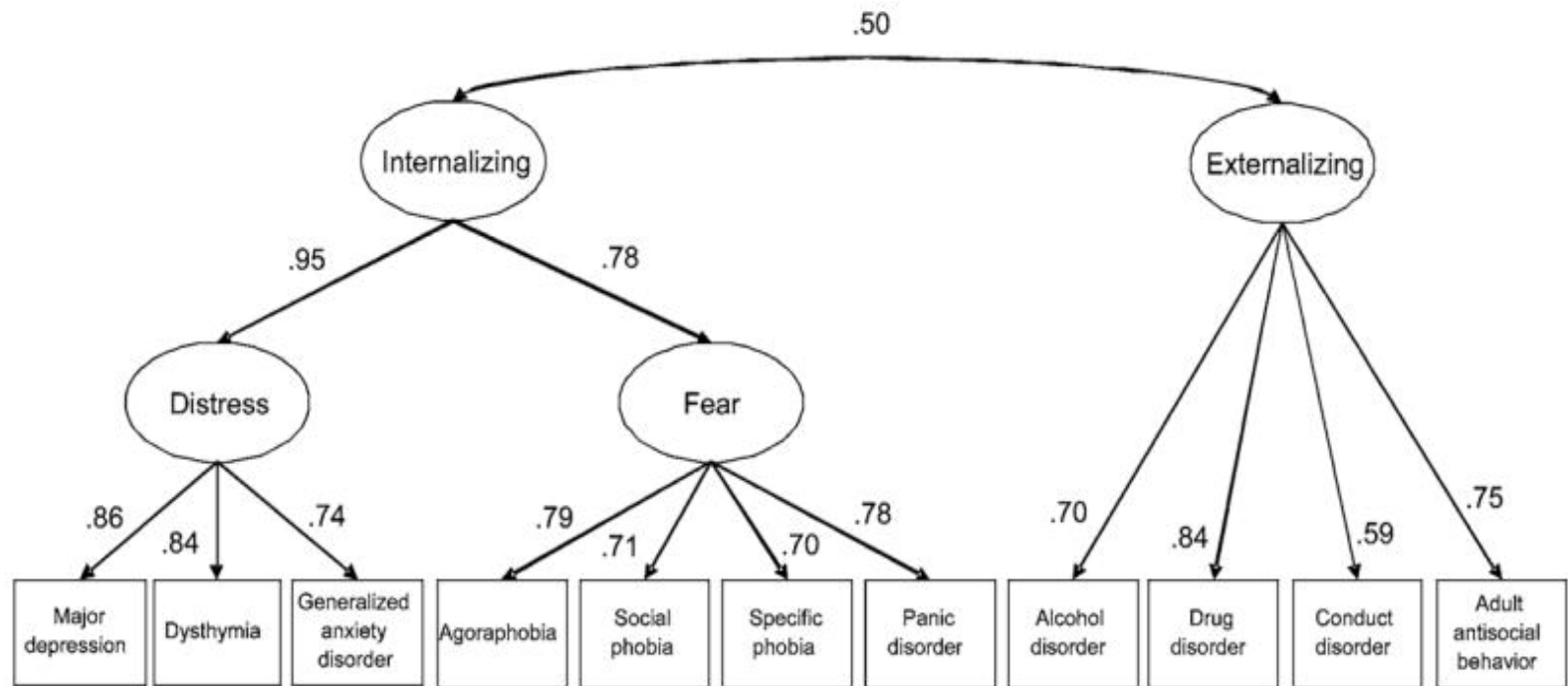
- **Modification index (MI)** allow you to examine which model restrictions may be relaxed to obtain better fitting model.
- MI reflects an approximation of how much the overall χ^2 would change if a fixed parameter was freely estimated.
- MI are obtained in Lavaan by:

```
fit ← cfa(CFA1F, data = myData)
modindices(fit, minimum.value = 10, sort=TRUE)
```

- Model modification is a controversial topic in SEM.
 - Respesification is usually empirically, not theoretically derived.
 - Typically both original and respecified model are tested against the same data (not ideal).

See the syntax file

EXAMPLE: 2ND ORDER CFA. DSM-METASTRUCTURE



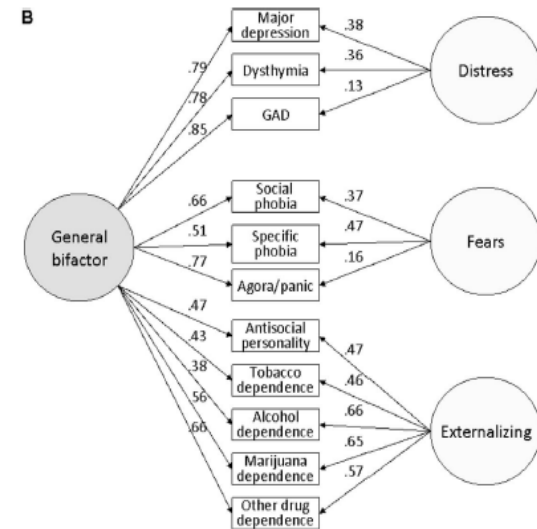
Is there a higher order structure to mental disorders?

PURPOSE OF 2ND ORDER CFA

- The second-order factors may be hypothesized to account for the variation among the first-order factors.
 - Often more parsimonious than a correlated factors structure.
- Empirical support for the first-order factor structure is a prerequisite for testing higher order structures.
- The rules of identification used in first-order CFA apply to the higher-order component of a hierarchical solution.
 - Assignment scale to the 2-order construct
 - 2nd-order must load on >2 1st order factors.

BIFACTOR MODELS

- Bifactor models are alternatives to second order models.
- According to the bifactor representation, each indicator has three sources of influences:
 1. A common factor,
 2. A specific factor for which the indicator is the unique representative.
 3. Measurement error.
 - The common factor is not correlated with specific factors, which are not correlated among themselves.



(Lahey et al., 2012)

BENEFITS OF BIFACTOR MODELS

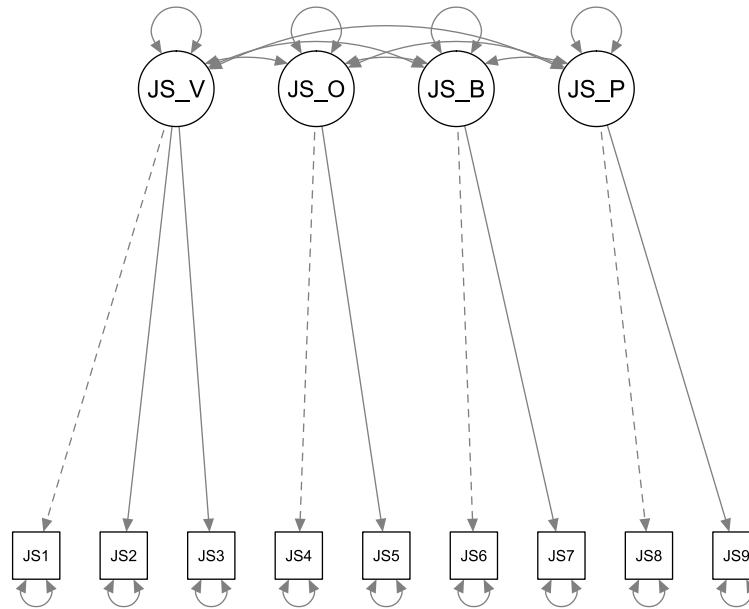
- Helps judge the relevance/presence of sub-factors in a measure.
- Makes it possible to work with models with fewer than four first order factors.

JUSTICE SENSITIVITY ITEMS

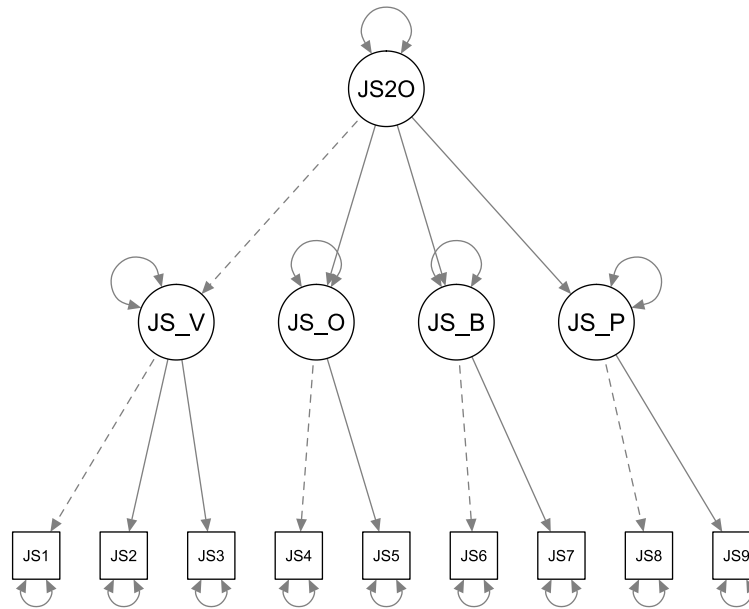
1. It makes me angry when others are undeservedly better off than me (Victim)
2. It worries me when I have to work hard for something that others easily achieve (Victim)
3. If others are treated better than me, it is difficult for me to stop thinking about it (victim)
4. I get upset when some are undeservedly worse off than others (observer)
5. It worries me when someone has to work hard for something that others easily get (observer)
6. I feel guilty when I am better off than others for no reason (beneficiary)
7. It bothers me when I easily get to something that others have to work hard for (beneficiary)
8. I feel guilty when I enrich myself at the expense of others (perpetrator)
9. It bothers me when I use tricks to achieve something that others must strive for (perpetrator)

EXERCISE: COMPARE ALTERNATIVE MULTIDIMENSIONAL FACTOR MODELS

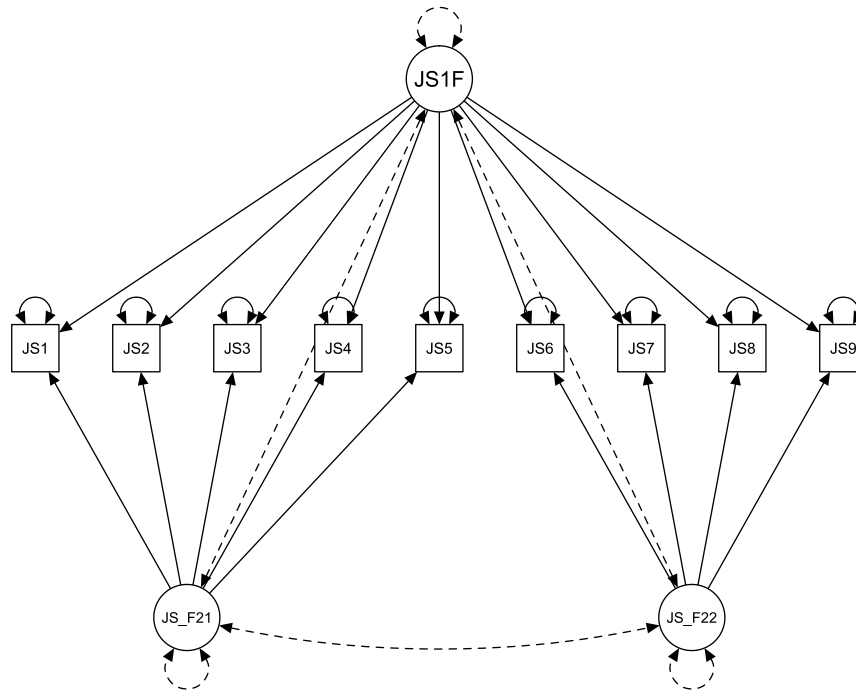
MODEL 1: FOUR FACTOR CFA



MODEL 2: SECOND ORDER CFA



MODEL 3: BIFACTOR MODEL



Notice: The "dotted" correlations are all equal to zero.

VARIOUS SEM ISSUES

- **Factor scores** are used for various purposes, such as to serve as proxies for latent variables, and to determine a participant's relative standing on the latent dimension.
- Conceptually, a factor score is the score that would have been observed for a person if it had been possible to measure the latent factor directly. In applied research, factor scores are often computed by creating coarse factor scores, which are simple unweighted composites of the raw scores of indicators (e.g., averaging or summing) found to have salient loadings on the factor.

GETTING FACTOR SCORES IN LAVAAN

```
# First fit model
myModel.Fit ← cfa(myModel, data=myData)

# "Predict function" gives factor scores
pred = predict(myModel.Fit)
```

SOME CAUTIONARY POINTS REGARDING SEM

- **SEM usually requires large samples**
 - Rule of thumb is that the sample size-to-parameters ratio should be 20:1.
 - Reviewers routinely reject publications with $n < 200$ (Barret, 2007)
- **Models, even in published ones, often contain errors**
 - MacCallum (2000) reviewed 500 applications of sem in 16 journals, and found many to have one flaw so serious that it compromised the scientific value [1].
- **Be aware of confirmation bias**
 - Good fit to your data does not mean the results will replicate in a new sample.
 - Remember to consider *alternative* models for your data.

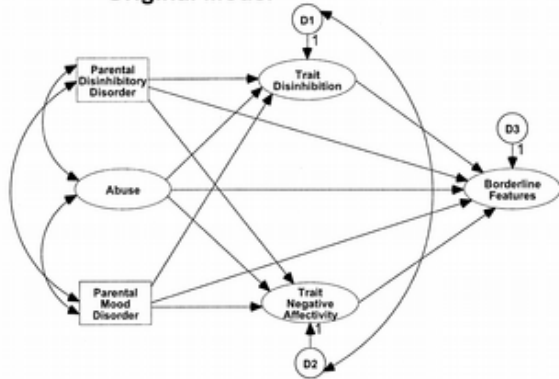
[1] <https://doi.org/10.1146/annurev.psych.51.1.201>

EQUIVALENT MODELS

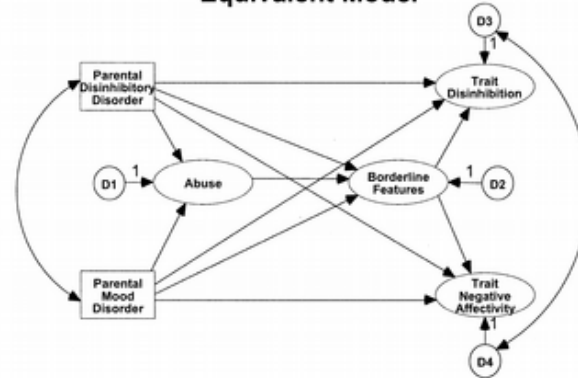
- If two different sets of parameter values generate the same expected covariance matrix, then the models are said to be **equivalent**.
- Complicated models can have thousands of equivalent models.
- They have identical fit, and selection must be based on theoretical rather than statistical grounds.

EXAMPLE: EQUIVALENT MODELS

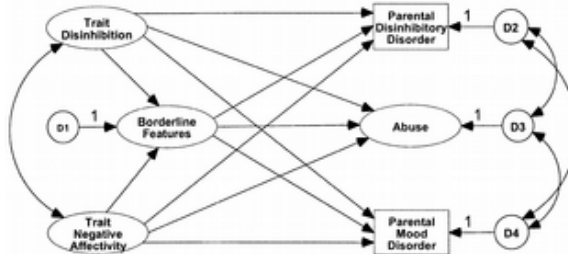
**Model 3A
Original Model**



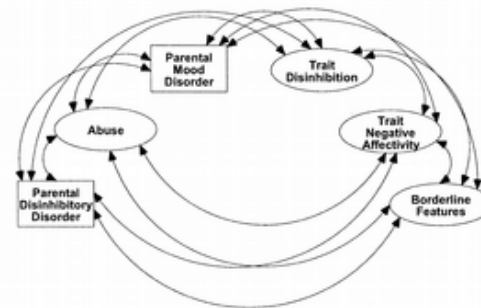
**Model 3B
Equivalent Model**



**Model 3C
Equivalent Model**



**Model 3D
Equivalent Model**



INPUT DATA: FITTING A MODEL TO SUMMARY STATISTICS VS. RAW DATA

```
5.700 5.600 6.400 5.700 6.000 6.200 5.700 5.600
1.000
0.767 1.000
0.731 0.709 1.000
0.778 0.738 0.762 1.000
-0.351 -0.302 -0.356 -0.318 1.000
-0.316 -0.280 -0.300 -0.267 0.675 1.000
-0.296 -0.289 -0.297 -0.296 0.634 0.651 1.000
-0.282 -0.254 -0.292 -0.245 0.534 0.593 0.566 1.000
```

Summary statistics input file

```
1 1 0 13 1 1 1
1 1 7 15.5 1 1 1
1 1 3 19 1 1 1
1 1 6 20 1 1 1
1 2 10 20.5 1 1 1
1 1 7 22.5 1 1 1
1 1 11 27.5 1 1 1
1 1 6 36 1 1 1
1 1 4 125 1 1 1
1 1 4 4.5 1.1 1.2 1
1 1 8 44.5 1.1 1.2 1
1 1 2 8 1.2 1.4 1
1 1 4 19.5 1.1 1 1.2
1 1 4 98.5 1.1 1 1.2
1 1 7 208.5 1.4 1.6 1.2
1 1 7 6 1.2 1 1.4
1 1 8 20 1.5 1.6 1.4
1 1 5 6 1.6 1.8 1.4
1 1 12 12.5 1.5 1.4 1.6
1 1 8 30 1.6 1.6 1.6
1 2 3 32.5 1.6 1.6 1.6
1 2 7 94 1.8 2 1.6
1 2 5 14 1.7 1.6 1.8
```

Raw data input file

- SEM models can be fitted to either summary statistics or raw data.
- Fit statistics on summary statistics require data on the number of observations.

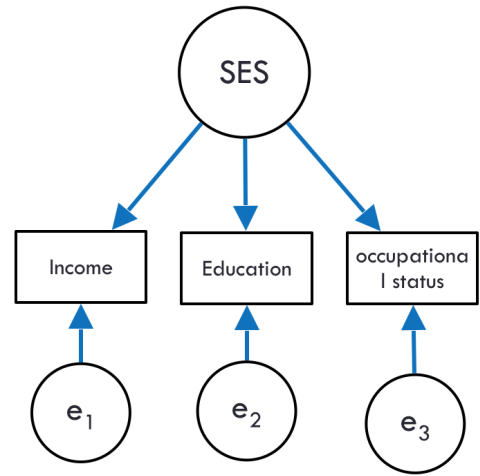
FITTING MODELS TO SUMMARY STATISTICS

```
# Make a covariance matrix from cormatrix and mean vec
MyCovarMatrix ← getCov(MyLocerCormatrix, sds=MyMeans)

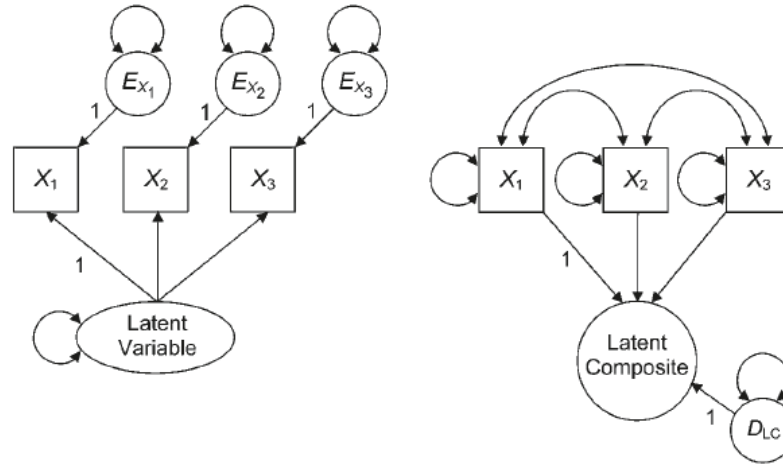
# Fit model to summary statistics
MyModel.fit=cfa(MyModel,
                sample.cov=MyCovarMatrix,
                sample.nobs = 400)
```


FORMATIVE INDICATORS

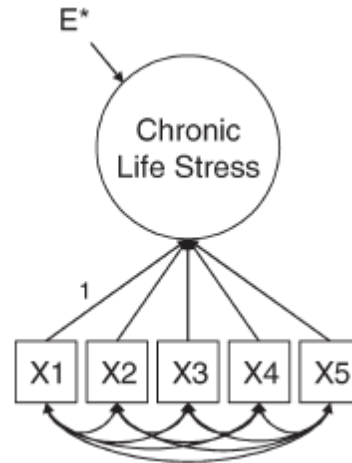
A REFLECTIVE FACTOR



A REFLECTIVE FACTOR



FORMATIVE VS REFLECTIVE FACTORS



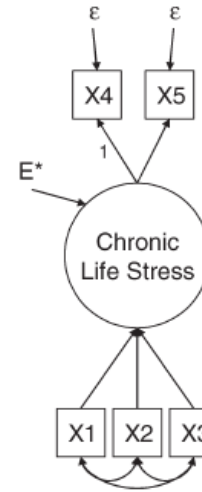
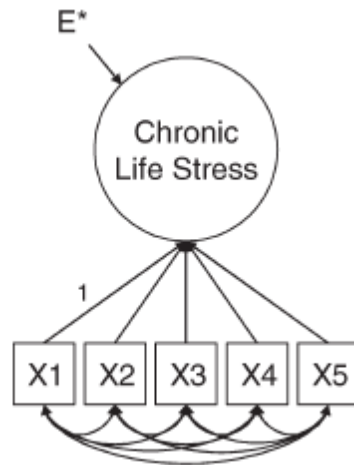
In a **reflective** measurement model, indicators are caused by the construct.

- High IQ will "cause" the correct answer in a set of intelligence indicators.
- Indicator residual covariances expected to be zero.

In a **formative** measurement model, the factor is caused by the indicators.

- The value of a car is a function of make, age and condition.
- We make no assumptions about the item covariances.

FORMATIVE VS REFLECTIVE FACTORS



A formative measurement model is not identified without additional information.

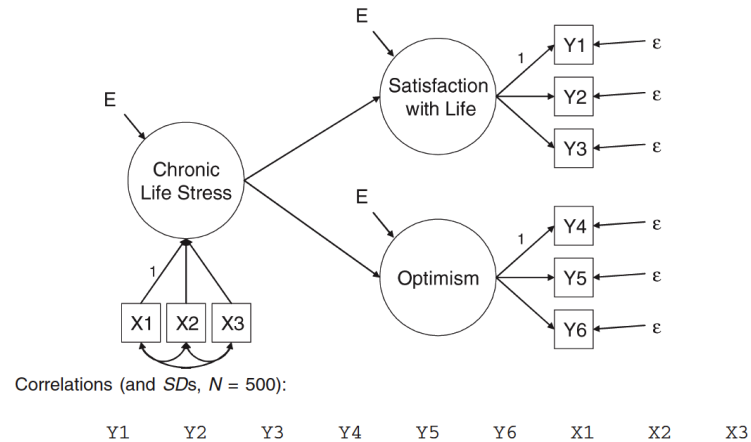
- Add two downstream measures.
- constrain disturbance (to zero).

FORMATIVE FACTORS IN LAVAAN

- Formative factors

```
# Defining a formative factor in Lavaan  
stress ← X1+X2+X3
```

IMPLEMENT THIS MODEL IN LAVAAN



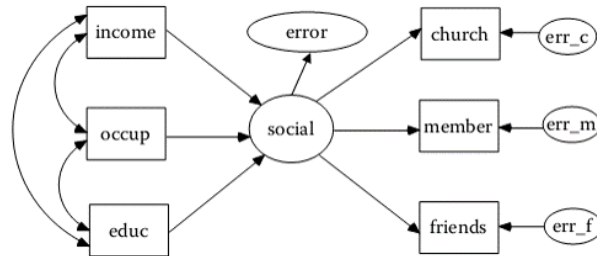
Compare a formative and a reflective model of chronic life stress on Satisfaction with life and optimism.

MIMIC MODELS

MULTIPLE INDICATORS MULTIPLE CAUSES (MIMIC) MODELS

In **MIMIC** models, latent variables that are predicted by observed variables.

- Social integration assessed in a sample of n=530.

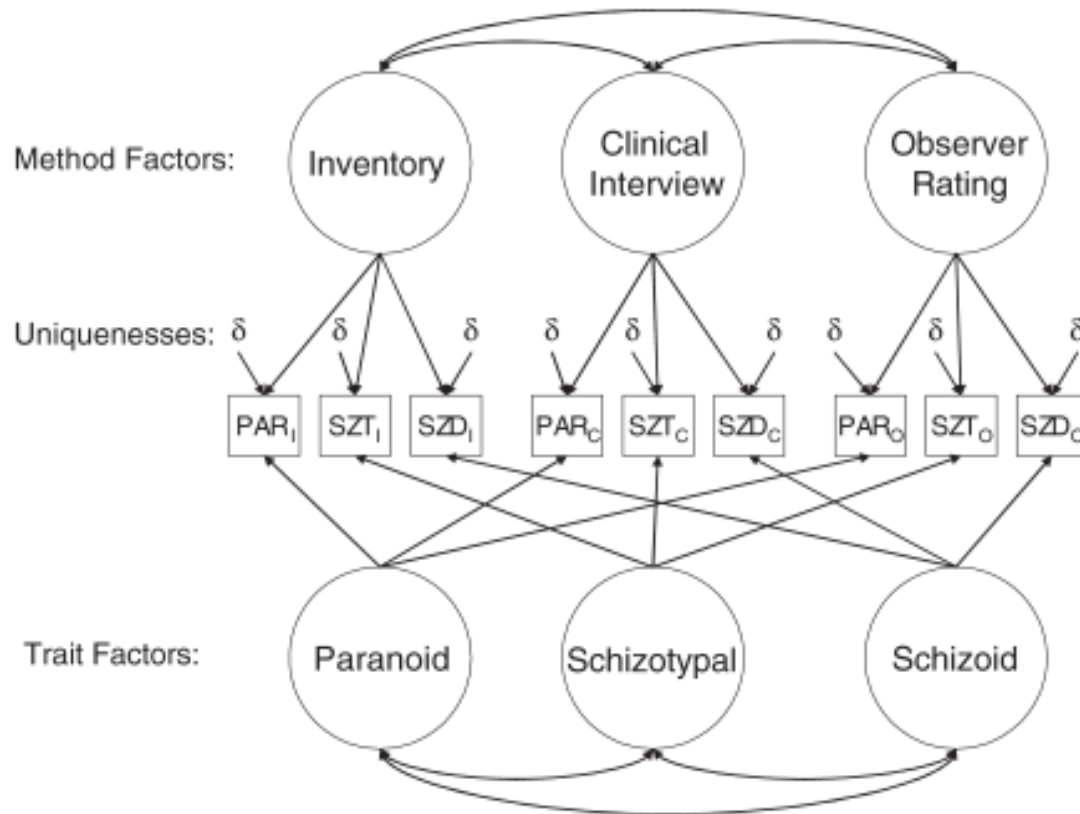


- Multiple indicators reflect the underlying latent variables/factors
- Multiple causes (observed predictors) affect latent variables/factors
- MIMIC factor have both effect and cause indicators.

MULTI-TRAIT MULTI-METHOD

MULTITRAIT-MULTIMETHOD (MTMM)

- Requires measuring more than one construct using more than one measure.
- Attempts to separate out *true* variance on psychological traits from variance due to measurement methods.
- MTMM can allow you to make claims about:
 - **Convergent validity**, the tendency for different measurement operations to converge on the same underlying trait.
 - **Discriminant validity**, the ability to discriminate among different traits.
 - **Common method variance**, the spurious variance that is attributable to the measurement method rather than to the constructs the measures are assumed to represent.



Run the model above. What do you learn about the constructs and the measurement methods?

MTMM_CLUSTER_A_CORRELATED_METHODS_EXERCISE.INP (2)

	Inventory			Clinical Interview			Observer Ratings		
	PAR	SZT	SZD	PAR	SZT	SZD	PAR	SZT	SZD
Inventory									
Paranoid (PAR)	(.930)								
Schizotypal (SZT)	.290	(.900)							
Schizoid (SZD)	.372	.478	(.910)						
Clinical Interview									
Paranoid (PAR)	.587	.238	.209	(.800)					
Schizotypal (SZT)	.201	.586	.126	.213	(.870)				
Schizoid (SZD)	.218	.281	.681	.195	.096	(.830)			
Observer Ratings									
Paranoid (PAR)	.557	.228	.195	.664	.242	.232	(.820)		
Schizotypal (SZT)	.196	.644	.146	.261	.641	.248	.383	(.790)	
Schizoid (SZD)	.219	.241	.676	.290	.168	.749	.361	.342	(.800)

Monomethod blocks: correlations among indicators derived from the same assessment method;

Heteromethod blocks: correlations among indicators assessed by different methods

MTMM_CLUSTER_A_CORRELATED_METHODS_EXERCISE.INP (3)

	Inventory			Clinical Interview			Observer Ratings			
	PAR	SZT	SZD	PAR	SZT	SZD	PAR	SZT	SZD	
Inventory										
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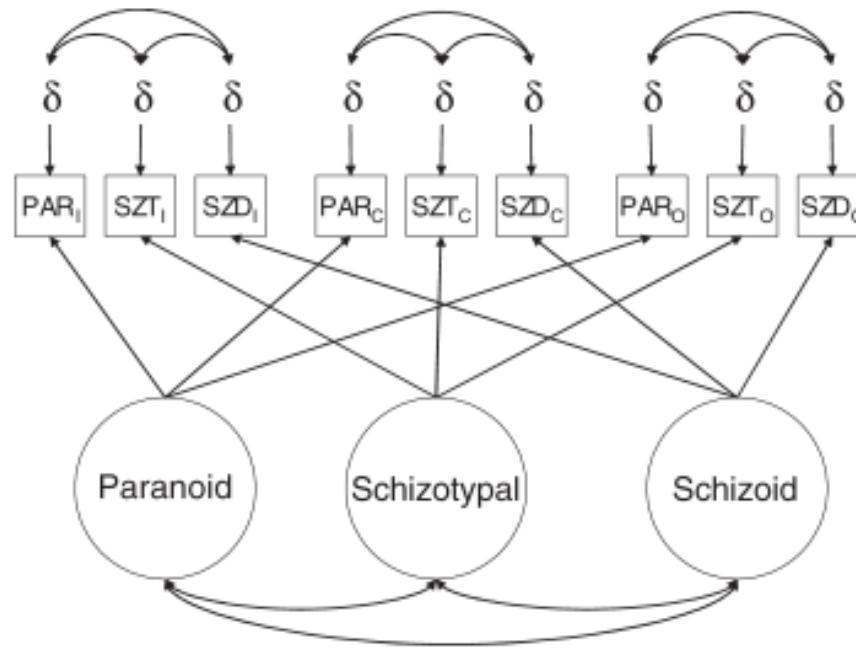
Validity diagonal: Within trait, cross method (High values indicate convergent validity)

MTMM_CLUSTER_A_CORRELATED_METHODS_EXERCISE.INP (4)

	Inventory			Clinical Interview			Observer Ratings		
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(Low values in all triangles indicate discriminant validity)

(Higher values in blue triangles than orange triangles indicate method variance)



Run the model. Do you find evidence for: Convergent validity, discriminant validity, method effects?

	Trait Factor Loadings			SMC	Uniqueness	Correlated Uniquenesses		
	Paranoid	Schizotypal	Schizoid					
PAR _I	.712			.507	.493	1.000		
SZT _I		.788		.621	.379	.094	1.000	
SZD _I			.769	.592	.408	.198	.293	1.000
PAR _C	.841			.707	.293	1.000		
SZT _C		.768		.589	.411	-.037	1.000	
SZD _C			.860	.740	.260	-.073	-.108	1.000
PAR _O	.788			.620	.380	1.000		
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SZD _O			.872	.760	.240	.111	.126	1.000
Trait Factor Correlations								
	Paranoid	Schizotypal	Schizoid					
Paranoid	1.000							
Schizotypal	.381	1.000						
Schizoid	.359	.310	1.000					

Note. SMC, squared multiple correlation (i.e., λ^2). All unstandardized parameters are significantly different ($p < .05$) from zero except for the correlation between the unique variances of SZT_C and PAR_C.

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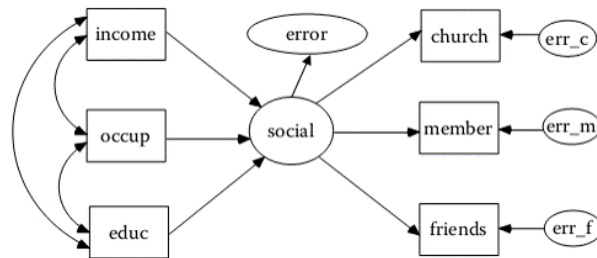


VARIOUS SEM ISSUES

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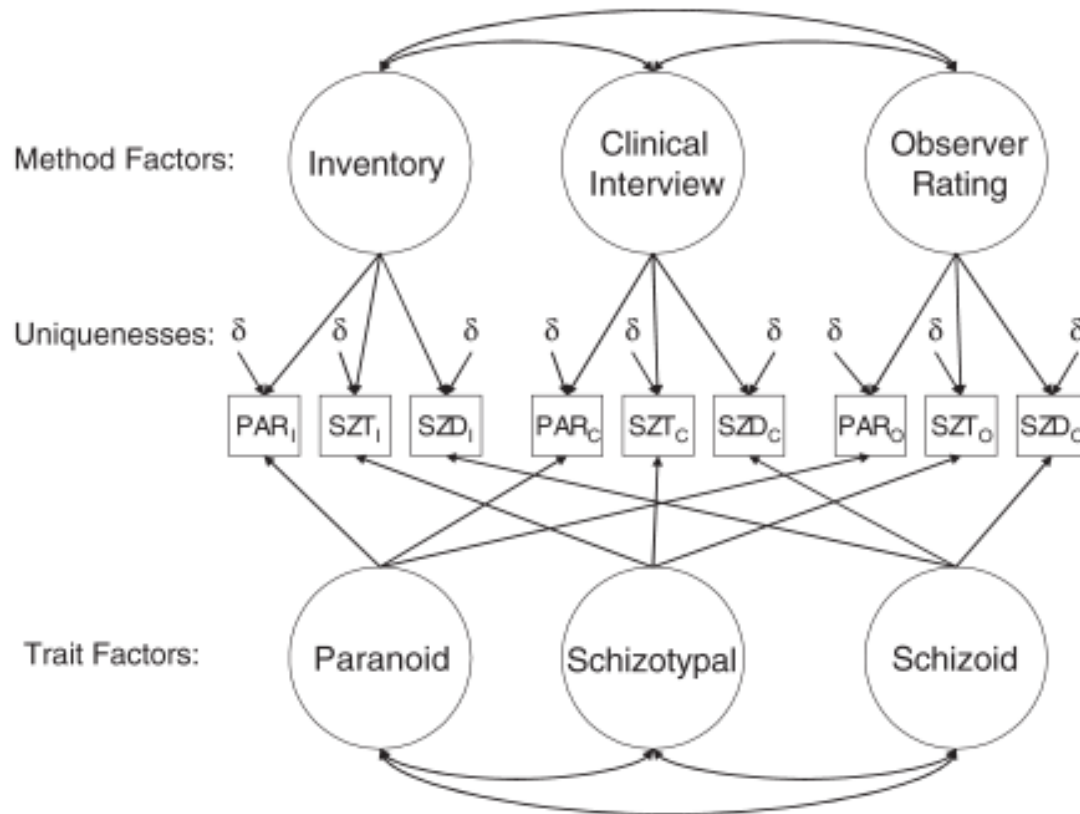


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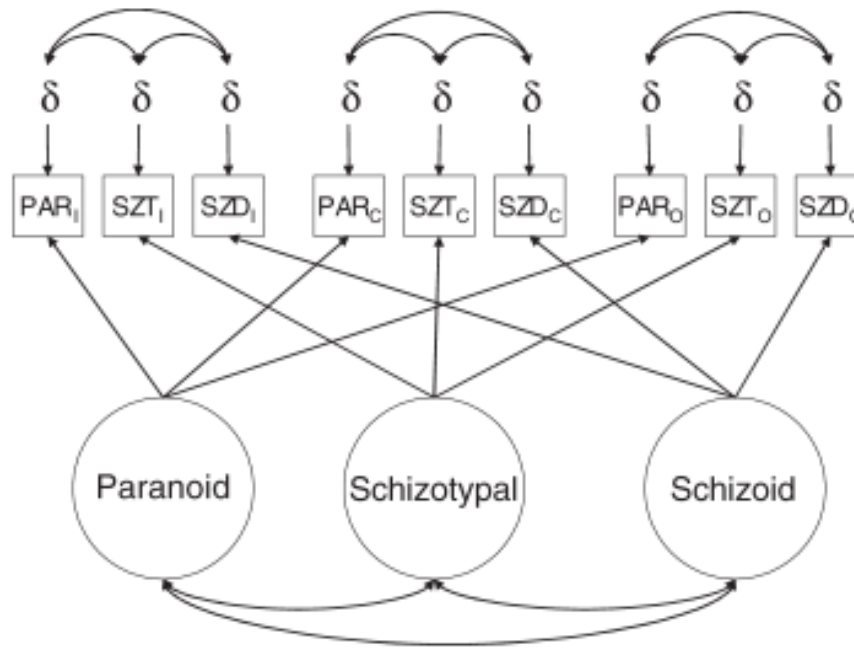
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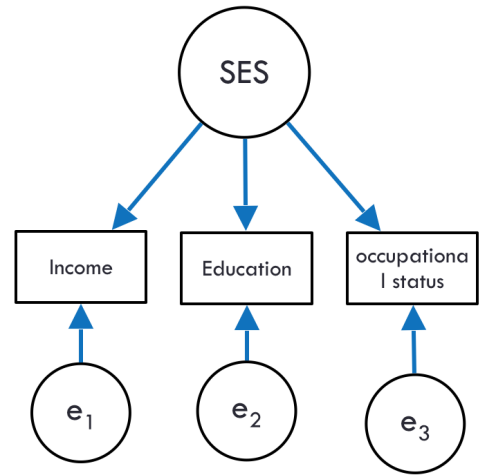
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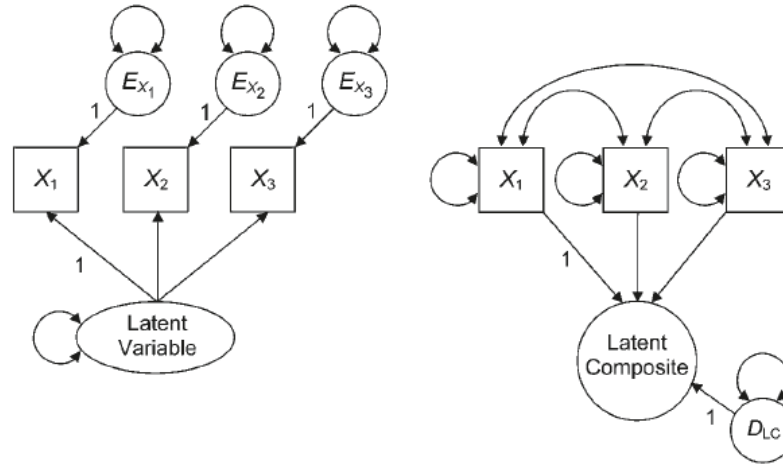
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FORMATIVE INDICATORS

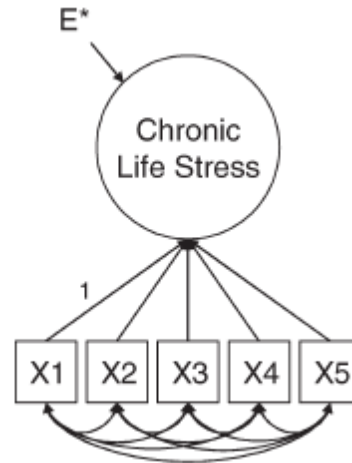
A REFLECTIVE FACTOR



A REFLECTIVE FACTOR



FORMATIVE VS REFLECTIVE FACTORS



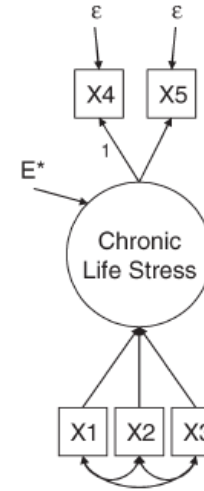
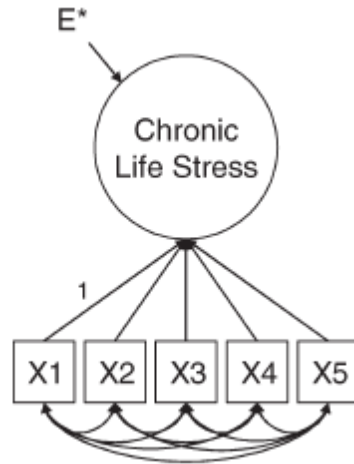
In a **reflective** measurement model, indicators are caused by the construct.

- High IQ will "cause" the correct answer in a set of intelligence indicators.
- Indicator residual covariances expected to be zero.

In a **formative** measurement model, the factor is caused by the indicators.

- The value of a car is a function of make, age and condition.
- We make no assumptions about the item covariances.

FORMATIVE VS REFLECTIVE FACTORS



A formative measurement model is not identified without additional information.

- Add two downstream measures.
- constrain disturbance (to zero).

KEY DISTINCTIONS: FORMATIVE VS REFLECTIVE MODELS

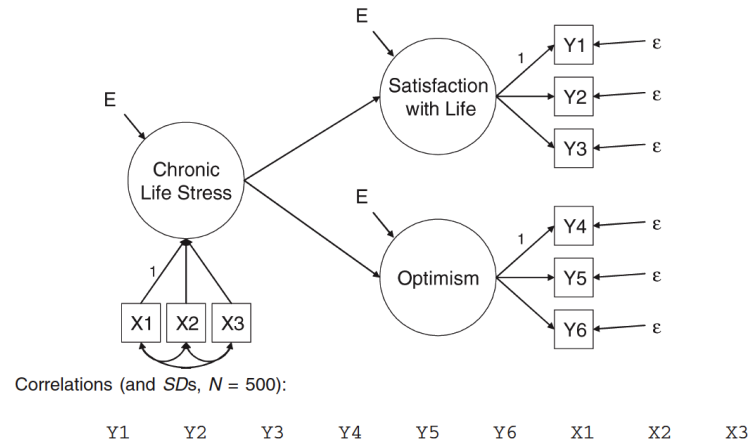
- **Direction of Causality:**
 - Reflective: Latent variable → Indicators
 - Formative: Indicators → Latent variable
- **Indicator Inter-correlation:**
 - Reflective: Indicators are often correlated
 - Formative: No assumption of indicator inter-correlation
- **Measurement Error:**
 - Reflective: Associated with each indicator
 - Formative: Considered at the construct level
- **Model Identification:**
 - Reflective: Generally straightforward
 - Formative: Often requires additional constraints or outcomes

FORMATIVE FACTORS IN LAVAAN

- Formative factors

```
# Defining a formative factor in Lavaan  
stress ← X1+X2+X3
```

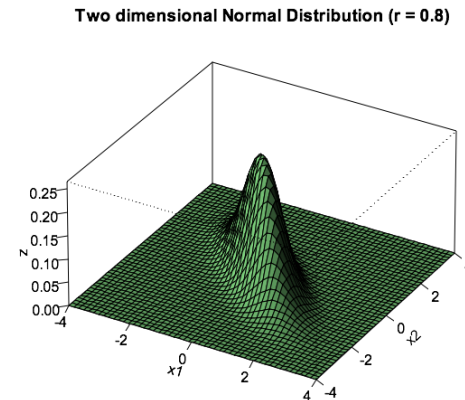
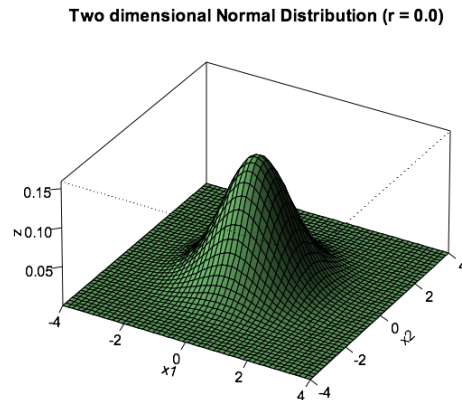
IMPLEMENT THIS MODEL IN LAVAAN



Compare a formative and a reflective model of chronic life stress on Satisfaction with life and optimism.

VARIOUS PRACTICAL COMPLICATIONS

THE ISSUE OF NON-NORMALITY IN SEM



- The vast majority of SEM models reported are fitted by maximum likelihood (ML).
- Correct inference by ML requires the data to be *multivariate normal*.
- Multivariate *kurtotic* data are particularly problematic.
 - Kurtosis can strongly impacts variance and covariances.
 - No clear consensus how much kurtosis constitutes a problem.
- Checking for non-normality is important before conducting an SEM analysis, as assumptions of normality are commonly violated in real-world data.

STATISTICAL CONSEQUENCES OF NON-NORMALITY IN SEM

- Inflated model χ^2 values (over rejection of models), and modest underestimation of fit indices such as the TLI and CFI.
- Underestimation of the standard errors of parameter.
 - Inflated risk of Type I-error, concluding that a parameter is significantly different from zero when that is not the case in the population.

- **Use a robust estimator**

- Using a robust estimator, such as the Satorra Bentler estimator, is a good strategy for handling non-normal data in SEM. This estimator adjusts the standard errors and chi-square test statistic to account for non-normality, providing more accurate model fit indices.

- **Bootstrapping**

- Bootstrapping is another useful technique to handle non-normal data in SEM. It involves repeatedly resampling the data to estimate sampling distributions. By using resampling, bootstrapping allows for the calculation of robust standard errors and confidence intervals, even in the presence of non-normality.

- **Item parcelling**

- Item parcelling is a technique where multiple items are combined or averaged to create parcels or composite variables. This helps to reduce the impact of non-normality and improve measurement reliability. However, care should be taken when using item parcelling, as it may also introduce potential issues such as loss of information or measurement bias.

Maximum Likelihood (ML)

- *Default for Continuous Data*
- Efficient, consistent, normally distributed estimates in large samples.
- Sensitive to normality violations.

Generalized Least Squares (GLS)

- *For Continuous and Complete Data*
- Efficient under normality, less so than ML in some conditions.

Weighted Least Squares (WLS)

- *For Categorical Data*
- Weights observations to account for non-normality of categorical data.
- Accurate but requires large, complete datasets.

Unweighted Least Squares (ULS)

- *Does Not Assume Normality*
- Ignores scale information, can be inefficient.

In Mplus

- **Corrects Chi-Square Values for Non-Normality**
- **MLM (Satorra-Bentler)** and **MLR (Robust ML)** available.
- MLM uses listwise deletion; MLR handles missing data more flexibly.

Chi-Square Difference Testing

- Conducted using the Satorra-Bentler (SB) (χ^2) statistic.
- Unlike ML, requires a scaled difference in chi-square for accurate testing.
- Detailed procedure at: [StatModel Chi-Square Difference Testing](#).

Standard Errors

- Standard errors are larger with MLM/MLR compared to ML.
- Reflects more realistic estimation of parameter variability under non-normality.

Various estimators in Mplus allow you to correct the chi-square values for non-normality.

- Available in Mplus as the **MLM** (Satorra-Bentler) **MLR** (robust ML).
- MLM requires listwise deletion, MLR does not.
- Chi-square *difference testing* can be conducted using the SB χ^2 statistic.
 - Unlike ML-based analysis, this test cannot be conducted by simply calculating the difference in χ^2 . Instead a *scaled* difference in chi-square difference must be used.
- The procedure is described in detail at: <http://www.statmodel.com/chidiff.shtml>.
- Standard errors are generally larger with MLM/MLR compared to the ML estimator.

COMPARING MODELS WITH ROBUST ESTIMATORS IN R

Key Functions

- `anova()`: Compares nested models using chi-square difference tests.
- `lavTestLRT()`: Specifically for likelihood ratio tests (equivalent to chi-square difference tests in this context).

When to Use

- Both functions are applicable for models estimated with robust methods (e.g., MLM, MLR).
- Automatically apply Satorra-Bentler scaling for accurate comparison under non-normality.

STEPS FOR COMPARISON

1. Fit Models:

- Fit both full and nested models using robust estimation.
- `fit_full ← sem(full_model_spec, data = dataset, estimator = "MLM")`
- `fit_nested ← sem(nested_model_spec, data = dataset, estimator = "MLM")`

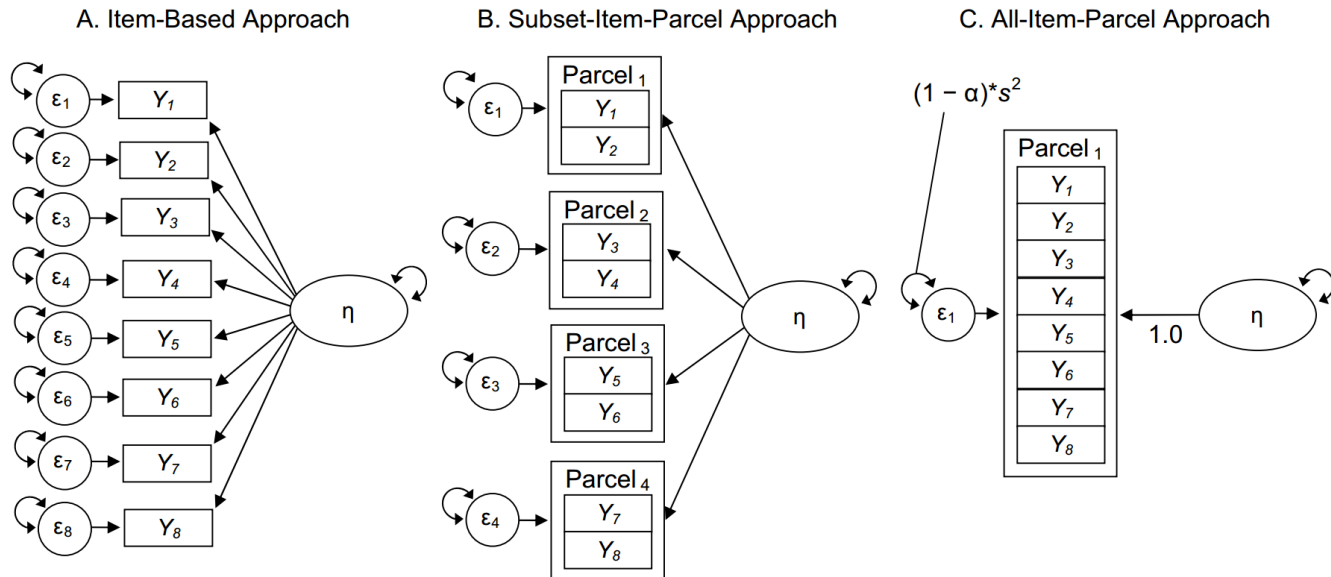
2. Compare Models:

- `anova(fit_nested, fit_full)` or `lavTestLRT(fit_nested, fit_full)`

3. Interpret Results:

- Output includes chi-square values, degrees of freedom difference, and p-value.
- Significant p-value indicates nested model significantly worsens the fit.

PARCELLING



.footnote[

[1] Little, 2002 [2] Matsunaga, 2002

ITEM PARCELLING IN SEM

Combining multiple items (indicators) into a single parcel

- Often used when items are believed to measure the same construct.

How to Perform Parcelling

1. **Group Items:** Combine items that theoretically measure the same construct.
2. **Create Parcels:** Calculate the mean or sum of grouped items to create parcels.

Tools in R

- No specific package for parcelling, but can be done using base R functions (`mean`, `sum`) or `dplyr` for data manipulation.

Example in R

```
library(dplyr)
data <- data %>%
  mutate(parcel1 = mean(c(item1, item2, item3), na.rm = TRUE))
```

- **Reduction in Information:** Parcelling may lead to loss of information and potential biases.
- **Assumption of Unidimensionality :** Assumes that all items in a parcel measure the same underlying construct.
- **Impact on Model Fit :** Can artificially improve model fit indices and affect parameter estimates.

- If the data contain missing values, the default behavior is listwise deletion.
- If the missing mechanism is MCAR (missing completely at random) or MAR (missing at random), the lavaan package provides case-wise (or ‘full information’) maximum likelihood estimation.
- You can turn this feature on, by using the argument `missing = "ML"` when calling the fitting function.
- The semTools package provides functions to perform multiple imputation with SEM models.

ALTERNATIVE APPROACHES FOR INFERENCE: BOOTSTRAPPING



The idea is to use the empirical distribution function as an approximation of the population.

- Draw a large number of new samples (>1000) of same size as the original from the observed data (with replacement).
- In each sample recalculate the statistic of interest.
- The resulting (bootstrap) distribution is an approximation of the sampling distribution.

ALTERNATIVE APPROACHES FOR INFERENCE: BOOTSTRAPPING, EXAPLE

$Y_0 = 79\ 91\ 131\ 109\ 108\ 107\ 99\ 118\ 114\ 94\ 117\ 106\ 106\ 96\ 103.$ $\bar{Y}_0 = 105.2$

Now, sample with replacement from the original dataset:

$Y_{b1} = 107\ 131\ 108\ 117\ 108\ 94\ 103\ 94\ 106\ 107\ 131\ 109\ 91\ 103\ 94.$ $\bar{Y}_{b1} = 106.9$

$Y_{b2} = 107\ 99\ 106\ 99\ 114\ 106\ 91\ 118\ 94\ 96\ 96\ 107\ 131\ 107\ 114.$ $\bar{Y}_{b2} = 105.7$

$Y_{b3} = 96\ 131\ 106\ 109\ 96\ 107\ 94\ 103\ 96\ 96\ 79\ 131\ 91\ 118\ 94.$ $\bar{Y}_{b3} = 103.1$

$Y_{b4} = 118\ 106\ 91\ 109\ 91\ 108\ 99\ 114\ 131\ 103\ 131\ 99\ 106\ 106\ 107.$ $\bar{Y}_{b4} = 107.9$

ALTERNATIVE APPROACHES FOR INFERENCE: BOOTSTRAP DISTRIBUTION

Bootstrap interval (98.9, 111.3)

There are two ways for using the bootstrap in lavaan.

1. Either you can set `se = "bootstrap"` or `test = "bootstrap"` when fitting the model (and you will get bootstrap standard errors, and/or a bootstrap based p-value respectively).
2. Use the `bootstrapLavaan()` function on an already fitted lavaan object. The latter function can be used to 'bootstrap' any statistic (or vector of statistics) that you can extract from a fitted lavaan object.

PRACTICAL COMPLEXITIES II: CATEGORICAL DATA

CATEGORICAL DATA

- A **continuous** variable can take infinitely many values (on a range).
- A **categorical** variable can take a fixed and limited number of possible values.
 - Ordinal variables have ordered categories (ex. weight classes), while nominal variables do not (ex. car brands).
- Both ML and robust ML (MLM) estimators require continuous outcome variables.
- If you have a binary exogenous covariate (say, gender), all you need to do is to recode it as a dummy (0/1) variable.

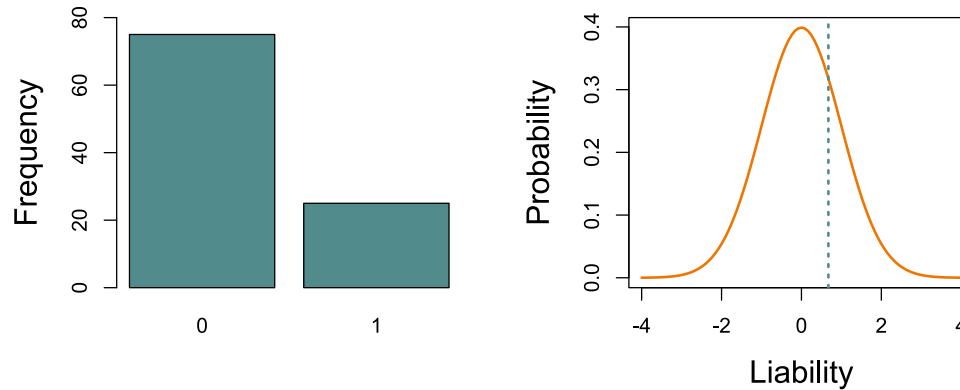
THREE WAYS OF HANDLING ORDINAL DATA

1. Treat it as continuous (historical solution)
2. Liability threshold model
3. Item response theory (IRT) approach

1. TREATING ORDINAL DATA AS CONTINUOUS

- This has historically been the way to treat ordinal data in SEM.
- Pearson correlations are typically higher when the variables are continuous.
 - The greatest attenuation occurs when the variables have <5 categories.
 - A high degree of skewness makes attenuation much worse.
 - There is negligible bias in non-skewed data with many (≥ 7) categories.

2. LIABILITY-THRESHOLD APPROACH



Underlying each observed ordinal variable is an unobserved latent counterpart, which is continuous and normally distributed.

2. THRESHOLD MODEL IN LAVAAN

- Categorical variables must be identified using the function `ordered`.
- Numerous estimators can be used
 - WLSMV is default estimator for categorical data, developed by Muthén, and previously available only in Mplus.
 - ML can be used, but is extremely slow on high dimensional ordinal data.

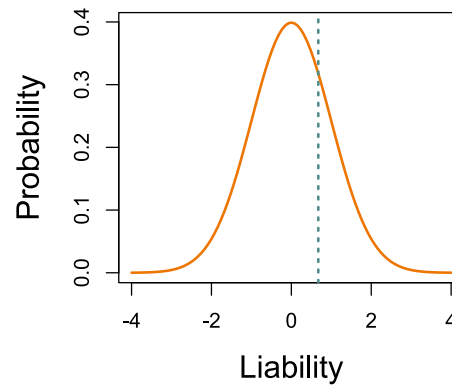
Endogenous categorical variables

-

```
fit ← cfa(myModel, data = myData,  
          ordered=c("item1", "item2",  
                   "item3", "item4"))
```

Lavaan will automatically switch to the WLSMV estimator:

2. THRESHOLD MODEL; SOME DETAILS



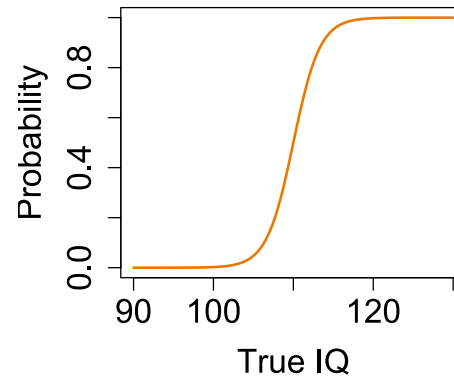
- Because estimates are based on the correlation matrix, residual variances with are not estimated for categorical data.

ITEM RESPONSE MODELS (IRT)

ITEM RESPONSE MODELS (IRT)

- A paradigm for the design, analysis, and scoring of tests, questionnaires, and similar instruments.

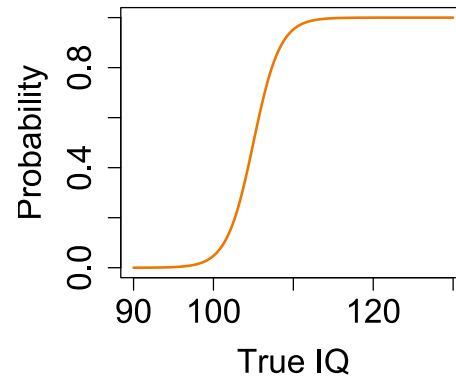
ITEM CHARACTERISTIC CURVES)



The **item characteristic curve** relates levels on a trait (e.g. IQ) to the probability of correctly answering (or endorsing) an item.

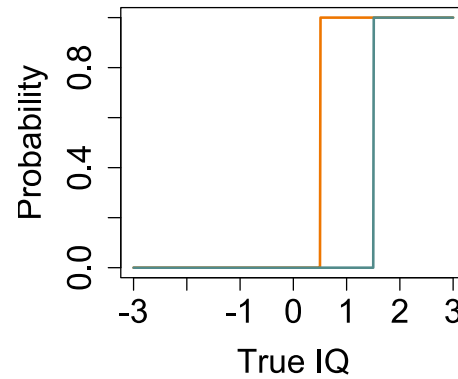
- *difficulty* is the value where probability is 0.5.

PLACEMENT ON THE LATENT DIMENSION



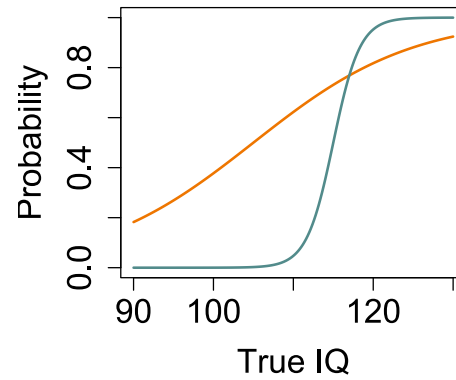
Multiple itmes help us better position people on the latent dimension.

STEP FUNCTIONS



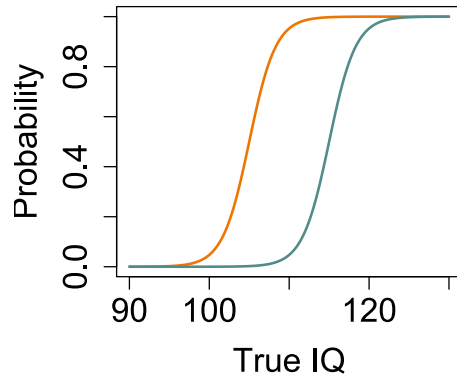
Step functions which transition from 0 to 1 at a single point could help precisely estimate latent value.

ITEM DISCRIMINABILITY

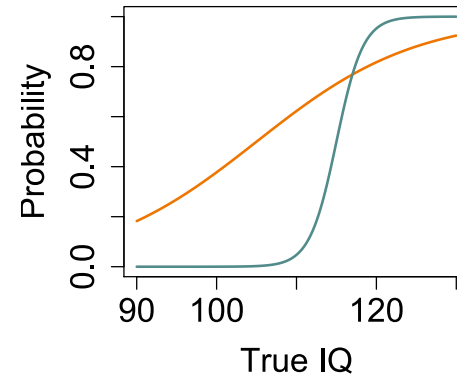


More flexible IRT models allow different levels of discriminability.

ONE VS TWO-PARAMETER IRT MODELS



$$p = \frac{e^{(\theta-\beta)}}{1 + e^{(\theta-\beta)}}$$



$$p = \frac{e^{\alpha(\theta-\beta)}}{1 + e^{\alpha(\theta-\beta)}}$$

ITEM INFORMATION

png

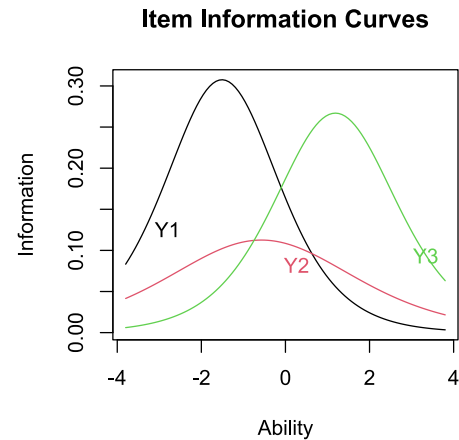
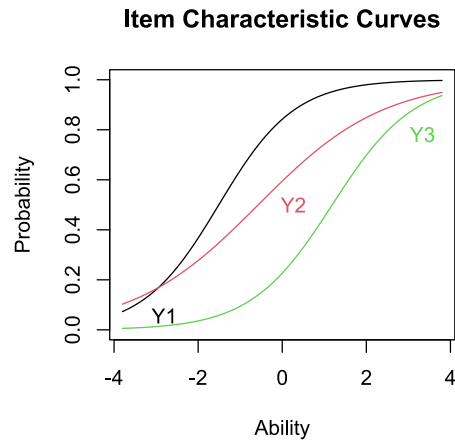
2

png

2

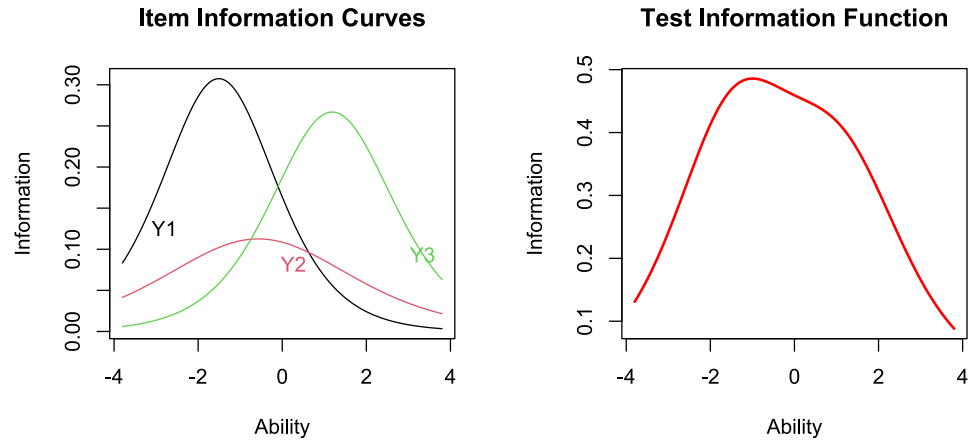
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2



Information extends the concept of reliability. Items contribute information to different degrees at different ranges of the latent trait.

TEST INFORMATION FUNCTION



Reliability is a single value, but in IRT models precision is not uniform across the entire range of test scores. *Test information* is the sum of the individual item information curves.

SUMMARIZING THE BENEFITS OF IRT MODELS

Parameters of IRT models are generally not sample- or test-dependent whereas true-score is defined in CTT in the context of a specific test.

- Scores on different instruments can be compared.
- Computerized adaptive testing; test items can be tailored to the individual to maximize precision.